# Exercícios Fourier Lista 2

Note Title 5/3/2010

0.1 (3,14)

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k] \longrightarrow \boxed{H(e^{j\omega})} \longrightarrow y[n] = \cos\left(\frac{5\pi}{2}n + \pi/4\right)$$

Solução:

$x[n]$ é periódico $N=4 \Longrightarrow \Omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$X[k] = \frac{1}{N} \sum_{k=\langle N \rangle} x[n]\, e^{-jk\Omega_0 n}$$

$$= \frac{1}{4} \quad \text{para todo } \underline{k}.$$

A saída é:

$$y[n] = \sum_{k=0}^{3} H\left(e^{j\frac{\pi}{2}k}\right) X[k]\, e^{jk\frac{\pi}{2}n} =$$

$$= \frac{1}{4} H(e^{j0})\, e^{j0} + \frac{1}{4} H\left(e^{j\pi/2\, n}\right) e^{j\pi/2\, n} +$$

$$+\frac{1}{4}H(e^{j\pi})e^{j\pi} + \frac{1}{4}H(e^{j3\pi/2})e^{j3\pi/2}$$

Mas $y[n]$ é tb igual a

$$y[n] = \cos\left(\frac{5\pi}{2}n + \pi/4\right) =$$

$$= \cos\left(2\pi n + \frac{\pi}{2}n + \pi/4\right) =$$

$$= \cos(\pi/2\, n + \pi/4) = \frac{e^{j\pi/2 n}e^{j\pi/4}}{2} +$$

$$+ \frac{e^{-j\pi/2 n}\cdot e^{-j\pi/4}}{2}$$

$$= \frac{e^{-j\pi/4}}{2}\cdot e^{-j\pi/2 n} + \frac{e^{j\pi/4}}{2}e^{j\pi/2 n} \quad \text{ou}$$

$$= \frac{e^{-j\pi/4}}{2}e^{j\frac{3\pi}{2}n} + \frac{e^{j\pi/4}}{2}e^{j\pi/2 n}$$

Comparando, temos:

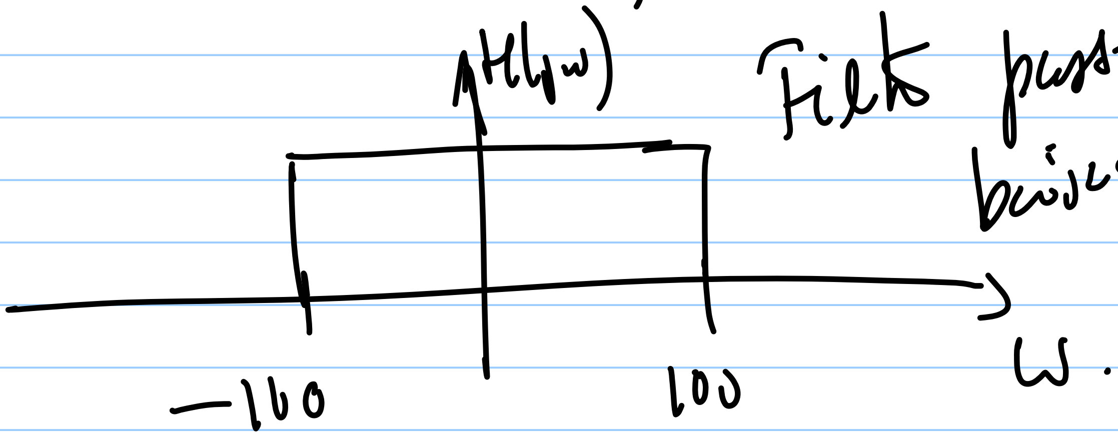
$H(e^{j0}) = H(e^{j\pi}) = 0.$

$H(e^{j\pi/2}) = 2e^{j\pi/4}$ e

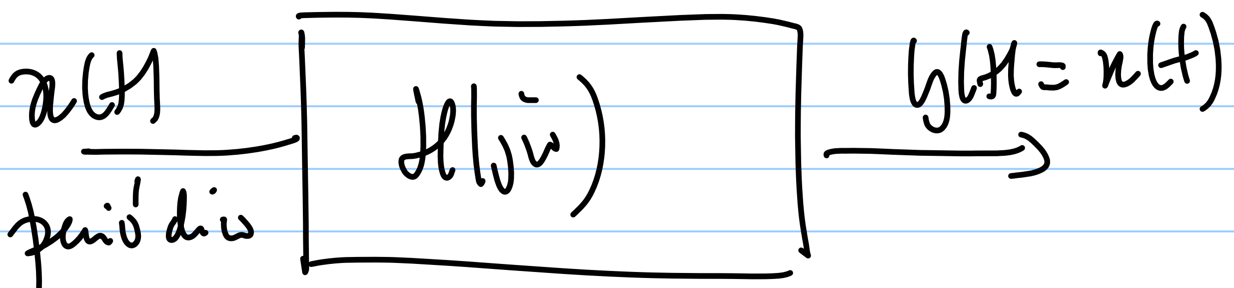
$H(e^{j3\pi/2}) = 2e^{-j\pi/4}$

0.2 (3.15)

$$H(j\omega) = \begin{cases} 1, & |\omega| \leq 100 \\ 0, & |\omega| > 100 \end{cases}$$




Filtro passa-baixo




= periódico

com $\pi/6$ e

coef. $X[k]$

Solução:

$$y(t) = \sum_{k=-\infty}^{\infty} H(jk\Omega_0) \cdot X[k] \, e^{jk\Omega_0 t}$$

Mas

$$\Omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi/6} = 12.$$

$$|w| > 100 \rightarrow 0.$$

$$\Downarrow$$

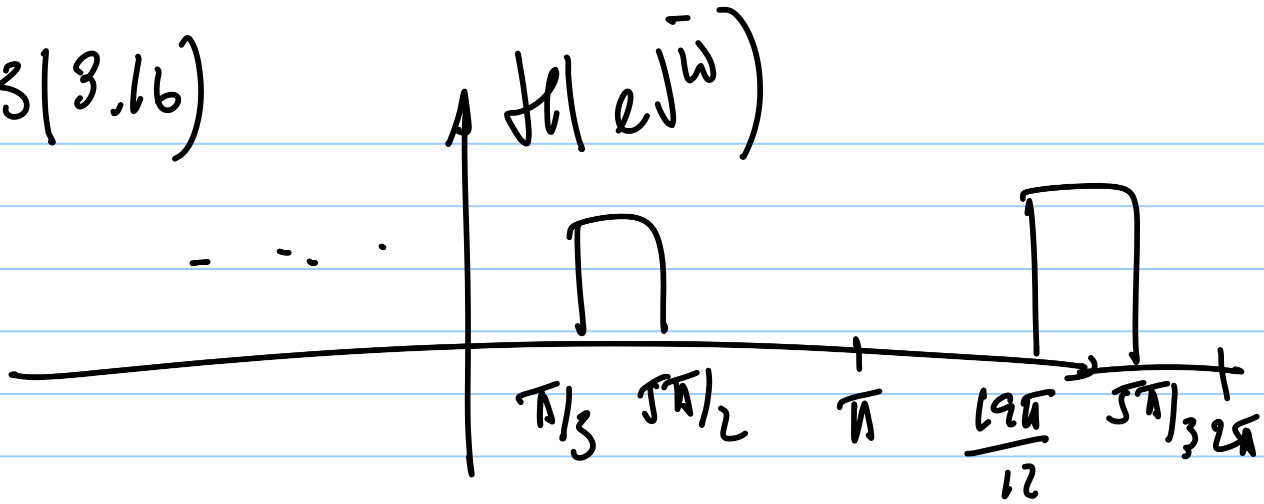
$$|k\Omega_0| \leq 100$$

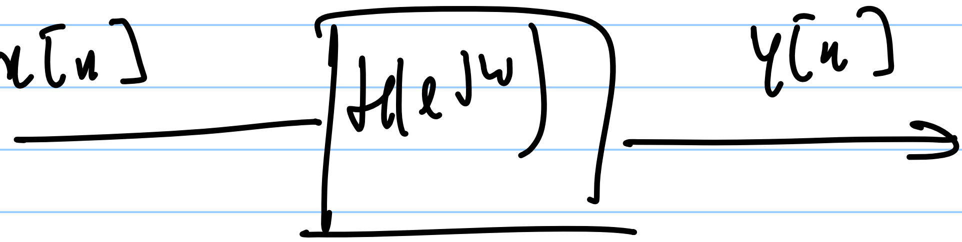
$$|k| \cdot 12 \leq 100.$$

$$|k| \leq \frac{100}{12} \rightarrow |k| \leq 8$$

Para $|k| > \infty \rightarrow \underline{y[k] = 0}$.

0.3 (3.16)




a) $x_1[n] = (-1)^n = \left(e^{j\pi}\right)^n = e^{j\pi n} =$

$= e^{j\frac{2\pi}{2}n} \Rightarrow N = 2$

$\Downarrow$

2 coeficientes

$x_1[n] = 0 . e^{j\overset{\uparrow}{0}\frac{2\pi}{2}n} + 1 . e^{j\overset{\uparrow}{1}.\frac{2\pi}{2}n}$

$k \qquad\qquad k$

$X[0] = 1$ e $X[1] = 1$

Sabemos que

$$y[n] = \sum_{k=0}^{1} H\left(e^{j\frac{2\pi k}{2}}\right) . X[k] . e^{j\frac{2\pi}{2}n}$$

$$= H(e^{j\pi}) . 1 . e^{j\pi n}$$

Mas a frequência $\pi$ não passa no filtro logo: $H(e^{j\pi}) = 0$, portanto $y[n] = 0$.

b) $x_2[n] = 1 + \operatorname{sen}\left(\frac{3\pi}{8}n + \pi/4\right)$.

Olhando a frequência

$$\frac{3\pi}{8} = 3 \cdot \frac{2\pi}{16}$$

ou desprezando a fase.

$$\operatorname{sen}\left(\frac{3\pi}{8}(n+N)\right) = \operatorname{sen}\left(\frac{3\pi}{8}n\right).$$

$$\operatorname{sen}\frac{3\pi n}{8}\cos\frac{3\pi N}{8} + \cos\frac{3\pi n}{8}\operatorname{sen}\frac{3\pi N}{8} = \operatorname{sen}\left(\frac{3\pi n}{8}\right)$$

Para que isso seja verdade

$$\cos\frac{3\pi N}{8} = 1 \quad \text{ou} \quad \frac{3\pi N}{8} = 2k\pi$$

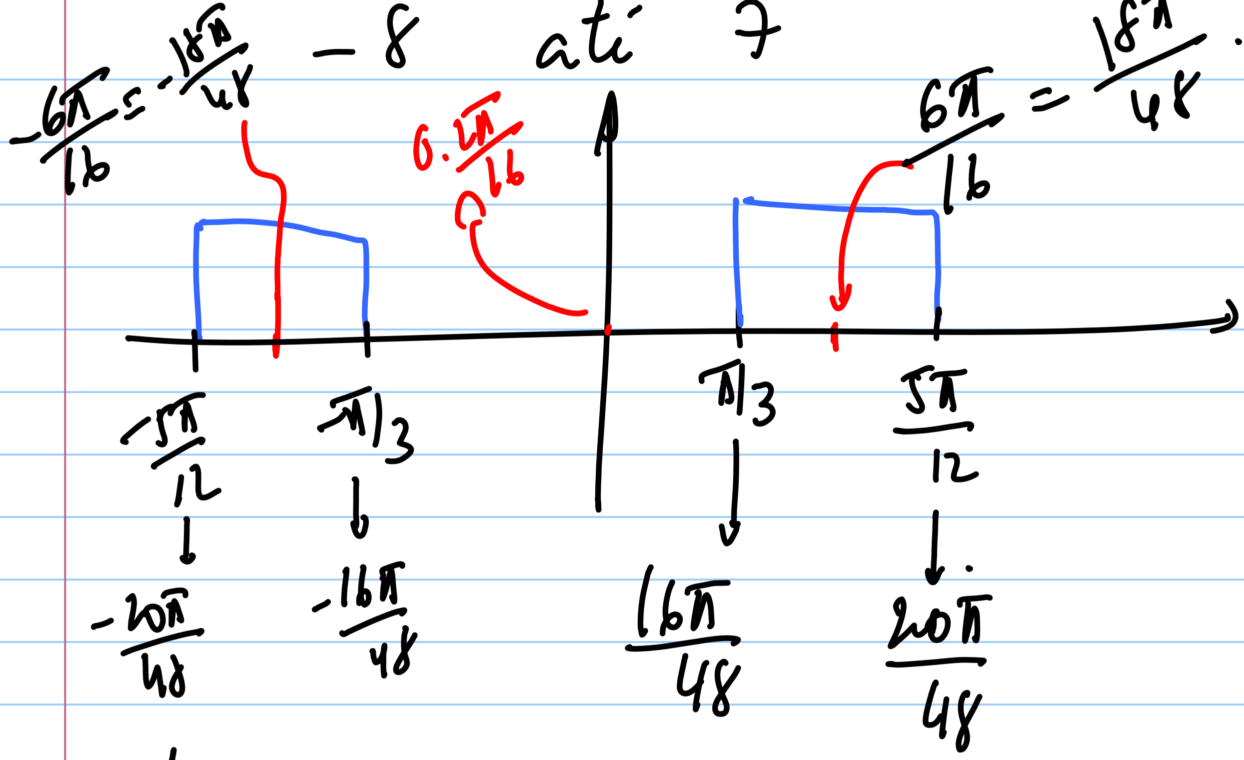
$$N = \frac{16k}{3}, \quad k=3 \rightarrow \underline{N=16}.$$

Podemos escrever $x_2[n]$ como

$$x_2[n] = e^{j\,0\,\frac{2\pi}{16}n} + \frac{1}{2j}e^{j\pi/4}e^{j\,3\cdot\frac{2\pi}{16}n} - \frac{1}{2j}e^{-j\pi/4}e^{j(-3)\cdot\frac{2\pi}{16}n} \quad \text{com } N=16$$

Escolhendo o intervalo $\langle N \rangle$.

$-8$ até $7$




Logo:

$$y_2[n] = \text{sen}\left(\frac{3\pi}{16}n + \frac{\pi}{4}\right)$$

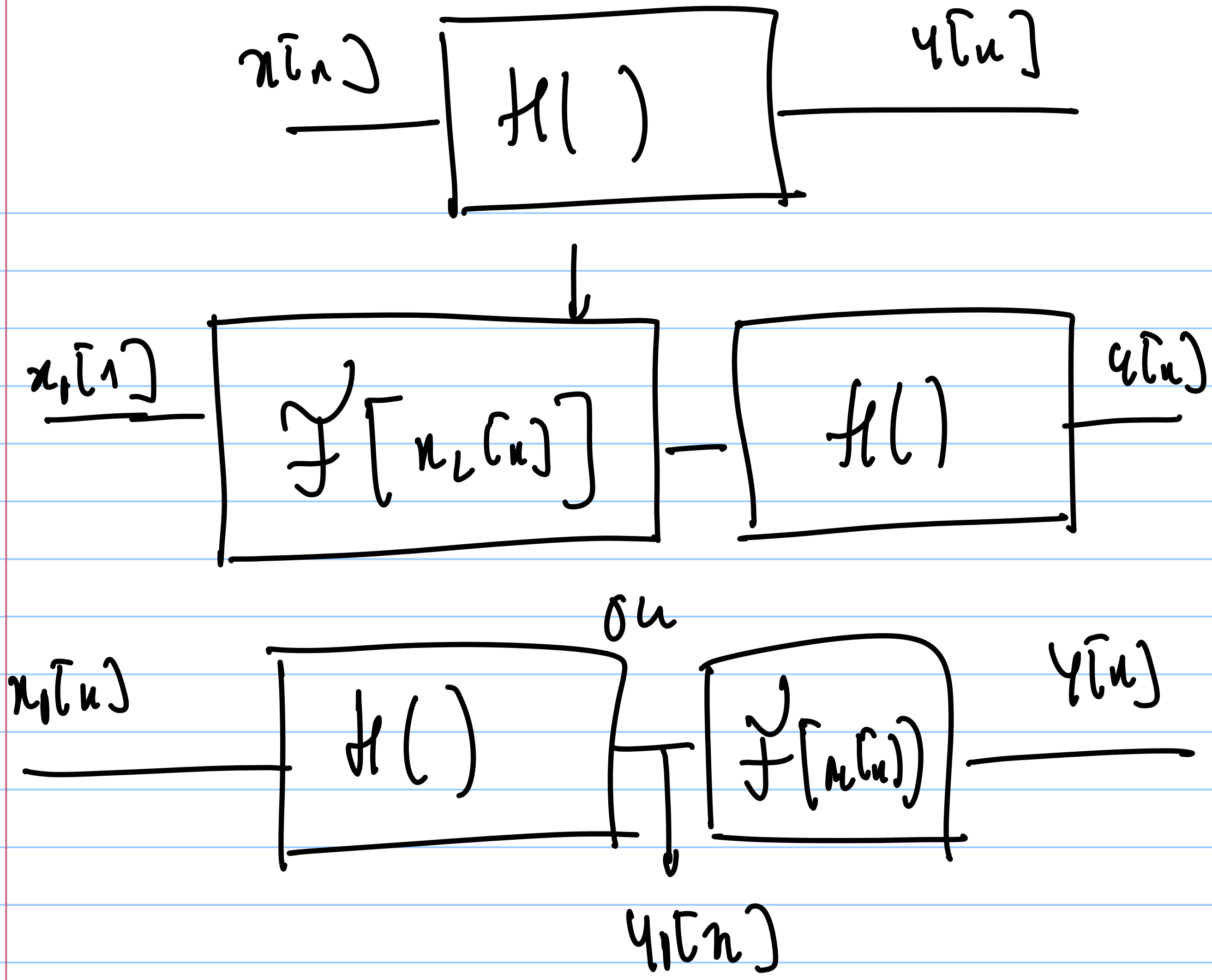
c) $$x_3[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{k-4} u[n-4k]$$

Repare que são cópias de $\left(\frac{1}{2}\right)^n u[n]$,

Logo:

$$\sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{k-4} u[n-4k] = \underbrace{\left[\left(\frac{1}{2}\right)^n u[n]\right] * \sum_{k=-\infty}^{\infty} \delta[n-4k]}_{x_a[n] \, * \, x_b[n]}$$

Então:

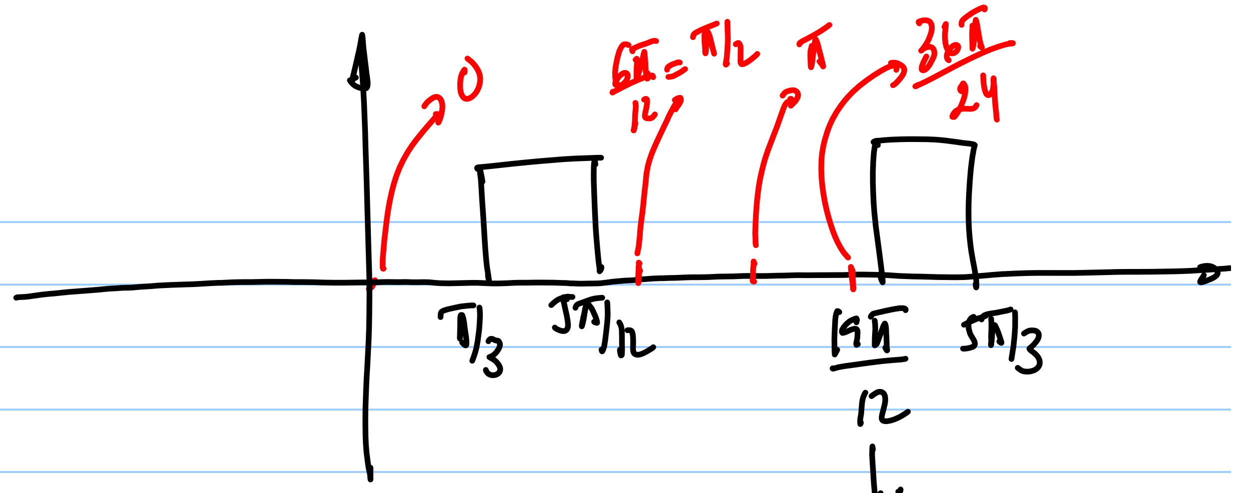



$\mathcal{F}[x_1[n]]$ ?

$x_1[n] \longrightarrow N=4 \rightarrow X_1[k] = \frac{1}{4}$

$$y_1[n] = \sum_{k=0}^{4} H(e^{jk\frac{2\pi}{4}}) . X_1[k]\, e^{jk2\pi/4\, n}$$

$$= \frac{1}{4}\Big( H(e^{j0}).e^{j0} + H(e^{j\pi/2})e^{j\pi/2\, n} + H(e^{j\frac{2\pi}{2}})e^{j\frac{2\pi}{2}n} + H(e^{j\frac{3\pi}{2}}).e^{j\frac{3\pi}{2}n}\Big)$$

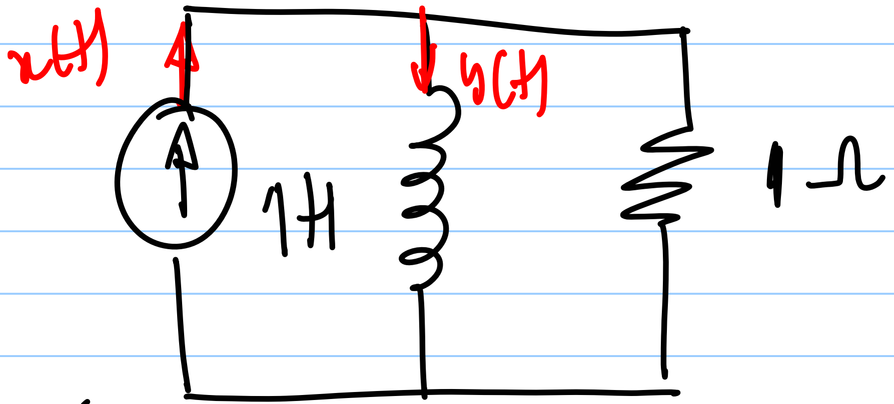



$$\frac{3\pi}{2} = \frac{3\pi \times 12}{2 \times 12} = \frac{36\pi}{24} \qquad \frac{38\pi}{24}$$

On sig:

$$y_1[n] = 0 \Rightarrow y[n] = 0.$$

# 3.19




Solução:

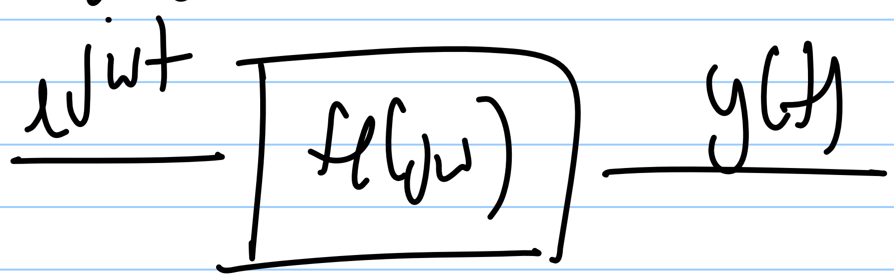
$$v_L = L\dot{y}$$

$$i_R = \frac{v_L}{R} = \frac{L}{R}\dot{y}$$

$$x(t) = y + \frac{L}{R}\dot{y} \quad , \quad \text{mas } L = R = 1$$

$$\boxed{\dot{y} + y = x.}$$

Considerando:



ou seja,

$$y(t) = H(j\omega)\cdot e^{j\omega t} \quad (\text{Fourier})$$

e

$$\dot{y}(t) = j\omega H(j\omega)\, e^{j\omega t}$$

Substituindo $y(t)$ e $\dot{y}(t)$ na eq. dif., temos:

$$j\omega H(j\omega)\cdot \cancel{e^{j\omega t}} + H(j\omega)\,\cancel{e^{j\omega t}} = \cancel{e^{j\omega t}}$$

$$(j\omega + 1)\, H(j\omega) = 1.$$

$$H(j\omega) = \frac{1}{j\omega + 1}.$$

—//—

$x(t)$ é periódica com período $T = 2\pi$,

ou

$$\Omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1.$$

$$x(t) = \frac{1}{2} e^{-jt} + \frac{1}{2} e^{jt}$$

$$X[-1] = X[1] = 1/2 \quad \text{(Fourier)}$$

Mas

$$y(t) = \sum_{k=-\infty}^{\infty} H(jk\omega_0)\, X[k]\, e^{jk\omega_0 t}$$

$$= X[-1]\, H(-j)\, e^{-jt} +$$

$$+ X[1]\, H(j)\, e^{+jt}$$

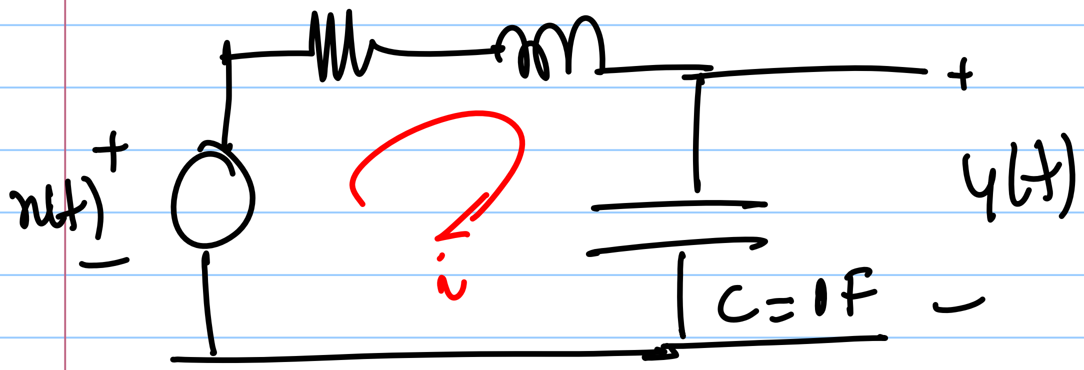
$$= \frac{1}{2}\left( \frac{1}{1-j} e^{-jt} + \frac{1}{1+j} e^{jt} \right)$$

$$= \frac{1}{2}\left( \frac{e^{j\pi/4}}{\sqrt{2}} e^{-jt} + \frac{e^{-j\pi/4}}{\sqrt{2}} e^{jt} \right)$$

$$= \frac{2}{2\sqrt{2}}\left( \frac{e^{-j(t-\pi/4)} + e^{j(t-\pi/4)}}{2} \right)$$

$$= \frac{1}{\sqrt{2}} \cos\left(t - \pi/4\right).$$

3.20

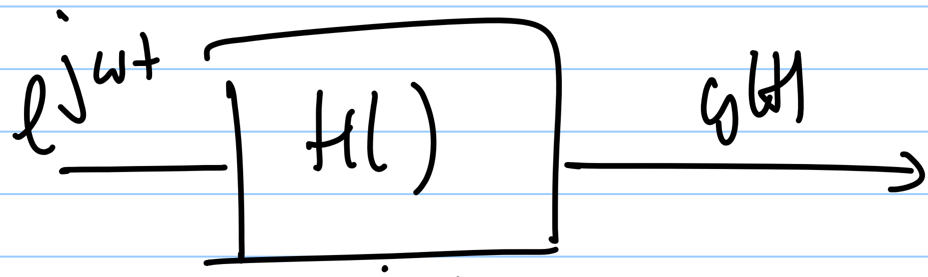



a) $i = C\dot{y}$

$$x = RC\dot{y} + LC\ddot{y} + y$$

$$\ddot{y} + \dot{y} + y = x.$$

b)




$$y = H(j\omega)e^{j\omega t}.$$

$$\dot{y} = j\omega H(j\omega)e^{j\omega t}$$

$$\ddot{y} = (j\omega)^2 H(j\omega)e^{j\omega t}$$

$$(j\omega)^2 H(j\omega)\cancel{e^{j\omega t}} + j\omega H(j\omega)\cancel{e^{j\omega t}} + H(j\omega)\cancel{e^{j\omega t}}$$
$$= \cancel{e^{j\omega t}}$$

$$[(j\omega)^2 + j\omega + 1]\, H(j\omega) = 1.$$

$$H(j\omega) = \frac{1}{(j\omega)^2 + j\omega + 1} =$$

$$= \frac{1}{j\omega + (1 - \omega^2)}$$

c) $x(t) = \cos(t) = \dfrac{e^{jt} + e^{-jt}}{2} =$

$$= \frac{1}{2} e^{-jt} + \frac{1}{2} e^{jt}$$

Logo:

$$y(t) = \frac{1}{2} H(-j) e^{-jt} + \frac{1}{2} H(j) e^{jt}$$

$$= \frac{1}{2} \frac{1}{-j} e^{-jt} + \frac{1}{2} \frac{1}{j} e^{jt}$$

$$= \frac{e^{jt} - e^{-jt}}{2j} = \operatorname{sen}(t)$$