Exercícios cap. 2 Lathi

Note Title 3/21/2010

2.2.4

$$(D^2+9)\,y(t) = (3D+2)\,x(t).$$

$$\ddot{y}+9y = 3\dot{x}+2x$$

$$y_0(0^-)=0 \quad e \quad \dot{y}_0(0^-)=6.$$

Solução:

Eq. característica:

$$\lambda^2+9=0 \Rightarrow \lambda=\pm j3$$

$$y_0 = A e^{j3t} + B e^{-j3t}$$

Como é real, faço:

$$A=\frac{c}{2}e^{j\theta} \quad e \quad B=\frac{c}{2}e^{-j\theta}$$

logo:

$$y_0 = \frac{c}{2}e^{j\theta}e^{j3t} + \frac{c}{2}e^{-j\theta}e^{-j3t}$$

$$= \frac{c}{2}\left(e^{j(3t+\theta)} + e^{-j(3t+\theta)}\right)$$

$$= c\cos(3t+\theta).$$

$\dot{y}_0 = -3c\,\text{sen}(3t+\theta)$

Para $t = 0^-$

$y_0(0^-) = c\cos(\theta) = 0$

$\dot{y}_0(0^-) = -3c\,\text{sen}(\theta) = 6$

$\underline{\theta = -\pi/2 \text{ e } c = 2}$

Logo:

$y_0(t) = 2\cos(3t - \pi/2) = 2\,\text{sen}(3t)$

<u>2.2.7</u>.

$(D+1)(D^2+5D+6)y(t) = Dx(t)$

$y_0(0^-) = 2$ $\qquad \ddot{y}_0(0^-) = 5$

$\dot{y}_0(0^-) = -1$

Solução:

Eq. característica

$(\lambda+1)(\lambda^2+5\lambda+6) = 0 \begin{cases} \lambda = -1 \\ \lambda = -2 \\ \lambda = -3 \end{cases}$

O fórmula é:

$$y_0(t) = Ae^{-t} + Be^{-2t} + Ce^{-3t}$$

$$\dot{y}_0(t) = -Ae^{-t} - 2Be^{-2t} - 3Ce^{-3t}$$

$$\ddot{y}_0(t) = +Ae^{-t} + 4Be^{-2t} + 9Ce^{-3t}$$

$$\begin{cases} A + B + C = 2 \\ -A - 2B - 3C = -1 \\ A + 4B + 9C = 5 \end{cases} \Rightarrow \begin{cases} A + B + C = 2 \\ -B - 2C = 1 \\ -3B - 8C = -3 \end{cases}$$

$$\begin{cases} B + 2C = -1 \xrightarrow{\times 3} -\; 3B + 6C = -3 \\ 3B + 8C = 3 \rightarrow +\; 3B + 8C = 3 \end{cases}$$

$$2C = 6$$

$$\boxed{C = 3}$$

$$B = -1 - 2 \times 3 \Rightarrow \boxed{B = -7}$$

$$A = 2 - 3 - (-7) = 6 \rightarrow \boxed{A = 6}$$

$$y_0(t) = 6e^{-t} - 7e^{-2t} + 3e^{-3t}$$

2.2.8

Resposta de entrada nula

$$y_0(t) = 2e^{-t} + 3$$

$$\Downarrow$$

$$Ae^{-1t} + Be^{+0t}$$

logo: $\begin{cases} \lambda = -1 \\ \lambda = 0 \end{cases}$

a) Não $\rightarrow$ $\underline{\lambda(\lambda+1) = 0}$ $\left\{ \lambda^2 + \lambda = 0 \right.$

b) Sim $\nearrow$

c) $\lambda(\lambda+1)^2 = 0$

$$\downarrow$$

$$y_0(t) = Ae^{0t} + Be^{-t} + Cte^{-t}$$

Para ficar da forma $Ae^{-t} + Be^{0t}$

C tem que ser zero.

Fim.

2.3.1

$$\ddot{y} + 4\dot{y} + 3y = \dot{x} + 5x$$

a equação acima está na forma

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

onde $N=2$, $M=1$ ou seja $N>M$

$$x(t) \longrightarrow \boxed{w(t) = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}} \longrightarrow w(t)$$

$$\longrightarrow \boxed{\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = w(t)} \longrightarrow y(t)$$

Podemos trocar a ordem

$$x(t) = \delta(t) \longrightarrow \boxed{\sum_{k=0}^{N} a_k \frac{d^k y_1(t)}{dt^k} = x(t)} \longrightarrow y_1(t) = h_1(t)$$

$$\longrightarrow \boxed{y(t) = \sum_{k=0}^{M} b_k \frac{d^k y_1(t)}{dt^k}} \longrightarrow y(t) = h(t)$$

Supondo que o sistema está em repouso inicialmente, temos:

$$y(0^-) = \frac{dy(0^-)}{dt} = \cdots = \frac{d^{N-1}y(0^-)}{dt^{N-1}} = 0$$

logo

$$\sum_{k=0}^{N} a_k \frac{d^k h_a(t)}{dt^k} = \delta(t)$$

No nosso caso:

$$\sum_{k=0}^{2} a_k \frac{d^k h_a(t)}{dt^k} = \delta(t)$$

$$\underset{1}{\underset{\|}{a_0}}\ \ddot{h}_a(t) + \underset{4}{\underset{\downarrow}{a_1}}\ \dot{h}_a(t) + \underset{3}{\underset{\downarrow}{a_2}}\ h_a(t) = \delta(t)$$

$$\ddot{h}_a(t) + 4\dot{h}_a(t) + 3h_a(t) = \delta(t)$$

Repare que só temos $\delta(t)$ e nenhuma derivada, logo

$$\int_{0^-}^{0^+} \frac{d^k h_c(t)}{dt^k}\, dt = \frac{d^{k-1} h_c(0^+)}{dt^{k-1}} = 0,$$

$$k = 1, \dots, N-1$$

$$\sum_{k=0}^{N} a_k \int_{0^-}^{0^+} \frac{d^k h_c(t)}{dt^k}\, dt =$$

$$= a_N \int_{0^-}^{0^+} \frac{d^N h_c(t)}{dt^N}\, dt = a_N \frac{d^{N-1} h_c(0^+)}{dt^{N-1}}$$

$$= 1 \quad \left(\int_0^{0^+} f(t)\, dt \right)$$

$$\frac{d^{N-1} h_c(0^+)}{dt^{N-1}} = \frac{1}{a_N}$$

Para $t = 0^+$

$$\sum_{k=0}^{N} a_k \frac{d^k h_c(t)}{dt^k} = 0$$

Supondo $Ae^{\lambda t}$ é a forma de solução, logo:

$$\sum_{k=0}^{N} a_k \frac{d^k(Ae^{\lambda t})}{dt^k} = 0$$

$$Ae^{\lambda t} \underbrace{\sum_{k=0}^{N} a_k \lambda^k}_{\text{polinômio característico}} = 0$$

e

$$h_c(t) = \sum_{k=1}^{N} A_k e^{\lambda_k t}$$

$$0 = h_c(0^+) = \sum_{k=1}^{N} A_k e^{\lambda_k 0^+} \Rightarrow \sum_{k=1}^{N} A_k$$

$$0 = \frac{dh_c(0^+)}{dt} = \sum_{k=1}^{N} A_k \lambda_k$$

$$\vdots$$

$$\frac{1}{a_N} = \frac{d^{N-1}h_c(0^+)}{dt^{N-1}} = \sum_{k=1}^{N} A_k \lambda_k^{N-1}$$

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1/c_N \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_N \\ \vdots & & & \\ \lambda_1^{N-1} & \lambda_2^{N-1} & \cdots & \lambda_N^{N} \end{bmatrix} \begin{bmatrix} A_1 \\ \vdots \\ A_N \end{bmatrix}$$

No nosso caso $N=2$.

- a eq. característica é
  $\lambda^2 + 4\lambda + 3 = 0 \begin{cases} \lambda = -1 \\ \lambda = -3 \end{cases}$

- $a_2 = 1$.

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

det( ) =
1 (-3) -
(-1)(1) =
-3+1 = -2

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -3 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} +1/2 \\ -1/2 \end{bmatrix}$$

$$h_a(t) = \left(1/2\, e^{-t} - 1/2\, e^{-3t}\right) u(t)$$

Precisamos, agora, usar a seguinte equação:

$$h(t) = \sum_{k=0}^{M=1} b_k \frac{d^k h_a(t)}{dt^k} \quad \begin{cases} b_0 = 5 \\ b_1 = 1 \end{cases}$$

$$\dot{h}_a(t) = -\frac{1}{2}e^{-t}u(t) + \cancel{\frac{1}{2}e^{-t}\delta(t)} + \frac{3}{2}e^{-3t}u(t) \cancel{-\frac{1}{2}e^{-3t}\delta(t)} = \left(-\frac{1}{2}e^{-t} + \frac{3}{2}e^{-3t}\right)u(t)$$

$$5\left(+\frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}\right)u(t)$$

$$+1\left(-\frac{1}{2}e^{-t} + \frac{3}{2}e^{-3t}\right)u(t)$$

$$= \left(2e^{-t} - e^{-3t}\right)u(t) \text{ logo:}$$

$$h(t) = \left(2e^{-t} - e^{-3t}\right)u(t)$$

2.3.2

$$(D^2 + 5D + 6)y(t) = (D^2 + 7D + 11)x(t)$$

$h(t) = ?$

Solução:

$N = 2$ e $M = 2$, $N \geq M$.

. Eq. característica.

$$\lambda^2 + 5\lambda + 6 = 0 \begin{cases} \lambda = -2 \\ \lambda = -3 \end{cases}$$

$$A_1 e^{-2t} + A_2 e^{-3t}$$

$a_2 = 1$, $a_1 = 5$, $a_0 = 6$

$b_2 = 1$, $b_1 = 7$, $b_0 = 11$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \frac{1}{-3-(-2)} \begin{bmatrix} -3 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$h_c(t) = \left(e^{-2t} - e^{-3t}\right)\mu(t)$$

$$h(t) = \sum_{k=0}^{2} b_k \frac{d^k h_c(t)}{dt^k} =$$

$$= b_0 h_c(t) + b_1 \dot{h}_c(t) + b_2 \ddot{h}_c(t)$$

$$= 11 h_c(t) + 7 \dot{h}_c(t) + 1 \ddot{h}_c(t)$$

$$11 \times \left(e^{-2t} - e^{-3t}\right) u(t) +$$

$$+ 7\left(-2e^{-2t}\mu(t) + \cancel{e^{-2t}\delta(t)} + 3e^{-3t}u(t) \cancel{- e^{-3t}\delta(t)}\right)$$

$$+ 1\left(+4e^{-2t}u(t) - 2e^{-2t}\delta(t) - 9e^{-3t}u(t) + 3e^{-3t}u(t)\right)$$

$$= \delta(t) + e^{-2t}u(t) + e^{-3t}u(t)$$

$$h(t) = \delta(t) + \left(e^{-2t} + e^{-3t}\right)\mu(t)$$

2-4-2.

$c(t) = x(t) * g(t)$

$x(at) * g(at) = ?$

Solução:

Usando a definição:

$$\int_{-\infty}^{\infty} x(a\tau)\, g(a(t-\tau))\, d\tau$$

Faço, $z = a\tau \Rightarrow dz = a\, d\tau \Rightarrow d\tau = \frac{1}{a} dz$

$\tau \to -\infty \Rightarrow z = \mp\infty$ depende do sinal de a.

$\tau \to +\infty \Rightarrow z = \pm\infty$

$$\frac{1}{|a|} \int_{-\infty}^{\infty} x(z)\, g(at - z)\, dz$$

$$\Downarrow$$

$c(at)$, logo.

$$x(at) * g(at) = \frac{1}{|a|} c(at)$$

2.4.3

Sei que

$$c(t) = \int_{-\infty}^{\infty} x(t)\, g(t-b)\, db \quad e$$
$$= x(t) * g(t)$$

que

$$x(at) * g(at) = \frac{1}{a} c(at)$$

Se $a = -1 \Rightarrow x(-t),\ g(-t)$ e $c(-t)$

e $x(-t) * g(-t) = c(-t)$

Se $x(t) = x(-t)$ PAR e
$g(t) = g(-t)$ PAR logo

$$x(t) * g(t) = c(-t) = c(t) \Rightarrow \underline{\text{PAR}}$$

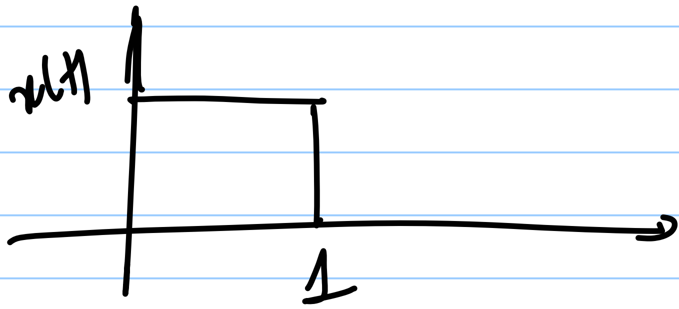
Se $x(t) = -x(-t) \Rightarrow x(-t) = -x(t)$

$$\underbrace{-x(t) * g(t)}_{-c(t)} = c(-t)$$

$-c(t) = c(-t) \Rightarrow$ ÍMPAR

2.4.11

$$h(t) = e^{-t} u(t)$$



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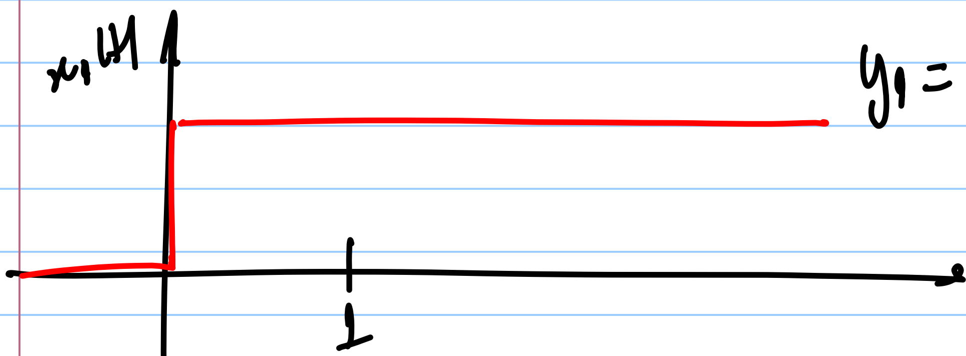

$$y_1 = (1 - e^{-t}) u(t)$$

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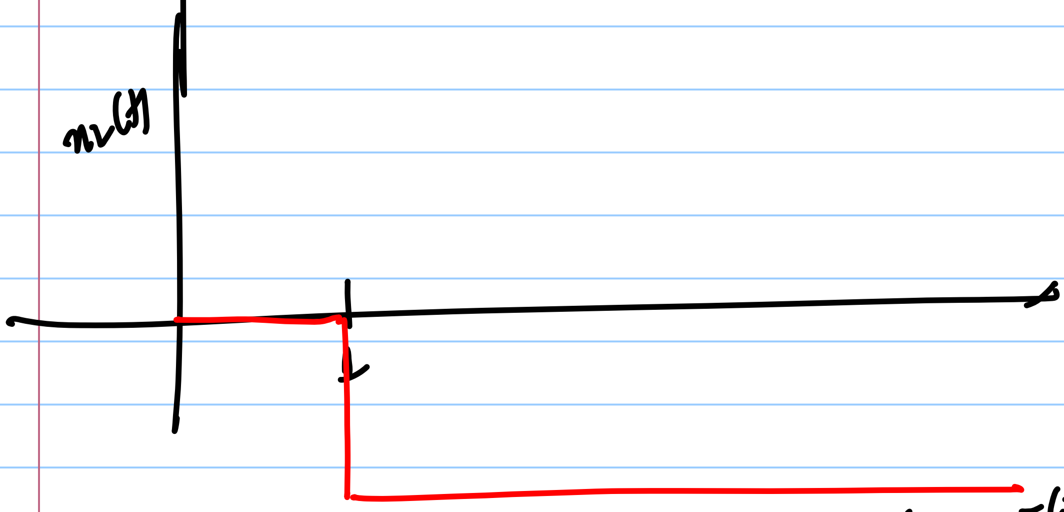
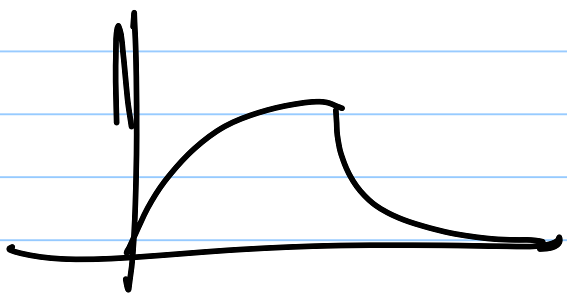
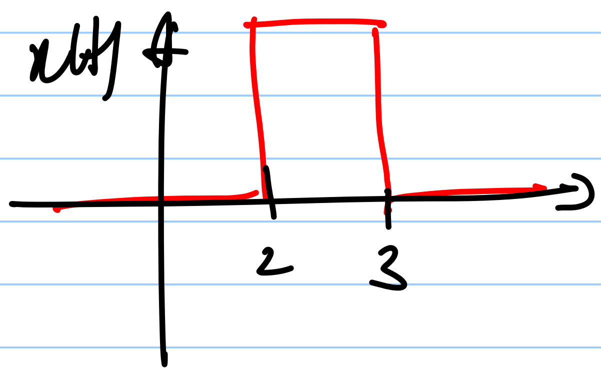


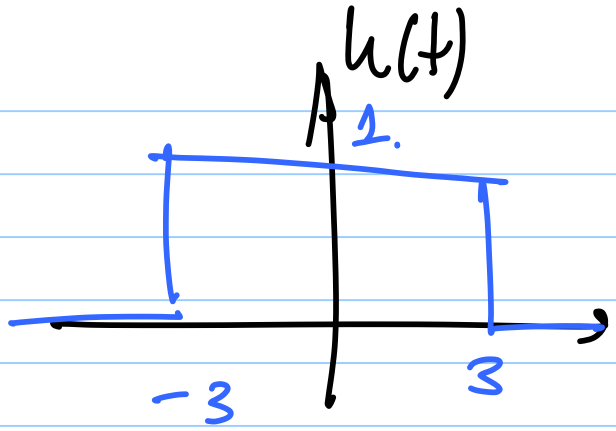
$$y_2 = \left(1 - e^{-(t-1)}\right) u(t-1)$$

$$y = y_1 + y_2.$$

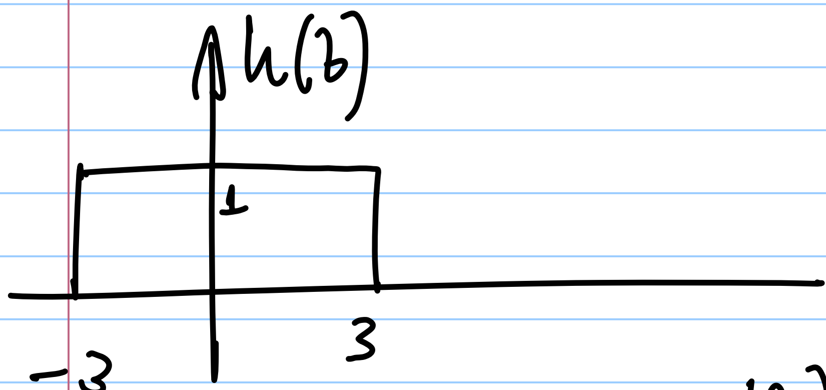


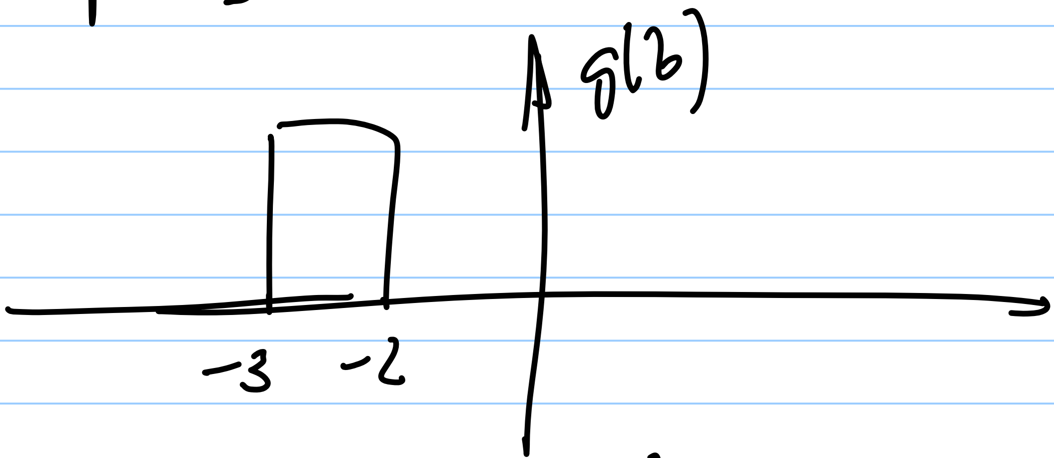
2.4.13

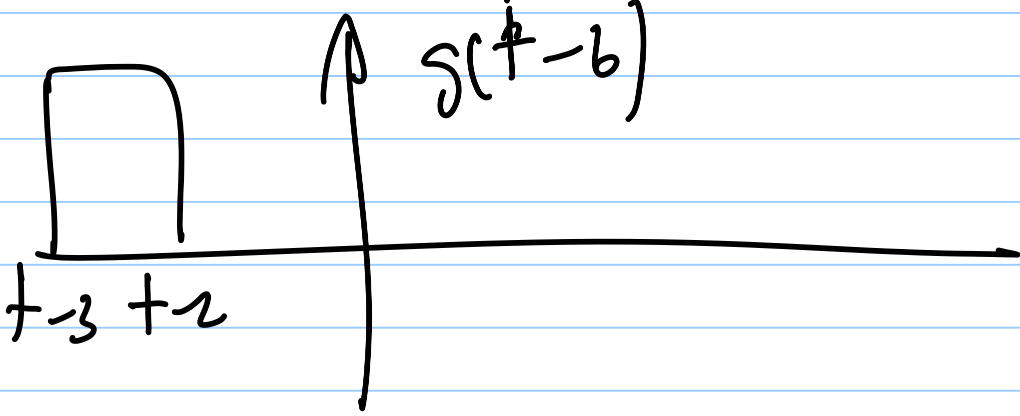


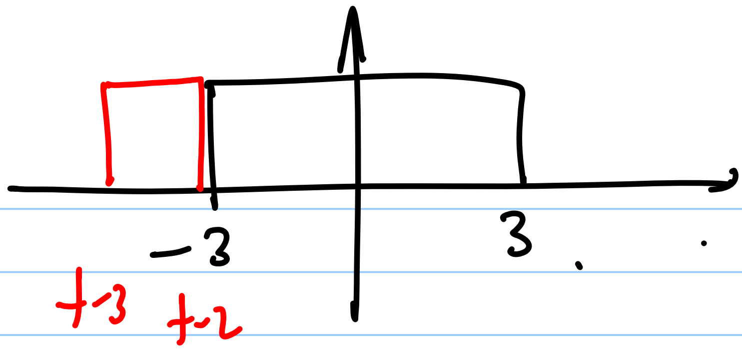


$$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \quad \text{ou} \quad \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$









$t-2=-3 \to t=-1$

$t-3=-3 \to t=0$