H∞ Disturbance Rejection for Continuous-Time Takagi-Sugeno Models based on Nested Convex Sums

R. Márquez[1], V. Campos[2], T. M. Guerra[1], R. Palhares[2], A. Kruszewski[3], M. Bernal[4]
[1]Université de Valenciennes et du Hainaut-Cambrésis
[2]Department of Electronics Engineering, Federal University of Minas Gerais
[3]LAGIS UMR CNRS 8146 Ecole Centrale de Lille
[4]Department of Electric and Electronics Engineering, Sonora Institute of Technology

[1]LAMIH UMR CNRS 8201, Le Mont Houy, 59313, Valenciennes Cedex 9, France.
Email :raymundo.marquez@itson.edu.mx, guerra@univ-valenciennes.fr.
[2]Av. Antônio Carlos, 6627 - Pampulha - Belo Horizonte – MG CEP 31270-901, Brazil.
Email: victor@cpdee.ufmg.br, palhares@cpdee.ufmg.br
[3]Cité Scientifique, BP 48, 59651, France. Email : alexandre.kruszewski@ec-lille.fr
[4]Ciudad Obregón, Mexico. Email: miguel.bernal@itson.edu.mx.

Abstract:
This paper is concerned with controller design for H-infinity disturbance rejection in continuous-time Takagi-Sugeno models. The control law belongs to a more general class than the well-known parallel distributed compensation: it is based on a recent approach which employs progressively more complex nested convex sums while preserving the use of a quadratic Lyapunov function. The results thus obtained are parameter-dependent linear matrix inequalities which allow logarithmic search of feasible solutions. Examples are provided to illustrate the aforementioned contributions.

Keywords:
Takagi-Sugeno models; Linear Matrix Inequalities; Linear Parameter Varying; Non-PDC Control Law; H∞ control.

1 Introduction

Originally appeared in the fuzzy context, Takagi-Sugeno models (TS) [1] have been increasingly used as a legitimate nonlinear control tool altogether with linear matrix inequalities (LMIs) and the direct Lyapunov method [2]. In the modern context, a TS model is an exact convex rewriting of a nonlinear model within a compact of the state space (region of interest), which can be systematically constructed via the sector nonlinearity approach [3]. Convexity is guaranteed by capturing the system nonlinearities within membership functions (MFs) which hold the convex sum property inside the region of interest [4]. Once a TS model is obtained, the direct Lyapunov method comes at hand to take advantage of the convex structure which allows stability conditions to be expressed in terms of LMIs [5]. Expressing stability conditions in terms of LMIs constitutes a numerical advantage over many other methods since they can be solved via convex optimization techniques which are already implemented in a number of toolboxes [6]–[8]. As for controller design, parallel distributed compensation (PDC) generalizes ordinary linear state feedback for TS models while preserving the LMI approach [5]. Thanks to the latter, performance requirements such as bounds on the control input, decay rate specification or disturbance rejection, can be easily included [2], [5].

The last two decades have witnessed a tremendous effort to extend the scope of the TS-LMI framework to larger families of problems, since most of the conditions are only sufficient [9], [10]. The continuous-time case have proved to be particularly difficult to deal with since piecewise generalizations of the quadratic Lyapunov function require extra conditions to guarantee its continuity [11], [12] while basis-dependent Lyapunov functions lead to the time-derivative of the MFs, which is hard to cast as a convex problem [13], [14]. Recently, several results which do not necessarily require a different sort of Lyapunov function have appeared; they are based on matrix properties and nested convex sums which progressively lead to better results in the form of parameter-dependent LMIs [15]–[20]. This work follows the same line of the latter for H infinity disturbance rejection; it is shown that multi-index controllers perform better than former approaches without any need of a different Lyapunov function, i.e., all the schemes are quadratic.

The results are organized as follows: section 2 introduces the sector nonlinearity methodology to construct TS models, provides notations for convex sums as well as several properties this work is based on; the main results on disturbance rejection are developed in section 3: first showing how to decouple the Lyapunov function from the control law design, then extending the previous result via an increasingly complex non-PDC control law; section 4 illustrates the advantages of the proposed control technique over some others; the paper concludes in section 5 with some final remarks.

2 Definitions and notations

Consider a nonlinear model of the form:

$$\begin{aligned} \dot{x}(t) &= f_1(t)x(t)+g(t)u(t)+d_1(t)w(t) \\ y(t) &= f_2(t)x(t)+d_2(t)w(t) \end{aligned} \tag{1}$$

with $f_i(\cdot)$, $g(\cdot)$, and $d_i(\cdot)$ being nonlinear functions, $x(t)\in\mathbb{R}^n$ the state vector, $u(t)\in\mathbb{R}^m$ the input vector, $y(t)\in\mathbb{R}^o$ the output vector, $w(t)\in\mathbb{R}^q$ an external disturbance, and $z(x(t))\in\mathbb{R}^p$ the premise vector assumed to be bounded and smooth in a compact set C of the state space including the origin.

Let $nl_j(\cdot)\in\left[\underline{nl}_j,\overline{nl}_j\right]$, $j\in\{1,\cdots,p\}$ be the set of bounded nonlinearities in (1) belonging to C. Employing the sector nonlinearity approach [3], the following weighting functions can be constructed

$$w_0^j(\cdot)=\frac{\overline{nl}_j-nl_j(\cdot)}{\overline{nl}_j-\underline{nl}_j},\ w_1^j(\cdot)=1-w_0^j(\cdot)$$

with $j\in\{1,\cdots,p\}$.

From the previous weights, the following membership functions (MFs) are defined:

$$h_i=h_{1+i_1+i_2\times 2+\cdots+i_p\times 2^{p-1}}=\prod_{j=1}^{p} w_{i_j}^j(z_j) \tag{2}$$

with $i\in\{1,\cdots,2^p\}$, $i_j\in\{0,1\}$. These MFs satisfy the convex sum property $\sum_{i=1}^{r} h_i(\cdot)=1$ and $h_i(\cdot)\geq 0$ in C. For simplicity, convex sums will be denoted as $\Upsilon_h=\sum_{i=1}^{r} h_i(z(t))\Upsilon_i$ and their inverse as $\Upsilon_h^{-1}=\left(\sum_{i=1}^{r} h_i(z(t))\Upsilon_i\right)^{-1}$.

Based on the previous definitions, an exact representation of (1) in C is given by the following continuous-time TS model:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{r} h_i(z)(A_i x+B_i u+D_i w) \\ &= A_h x+B_h u+D_h w \\ y &= \sum_{i=1}^{r} h_i(z)(C_i x+E_i w) \\ &= C_h x+E_h w \end{aligned} \tag{3}$$

with $r=2^p\in\mathbb{N}$ representing the number of linear models in (3) and pairs (A_i,B_i,C_i,D_i,E_i), $i=1,\ldots,r$, the set of matrices of proper dimensions at the polytope vertex $h_i=1$.

The following non-PDC control law in [17] is adopted:

$$\begin{aligned} u(t) &= \sum_{i=1}^{r} h_i(z)F_i\left(\sum_{i=1}^{r} h_i(z)H_i\right)^{-1} x \\ &= F_h H_h^{-1} x \end{aligned} \tag{4}$$

with $F_i \in \mathbb{R}^{m\times n}$, $i=1,\ldots,r$ the controller gains and $H_i \in \mathbb{R}^{n\times n}$, $i=1,\ldots,r$ matrices which allow decoupling the Lyapunov function from the control law design, as shown later.

The closed-loop TS model is then written as:

$$\begin{aligned}\dot{x} &= \bar{A}_{hh}H_h^{-1}x + E_h w\\ y(t) &= C_h x + G_h w\end{aligned}, \tag{5}$$

with $\bar{A}_{hh} = A_h H_h + B_h F_h$.

2.1 Properties

We present some convenient properties to be used in the development of the main results of this study.

Property 1 (Schur complement) [6]**:** Let $P \in R^{m\times m}$ be a positive definite matrix, $X \in R^{m\times n}$ and $Q \in R^{n\times n}$, then

$$\begin{cases} Q - X^T P^{-1} X > 0, \\ P > 0 \end{cases} \Leftrightarrow \begin{bmatrix} Q & (*) \\ X & P \end{bmatrix} > 0 \tag{6}$$

Property 2 [19]**:** Given $P = P^T > 0$, then

$$Q^T P^{-1} Q \geq Q^T + Q - P \tag{7}$$

It is well-known that a TS-LMI based controller design usually leads to inequalities containing multiple nested convex sums. For instance, given matrix expressions $\Upsilon_{i_0 i_1 \ldots i_q}$, $i_0, i_1, \ldots, i_q \in \{1,\ldots,r\}$, the following inequality may arise:

$$\sum_{i_0=1}^{r}\sum_{i_1=1}^{r}\cdots\sum_{i_q=1}^{r} h_{i_0}(z) h_{i_1}(z)\ldots h_{i_q}(z)\Upsilon_{i_0 i_1 \ldots i_q} < 0. \tag{8}$$

The sign of such expressions should be established via LMIs, which implies that the MFs therein should be adequately retired: conditions thus obtained will be therefore only sufficient. This is why selecting a proper way to perform this task is important to reduce conservatism. When double sums are involved ($q=1$), a good compromise for guaranteeing (8) without adding slack variables is given by the following lemma:

Relaxation 1 [21]**:** Let $\Upsilon_{i_0 i_1}$, $i_0, i_1 \in \{1,\cdots,r\}$ be matrices of the same size. Condition (8) is verified for $q=1$ if:

$$\Upsilon_{i_0 i_1} + \Upsilon_{i_1 i_0} < 0, \quad \forall i_0, i_1 > i_0 \tag{9}$$

Should more than two nested convex sums be involved, a generalization of the sum relaxation in [22] will be used [23]:

Relaxation 2 [23]**:** Let $\Upsilon_{i_0 i_1 \ldots i_q}$, $i_0, i_1, \ldots, i_q \in \{1,\ldots,r\}$ be matrices of the same size and $\mathrm{P}(i_0, i_1, \cdots, i_q)$ be the set of all permutations of the indexes i_0, i_1,..., i_q. Condition (8) is verified if:

$$\begin{aligned}&\sum_{i_0 i_1 \cdots i_q \in \mathrm{P}(i_0, i_1, \ldots, i_q)} \Upsilon_{i_0 i_1 \cdots i_q} < 0,\\ &\forall (i_0, i_1, \ldots, i_q) \in \{1,\ldots,r\}^{q+1}\end{aligned} \tag{10}$$

An asterisk $(*)$ for inline expressions denotes the transpose of the terms on its left-hand side; for matrix expressions denotes the transpose of its symmetric block-entry. When convenient, arguments will be omitted.

3 Main results

Consider the following quadratic Lyapunov function candidate:

$$V(x(t)) = x(t)^T P^{-1} x(t), \tag{11}$$

with $P = P^T > 0$.

The TS model (5) satisfies the H infinity attenuation criterion $\gamma > 0$ if the following well-known condition holds [2]

$$\dot{V} + y(t)^T y(t) - \gamma^2 w(t)^T w(t) \leq 0. \tag{12}$$

3.1 Quadratic Lyapunov function

Theorem 1: The TS model (3) under the control law (4) is globally asymptotically stable with disturbance attenuation γ if there exists $\varepsilon > 0$ and matrices $P = P^T > 0$, and F_{i_1}, H_{i_1}, $i_1 = 1, \ldots, r$ of proper dimensions such that (9) holds where

$$\Upsilon_{i_0 i_1} = \begin{bmatrix} P & \Gamma_{i_0 i_1}^{12} & \varepsilon D_{i_0} & \varepsilon P C_{i_0}^T \\ (*) & \Gamma_{i_0 i_1}^{22} & 0 & 0 \\ (*) & (*) & -\varepsilon\gamma^2 I & \varepsilon E_{i_0}^T \\ (*) & (*) & (*) & -\varepsilon I \end{bmatrix}, \quad (13)$$

with $\Gamma_{i_0 i_1}^{12} = H_{i_1} + \varepsilon\left(A_{i_0} H_{i_1} + B_{i_0} F_{i_1}\right)$ and $\Gamma_{i_0 i_1}^{22} = P - H_{i_1}^T - H_{i_1}$.

Proof: The time-derivative of (11) is:

$$\dot{x}^T P^{-1} x + x^T P^{-1} \dot{x} < 0 \quad (14)$$

and taking into account (5), (12) can be rewritten as:

$$\begin{pmatrix} x^T P^{-1}\left(\bar{A}_{hh} H_h^{-1} x + D_h w\right) x + (*) \\ \quad + \left(C_h x + E_h w\right)^T (*) - \gamma^2 w^T w) \end{pmatrix} < 0,$$

which can be rewritten as:

$$\begin{bmatrix} x \\ w \end{bmatrix}^T \left\{ \Phi + \begin{bmatrix} C_h^T \\ E_h^T \end{bmatrix} \begin{bmatrix} C_h & E_h \end{bmatrix} \right\} \begin{bmatrix} x \\ w \end{bmatrix} < 0$$

with $\Phi = \begin{bmatrix} P^{-1}\bar{A}_{hh} H_h^{-1} + (*) & P^{-1} D_h \\ D_h^T P^{-1} & -\gamma^2 I \end{bmatrix}$.

The condition in (12) is satisfied if:

$$\Phi + \begin{bmatrix} C_h^T \\ E_h^T \end{bmatrix} \begin{bmatrix} C_h & E_h \end{bmatrix} < 0,$$

and applying Schur's complement gives:

$$\begin{bmatrix} P^{-1}\bar{A}_{hh} H_h^{-1} + (*) & P^{-1} D_h & C_h^T \\ (*) & -\gamma^2 I & E_h^T \\ (*) & (*) & -I \end{bmatrix} < 0.$$

By congruence property with $diag\{P, I, I\}$ the previous inequality yields,

$$\begin{bmatrix} \bar{A}_{hh} H_h^{-1} P + (*) & D_h & P C_h^T \\ (*) & -\gamma^2 I & E_h^T \\ (*) & (*) & -I \end{bmatrix} < 0. (15)$$

Considering a small enough $\varepsilon > 0$, (15) holds if:

$$\begin{bmatrix} \begin{pmatrix} \bar{A}_{hh} H_h^{-1} P + (*) \\ \quad + \varepsilon \bar{A}_{hh} H_h^{-1} P (*) \end{pmatrix} & D_h & P C_h^T \\ (*) & -\gamma^2 I & E_h^T \\ (*) & (*) & -I \end{bmatrix} < 0$$

which is straightforwardly equivalent to:

$$\begin{bmatrix} \begin{pmatrix} \varepsilon \bar{A}_{hh} H_h^{-1} P + (*) \\ \quad + \varepsilon^2 \bar{A}_{hh} H_h^{-1} P (*) \end{pmatrix} & \varepsilon D_h & \varepsilon P C_h^T \\ (*) & -\varepsilon\gamma^2 I & \varepsilon E_h^T \\ (*) & (*) & -\varepsilon I \end{bmatrix} < 0$$

or rewritten:

$$\begin{bmatrix} \left(I + \varepsilon \bar{A}_{hh} H_h^{-1}\right) P (*) - P & \varepsilon D_h & \varepsilon P_v C_h^T \\ (*) & -\varepsilon\gamma^2 I & \varepsilon E_h^T \\ (*) & (*) & -\varepsilon I \end{bmatrix} < 0.$$

Thus by Schur complement the previous inequality is equivalent to:

$$\begin{bmatrix} -P & I+\varepsilon \bar{A}_{hh} H_h^{-1} & \varepsilon D_h & \varepsilon P C_h^T \\ (*) & -P^{-1} & 0 & 0 \\ (*) & (*) & -\varepsilon\gamma^2 I & \varepsilon E_h^T \\ (*) & (*) & (*) & -\varepsilon I \end{bmatrix} < 0 . \quad (16)$$

Congruence property with full-rank matrix $diag\left\{I, H_h^T, I, I\right\}$ gives:

$$\begin{bmatrix} -P & H_h+\varepsilon \bar{A}_{hh} & \varepsilon D_h & \varepsilon P C_h^T \\ (*) & -H_h^T P^{-1} H_h & 0 & 0 \\ (*) & (*) & -\varepsilon\gamma^2 I & \varepsilon E_h^T \\ (*) & (*) & (*) & -\varepsilon I \end{bmatrix} < 0 . \quad (17)$$

Using property (7) then (17) holds if:

$$\begin{bmatrix} -P & H_h+\varepsilon \bar{A}_{hh} & \varepsilon D_h & \varepsilon P C_h^T \\ (*) & P-H_h^T-H_h & 0 & 0 \\ (*) & (*) & -\varepsilon\gamma^2 I & \varepsilon E_h^T \\ (*) & (*) & (*) & -\varepsilon I \end{bmatrix} < 0 . \quad (18)$$

(18) holds if relaxation (9) is applied with $\Upsilon_{i_0 i_1}$ defined as in (13), which concludes the proof.□

3.2 Quadratic Lyapunov function: expanded indexes

Now, the main advantage of decoupling the control law from matrix P can be stated. Expanding F_h to $F_{h\cdots h}$ for a PDC law has very few interest because the first term will remain $A_h P$.

Consider now expanding using multiple indexes, the control law (4):

$$u(t) = F_{h\cdots h} H_{h\cdots h}^{-1} x(t) \quad (19)$$

with: $F_{\underbrace{h\cdots h}_{q}} = \sum_{i_1=1}^{r} \cdots \sum_{i_q}^{r} h_{i_1} \ldots h_{i_q} F_{i_1 \ldots i_q}$ the following theorem can be derived.

Theorem 2: The TS model (3) under the control law (19) is asymptotically stable with disturbance attenuation γ if it exists $\varepsilon > 0$, and matrices $P = P^T > 0$, $F_{i_1 \ldots i_q}$ and $H_{i_1 \ldots i_q}$, $i_1, \ldots, i_q \in \{1, \ldots, r\}$ such that (10) holds with:

$$\Upsilon_{i_0 i_1 \cdots i_q} = \begin{bmatrix} P & \Gamma_{i_0 i_1 \cdots i_q}^{12} & \varepsilon D_{i_0} & \varepsilon P C_{i_0}^T \\ (*) & \Gamma_{i_0 i_1 \cdots i_q}^{22} & 0 & 0 \\ (*) & (*) & -\varepsilon\gamma^2 I & \varepsilon E_{i_0}^T \\ (*) & (*) & (*) & -\varepsilon I \end{bmatrix} \quad (20)$$

with $\Gamma_{i_0 i_1 \cdots i_q}^{12} = H_{i_1 \cdots i_q} + \varepsilon\left(A_{i_0} H_{i_1 \cdots i_q} + B_{i_0} F_{i_1 \cdots i_q}\right)$ and $\Gamma_{i_0 i_1 \cdots i_q}^{22} = P - H_{i_1 \cdots i_q}^T - H_{i_1 \cdots i_q}$.

Proof: It follows directly from (14)-(18) considering the control law in (19)□.

Remark 1: Conditions in this work are parameter-dependent LMI; their result depend on the choice of ε. Nevertheless, it has been proved in [24] and [25] that a logarithmically spaced family of values, for instance $\varepsilon \in \left\{10^{-6}, 10^{-5}, \ldots, 10^{6}\right\}$, is adequate to avoid an exhaustive search of feasible solutions, thus outperforming existing results.

Remark 2: Conditions in (13) are a particular case of conditions in (20) and it occurs when $q = 1$.

4 Example

The proposed results are illustrated via the following numerical example.

Example 1: Consider the following TS model:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{4} h_i(z)\left(A_i x + B_i u + E_i w\right) \\ y &= \sum_{i=1}^{4} h_i(z)\left(C_i x + G_i w\right) \end{aligned} \quad (21)$$

with

$$A_1 = \begin{bmatrix} 1.59 & -7.29 \\ 0.01 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.02 & -4.64 \\ 0.35 & 0.21 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -5 & -4.33 \\ 0 & 0.05 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0.89 & -5.29 \\ 0.1 & 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 8 \\ 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 12 \\ -1 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -0.1 \\ -0.4 \end{bmatrix}^T, \quad C_2 = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}^T, \quad C_3 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}^T,$$
$$C_4 = \begin{bmatrix} -0.1 \\ -0.4 \end{bmatrix}^T, \quad E_1 = E_3 = \begin{bmatrix} -0.15 \\ 0.1 - 0.05\alpha \end{bmatrix},$$
$$E_2 = E_4 = \begin{bmatrix} -0.15 \\ 0.1 + 0.05\alpha \end{bmatrix}, \quad G_1 = G_3 = 0.1\alpha,$$
$$G_2 = G_4 = -0.1\alpha, \quad w_0^1 = x_1^2, \quad w_0^2 = \frac{x_2^2}{4},$$
$$w_1^1 = 1 - w_0^1, \quad w_1^2 = 1 - w_0^2, \quad h_1 = w_0^1 w_0^2,$$
$h_2 = w_0^1 w_1^2$, $h_3 = w_1^1 w_0^2$, $h_4 = w_1^1 w_1^2$, and α is a real-valued parameter.

Table 1.- Comparison of H_∞ performances.

Approach	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
QS	3.0260	2.4911	2.0262
Th. 1, Th. 2 ($q = 1$)	1.9039	1.6172	1.3764
Th. 2 ($q = 2$)	0.2363	0.1936	0.1748
Th. 2 ($q = 3$)	0.1794	0.1425	0.1394

The performance bounds obtained by Theorems 1 and 2 in this work as well as QS approach for differrents values of α are provided in Table 1 with $\varepsilon = 0.1$.
Table 1 shows that the performance of Theorem 2 is clearly better than results in Theorem 1 and the QS approach when the parameter q is increased. Also, it is possible to note that Theorem 1 is a particular case of Theorem 2 ($q = 1$).
Figure 1 is presented in order to illustrate the behavior of the parameter γ with respect to increasing the parameter q in Theorem 2. The minimal value for γ is calculated for $\alpha \in [0,1]$.

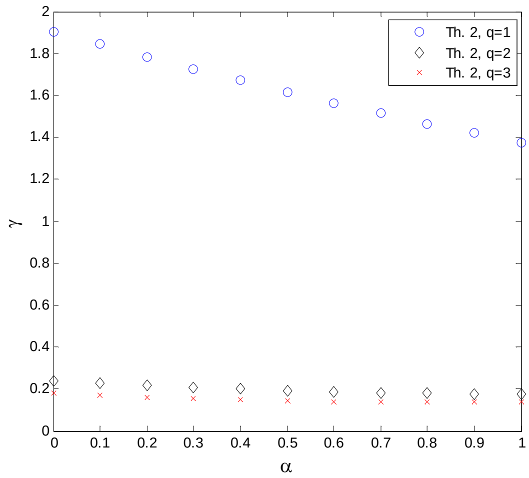

Figure 1. γ values in Example 1 for Theorem 2 with $q = 1$, $q = 2$, and $q = 3$.

It is possible to observe in Fig. 1 that if parameter q increase the minimal value of γ decrease.

5 Conclusions

A novel approach for controller design for H-infinity disturbance rejection for continuous-time nonlinear models in the TS form has been presented. Taking advantage of a Tustin-like transformation, the controller design has been decoupled from the quadratic Lyapunov function it is based on. The nested convex structure in the control law permits to obtain progressively better performance on disturbance attenuation. Examples are provided to illustrate the effectiveness of the proposed approach.

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