Introduction to Aircraft Control

Prof. Leo Torres

March, 2022

Section 1

An Overview of Aircraft Dynamics

Aircraft Dynamics: Dynamic Modes

- The eigenvalues (or the poles) associated with the Local Linear Model for the Aircraft dynamics are very important in determining how the aircraft will respond to changes in the inputs and/or to initial conditions close but not at the Trim condition.
- Indeed, we can associate dynamic modes of response to each real eigenvalue/pole and to each pair of complex conjugate eigenvalues/poles.
- This idea dates back to the work of the British Engineer George H. Bryan in 1911 (only 8 years after the the first flight by the Wright brothers)!

Aircraft Dynamics: Modes of Response

Aircraft Typical Set of States:

$$V_{\mathrm{T}}$$
$$\alpha$$
$$Q$$
$$\theta$$
$$h$$

$$\beta$$
$$P$$
$$R$$
$$\phi$$
$$\psi$$

$$p_{\mathrm{N}}$$
$$p_{\mathrm{E}}$$

Aircraft Dynamics: Modes of Response

Aircraft Typical Set of States:

V_{T}, α, Q, θ, h	Longitudinal Dynamics
β, P, R, ϕ, ψ	Lateral-Directional Dynamics
p_{N}, p_{E}	Navigation: (not important for stability analysis)

Aircraft Dynamics: Modes of Response

Aircraft Typical Set of States:

V_{T} α Q θ	Longitudinal Dynamics
h	Often ignored in stability analysis
β P R ϕ	Lateral-Directional Dynamics
ψ	Often ignored in stability analysis
p_{N} p_{E}	Navigation: (not important for stability analysis)

Aircraft Dynamics: Modes of Response

Aircraft Typical Dynamic Modes of Response:

Longitudinal Dynamics

- Phugoid $\longleftarrow V_{\mathrm{T}}$
- Short-period $\longleftarrow \alpha$
- Short-period $\longleftarrow Q$
- Phugoid $\longleftarrow \theta$
- Phugoid $\longleftarrow h$ — Often ignored in stability analysis

Lateral-Directional Dynamics

- Dutch Roll $\longleftarrow \beta$
- Roll Subsidence $\longleftarrow P$
- Dutch Roll $\longleftarrow R$
- Spiral $\longleftarrow \phi$
- Spiral + Dutch Roll $\longleftarrow \psi$ — Often ignored in stability analysis

p_{N}, p_{E} — Navigation: (not important for stability analysis)

Aircraft Longitudinal Dynamics

Consider the Flat-Earth equations for the state variables associated with the Longitudinal dynamics:

$$
\begin{aligned}
\dot{V}_{\mathrm{T}} &= \frac{1}{M_{\mathrm{A}}}\left[F_{\mathrm{T}}\cos(\alpha+\alpha_{\mathrm{T}})\cos\beta - D\right] + g_1(\alpha,\beta,\phi,\theta),\\
\dot{\alpha} &= \frac{1}{M_{\mathrm{A}}V_{\mathrm{T}}\cos\beta}\left[-F_{\mathrm{T}}\sin(\alpha+\alpha_{\mathrm{T}}) - L\right] + \frac{g_3(\alpha,\phi,\theta)}{V_{\mathrm{T}}\cos\beta} + \frac{Q_{\mathrm{W}}(\alpha,\beta,P,Q)}{\cos\beta},\\
\dot{Q} &= c_5 PR - c_6(P^2 - R^2) + c_7 m,\\
\dot{\theta} &= Q\cos\phi - R\sin\phi,\\
\dot{h} &= V_T\left[\cos\alpha\cos\beta\sin\theta - \sin\beta\sin\phi\cos\theta - \sin\alpha\cos\beta\cos\phi\cos\theta\right].
\end{aligned}
$$

where the constants c_5, c_6, c_7 depend on the Aircraft inertia matrix elements, F_{T} is the thrust force, α_{T} is angle of the thrust force vector with the body-fixed reference frame x-axis, and g_1, g_3, and Q_{W} are elements of the gravity and the angular velocity vectors represented in the Wind axis reference frame.

Aircraft Longitudinal Dynamics

Consider the Flat-Earth equations for the state variables associated with the Longitudinal dynamics:

$$
\begin{aligned}
\dot{V}_{\mathrm{T}} &= \frac{1}{M_{\mathrm{A}}}\left[F_{\mathrm{T}}\cos(\alpha+\alpha_{\mathrm{T}})\cos\beta - D\right] + g_1(\alpha,\beta,\phi,\theta),\\
\dot{\alpha} &= \frac{1}{M_{\mathrm{A}}V_{\mathrm{T}}\cos\beta}\left[-F_{\mathrm{T}}\sin(\alpha+\alpha_{\mathrm{T}}) - L\right] + \frac{g_3(\alpha,\phi,\theta)}{V_{\mathrm{T}}\cos\beta} + \frac{Q_{\mathrm{W}}(\alpha,\beta,P,Q)}{\cos\beta},\\
\dot{Q} &= c_5 PR - c_6(P^2 - R^2) + c_7 m,\\
\dot{\theta} &= Q\cos\phi - R\sin\phi,\\
\dot{h} &= V_T\left[\cos\alpha\cos\beta\sin\theta - \sin\beta\sin\phi\cos\theta - \sin\alpha\cos\beta\cos\phi\cos\theta\right].
\end{aligned}
$$

where the constants c_5, c_6, c_7 depend on the Aircraft inertia matrix elements, F_{T} is the thrust force, α_{T} is angle of the thrust force vector with the body-fixed reference frame x-axis, and g_1, g_3, and Q_{W} are elements of the gravity and the angular velocity vectors represented in the Wind axis reference frame.
Notice that there are many variables from the Lateral-Directional Dynamics appearing in these expressions.

Aircraft Longitudinal Dynamics

- The Longitudinal Dynamics is defined for small variations around the **Wings Level Flight without Sideslipping** condition:

$$\beta(t) = \phi(t) = P(t) = R(t) = 0.$$

Aircraft Longitudinal Dynamics

- The Longitudinal Dynamics is defined for small variations around the **Wings Level Flight without Sideslipping** condition:

$$\beta(t) = \phi(t) = P(t) = R(t) = 0.$$

- In this case, if the variables β, ϕ, P and R remain zero, the Aircraft Longitudinal movements (straight flight, climb, descent, pitching up, pitching down, etc) can be analyzed separately from other movements such as rolling, yawing, and sideslipping.

Aircraft Longitudinal Dynamics: Phugoid Mode

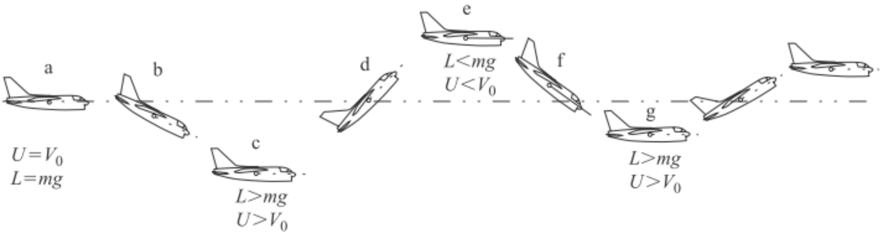


Figure 6.3 *The development of a stable phugoid.*

Michael V. Cook. *Flight Dynamics Principles*. (2007)

Very slow oscillations with a settling time in the order of tens of seconds.
Complex conjugate eigenvalues.

Aircraft Longitudinal Dynamics: Short-period Mode

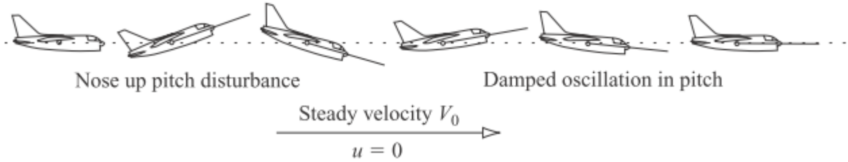


Figure 6.2 *A stable short period pitching oscillation.*

Michael V. Cook. *Flight Dynamics Principles*. (2007)

Very <u>fast</u> oscillations with a settling time in the order of <u>tenths</u> of seconds.
Complex conjugate eigenvalues.

Aircraft Lateral-Directional Dynamics

Consider the Flat-Earth equations for the state variables associated with the Lateral-Directional dynamics:

$$\dot{\beta} = \frac{1}{M_{\mathrm{A}} V_{\mathrm{T}}} \left[-F_{\mathrm{T}} \cos(\alpha + \alpha_{\mathrm{T}}) \mathrm{sen}\beta - C \right] + \frac{g_2(\alpha, \beta, \phi, \theta)}{V_T} - R_{\mathrm{W}}(\alpha, P, R),$$

$$\dot{P} = (-c_1 R + c_2 P) Q + c_3 \ell + c_4 N,$$

$$\dot{R} = (c_8 P - c_2 R) Q + c_4 \ell + c_9 N,$$

$$\dot{\phi} = P + Q \tan\theta \sin\phi + R \tan\theta \cos\phi,$$

$$\dot{\psi} = Q \frac{\sin\phi}{\cos\theta} + R \frac{\cos\phi}{\cos\theta}.$$

where the constants $c_1, \ldots, c_9$ depend on the Aircraft inertia matrix elements, F_{T} is the thrust force, α_{T} is angle of the thrust force vector with the body-fixed reference frame x-axis, and g_2 and R_{W} are elements of the gravity and the angular velocity vectors represented in the Wind axis reference frame.

Aircraft Lateral-Directional Dynamics

Consider the Flat-Earth equations for the state variables associated with the Lateral-Directional dynamics:

$$
\begin{aligned}
\dot{\beta} &= \frac{1}{M_{\mathrm{A}} V_{\mathrm{T}}} \left[-F_{\mathrm{T}} \cos(\alpha + \alpha_{\mathrm{T}}) \mathrm{sen} \beta - C \right] + \frac{g_2(\alpha, \beta, \phi, \theta)}{V_T} - R_{\mathrm{W}}(\alpha, P, R), \\
\dot{P} &= (-c_1 R + c_2 P) Q + c_3 \ell + c_4 N, \\
\dot{R} &= (c_8 P - c_2 R) Q + c_4 \ell + c_9 N, \\
\dot{\phi} &= P + Q \tan\theta \sin\phi + R \tan\theta \cos\phi, \\
\dot{\psi} &= Q \frac{\sin\phi}{\cos\theta} + R \frac{\cos\phi}{\cos\theta}.
\end{aligned}
$$

where the constants $c_1, \ldots, c_9$ depend on the Aircraft inertia matrix elements, F_{T} is the thrust force, α_{T} is angle of the thrust force vector with the body-fixed reference frame x-axis, and g_2 and R_{W} are elements of the gravity and the angular velocity vectors represented in the Wind axis reference frame.
Notice that there are many variables from the Longitudinal Dynamics appearing in these expressions.

Aircraft Lateral-Directional Dynamics

- The Lateral-Directional Dynamics is defined for small variations around the **Straight Wings Level Flight** condition, similarly to what was done for the Longitudinal Dynamics.

Aircraft Lateral-Directional Dynamics

- The Lateral-Directional Dynamics is defined for small variations around the **Straight Wings Level Flight** condition, similarly to what was done for the Longitudinal Dynamics.
- Notice that, when the initial condition is $\beta(0) = \phi(0) = P(0) = R(0) = 0$, even if the Longitudinal Dynamics variables change over time, the Lateral-Directional variables will remain zero, as long as the forces and moments $C = \ell = N = 0$.

Aircraft Lateral-Directional Dynamics

- However, differently from the Longitudinal Dynamics, nonzero changes in the Lateral-Directional variables would induce changes in the Longitudinal Dynamics variables, and this would subsequently break the decoupling between these two main dynamics: there is no real decoupling in the nonlinear context.

Aircraft Lateral-Directional Dynamics

- However, differently from the Longitudinal Dynamics, nonzero changes in the Lateral-Directional variables would induce changes in the Longitudinal Dynamics variables, and this would subsequently break the decoupling between these two main dynamics: there is no real decoupling in the nonlinear context.
- Therefore, we have to add the **assumption that the Longitudinal Dynamics variables are kept approximately constant regardless of the variations in β, P, R, ϕ, and ψ**. That is:

$$V_{\mathrm{T}}(t) \approx V_{\mathrm{T}}^{*}, \quad \theta(t) \approx \theta^{*} \approx \alpha(t), \quad Q \approx 0.$$

Aircraft Lateral-Directional Dynamics

- However, differently from the Longitudinal Dynamics, nonzero changes in the Lateral-Directional variables would induce changes in the Longitudinal Dynamics variables, and this would subsequently break the decoupling between these two main dynamics: there is no real decoupling in the nonlinear context.
- Therefore, we have to add the **assumption that the Longitudinal Dynamics variables are kept approximately constant regardless of the variations in β, P, R, ϕ, and ψ**. That is:

$$V_{\mathrm{T}}(t) \approx V_{\mathrm{T}}^{*}, \quad \theta(t) \approx \theta^{*} \approx \alpha(t), \quad Q \approx 0.$$

- On the other hand, this assumption is consistent with the assumptions already adopted for obtaining Local Linear Models via linearization: **the decoupling exists only in the context of local linear analysis**.

Aircraft Lateral-Directional Dynamics: Roll Subsidence Mode

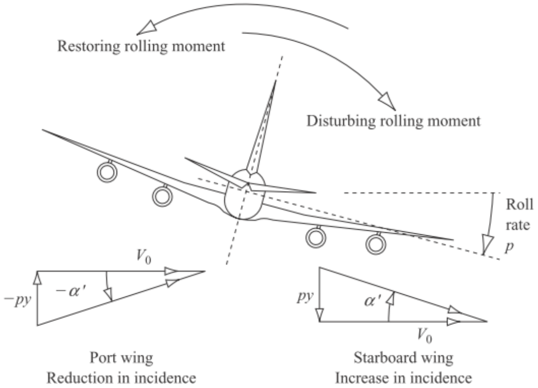


Figure 7.3 *The roll subsidence mode.*

Michael V. Cook. *Flight Dynamics Principles*. (2007)

Very fast and non-oscillatory, with a settling time in the order of tenths of seconds. Real Eigenvalue.

Aircraft Lateral-Directional Dynamics: Spiral Mode

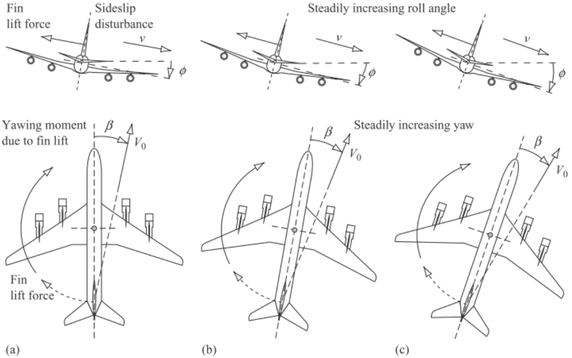


Figure 7.4 *The spiral mode development.*

Michael V. Cook. *Flight Dynamics Principles*. (2007)

Very slow and non-oscillatory, with a settling time in the order of tens of seconds, or mildly unstable. Real Eigenvalue.

Aircraft Lateral-Directional Dynamics: Dutch Roll Mode

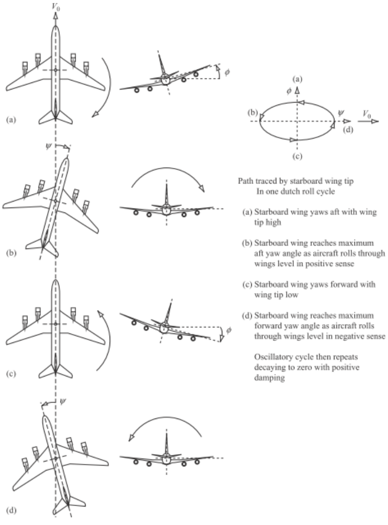


Figure 7.5 *The oscillatory dutch roll mode.*

Michael V. Cook. *Flight Dynamics Principles*. (2007)

Slow oscillations with a settling time in the order of a few seconds. Complex Conjugate Eigenvalues.

Alleged origin of the name "Dutch Roll": ice skating movement referred to by the Aeronautical Engineer Jerome C. Hunsaker in "Proceedings of the National Academy of Sciences of America". (1916), 2: 282.

Section 2

Aicraft Automatic Control

Changing the Aircraft Dynamics

- What are the objectives of changing the Aircraft Dynamics?
  1. To stabilize dynamically unstable modes (stability)
  2. To improve the damping of underdamped responses (stability).
  3. To make the aircraft respond faster to pilot demands (performance).
  4. To alleviate pilot's workload (performance).
  5. ...

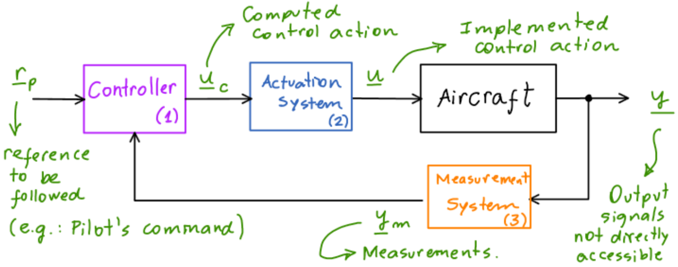
Changing the Aircraft Dynamics

- What are the objectives of changing the Aircraft Dynamics?
  1. To stabilize dynamically unstable modes (stability)
  2. To improve the damping of underdamped responses (stability).
  3. To make the aircraft respond faster to pilot demands (performance).
  4. To alleviate pilot's workload (performance).
  5. ...
- How can we change the Aircraft Dynamics?
  1. Either by **Changing the Airframe** to have appropriate stability and control derivatives,
  2. or by **Controlling the Aircraft** by feeding back information about the aircraft variables to determine the values of the inputs.

Aircraft Closed-Loop Control

Every time information is fed back to the pilot (e.g., by visual cues or by looking at the instruments in the cockpit panel), or to an *automatic device* (e.g., by using measurements), and this leads to changes in actions performed to fly the Aircraft, we have **Closed-Loop Control**.

In the second case, we have an **Automatic Closed-Loop Control**:



Aircraft Automatic Closed-Loop Control

- Notice that the "Aircraft" system is not really changed with respect to what it was before using closed-loop control, but instead three other subsystems were added: (1) Controller subsystem, (2) Actuation subsystem, and (3) Measurement subsystem. We are actullay creating a new relationship between the variables $\underline{r}_{\mathrm{p}}$ and $\underline{y}$.
- Even if $\underline{r}_{\mathrm{p}} = 0$, there can be deflections in the control surfaces and changes in the commanded thrust because $\underline{u}_c \neq 0$ due to the feedback from the measurements.
- This situation is similar to a "24/7 co-pilot" that is always helping the pilot to fly the Aircraft in a safer and effective manner.

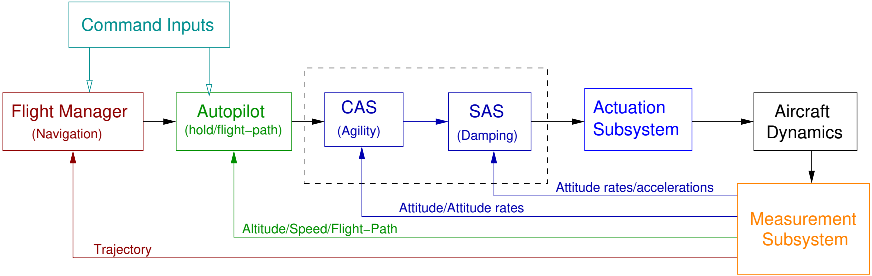
Automatic Control Functions I

Typical functions found in Aircraft Automatic Flight Control Systems (AFCS) can be grouped in the following categories, in increasing order of complexity, and from the innermost control loops to the outermost ones:

1. **SAS** (Stability Augmentation System)
2. **CAS** (Control Augmentation System)
3. **Autopilot**

Automatic Control Functions II

Typical Cascaded Structure for Aircraft Automatic Control:



Automatic Control Functions III

- **SAS** (Stability Augmentation System): improvement of the damping associated with longitudinal and lateral-directional modes:
  - Roll damper.
  - Pitch damper.
  - Yaw damper.

Automatic Control Functions IV

- **CAS** (Control Augmentation System): improvement of the aircraft speed of response to commands originated by the pilot to facilitate the achievement of mission goals:
  - Roll rate.
  - Pitch rate.
  - Normal acceleration.
  - Lateral-directional CAS (roll with high α).

Automatic Control Functions V

- **Autopilot**: alleviation of pilot's workload by providing flight trajectory control functions:
  - Pitch attitude hold.
  - Altitude hold.
  - Mach hold.
  - Automatic Landing.
  - Bank angle hold.
  - Turn coordination.
  - Heading hold/VOR hold.

Control Design Techniques I

- Many different Control Design Methods can be used to synthesize controllers (or control laws) to implement the aforementioned SAS, CAS and Autopilot AFCS functionalities.
- The differences are related to the assumptions considered for the Aircraft mathematical model and the controller structure, as well as the sensors and actuators available to design the control system, such as: time-domain or frequency-domain approaches; linear or nonlinear systems descriptions; SISO or MIMO points-of-view; state or output feedback.

Control Design Techniques II

- **Time-domain**: using State-Space Representations to describe how the signals evolve in time. Notice that this is not restricted to LTI systems. Consider the following State-Space Representation:

$$\dot{x} = f(x, u),$$
$$y = h(x, u),$$

  where $f : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ and $h : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^p$ are nonlinear functions.

- **Frequency-domain**: using Transfer Functions to describe the dynamics as a system with an underlying Frequency Response (e.g. high-gain at low frequencies and low-gain at high frequencies). It is intimately associated with LTI systems described by

$$\begin{array}{ll} \dot{x} & \approx Ax + Bu, \\ y & \approx Cx + Du, \end{array} \Rightarrow Y(s) = G(s)U(s), \quad G(s) = C(s\mathbb{1} - A)^{-1}B + D.$$

Control Design Techniques III

- **Linear**: the controller and the aircraft model are both considered LTI systems, such that the closed-loop system becomes:

$$\begin{cases} \dot{x} &= Ax + Bu, \\ y &= Cx + Du, \end{cases}$$
$$\begin{cases} \dot{x}_c &= A_c x_c + B_{c,y} y + B_{c,r} r, \\ u &= C_c x_c + D_{c,y} y + D_{c,r} r \end{cases}$$

  with x, u and y the states, inputs and outputs of the Aircraft; x_c the controller states; and r the reference signals to be tracked (e.g. pilot's commands). Notice that the overall system can be represented as a single LTI system with inputs r and output y. Inevitably one has to consider many different LTI models, each one valid around a specific Trim condition.

- **Nonlinear**: the controller and/or the aircraft model are nonlinear systems. This prevents the direct utilization of Linear Control Theory (e.g. Laplace Transform), demanding other strategies mainly based on **Lyapunov Stability Analysis** and related approaches.

Control Design Techniques IV

- **SISO** (Single-Input Single-Output) approach: the controller is viewed as a collection of independent control loops that operate in parallel. This allows the division of the work associated with controllers' design among different teams, and the utilization of simple principles to devise control strategies. However, clearly one controller actions do perturb the other control loops (they are not *really* independent).
- **MIMO** (Multiple-Input Multiple-Output) approach: the controller is considered as a single entity. All functionalities should be incorporated in the same control structure. The controller can become highly non-intuitive for a human being to grasp why its structure reached a particular final form. Usually optimization methods are used to find optimal parameters, and the intuition changes from the control structure selection to the choice of what cost function and constraints should be considered in the optimization.

Control Design Techniques V

- **Full State Feedback**: all states must be measured to be fed back to the controller (higher cost).
- **Static Output Feedback**: only the outputs – usually the variables to be controlled – are fed back to the controller (lower cost).
- **Dynamic Output Feedback**: only the outputs are utilized in the control loop, but there is an internal **State Observer** to estimate the states not directly measured.

Control Design Techniques VI

- And the techniques can incorporate or not concepts related to
  1. Optimization: control parameters chosen aiming the minimization of a meaningful cost function.
  2. Robustness: guaranteed reliability against parametric uncertainties and external disturbances.
  3. Adaptation: controller parameters are changed automatically as the aircraft changes (due to aging, failures, etc).

Control Design Techniques VII

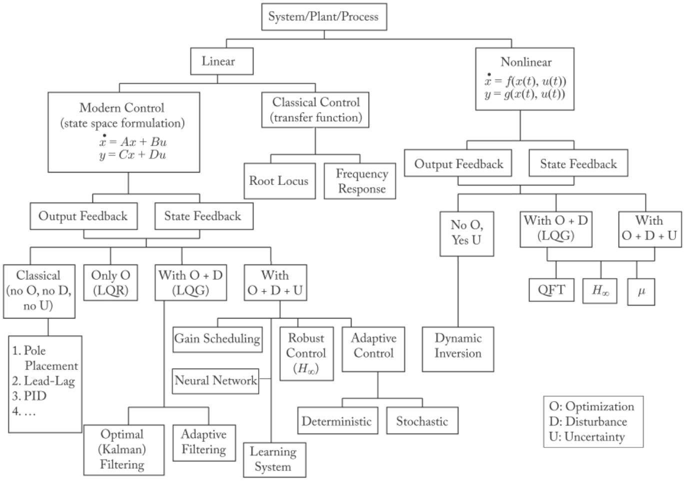
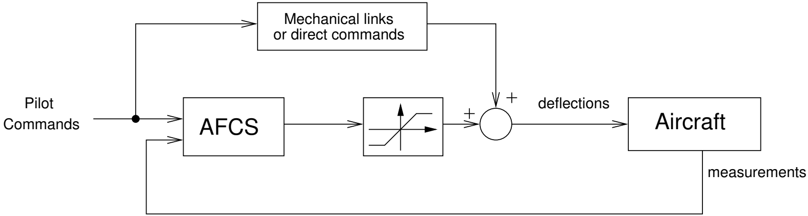


Figure 2.2: Control system design techniques.

Source: "Automatic Flight Control Systems", Mohammad Sadraey (2019)

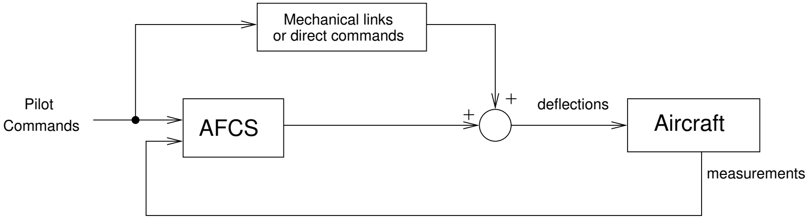
Full-Authority × Limited-Authority, and Direct Mode



Limited-Authority Automatic Control.

The deflections from the AFCS are limited on purpose, and there are direct commands from the pilot sent to the actuators (Direct Mode).

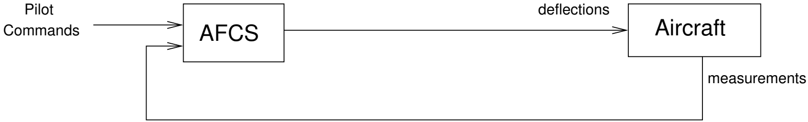
Full-Authority × Limited-Authority, and Direct Mode



Full-Authority Automatic Control.

The deflections are directly commanded by the AFCS which is capable of full deflections that can even overcome the pilot's direct commands.

Full-Authority × Limited-Authority, and Direct Mode



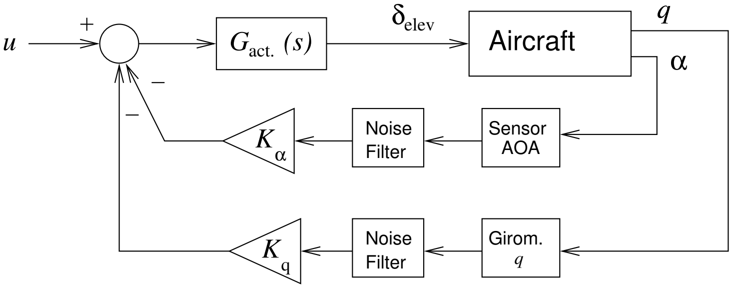
Full-Authority Automatic Control.

The deflections are fully commanded by the AFCS.

Section 3

Stability Augmentation Systems Examples

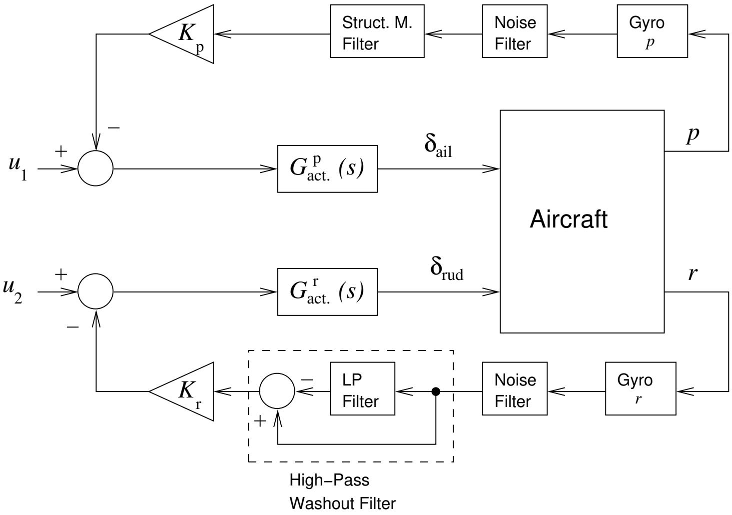
SAS: Pitch axis I



SAS: Pitch axis II

- Goal: increase damping for the short-period mode.
- To feedback only pitch-rate q should be tried first, since this signal is usually less noisy. The position of the gyro on the airframe is important to minimize interference from structural vibration modes.
- If necessary, α can also be fed back with the caveat of being usually a slower and noisier measurement signal. This can be improved by employing α estimators based on the less noisy normal acceleration a_{n} from the IMU (Inertial Measurement Unit).
- Be careful: a negative sign must be included, if necessary, to account for the sign convention adopted for the elevator deflection.

SAS: Yaw Damper I



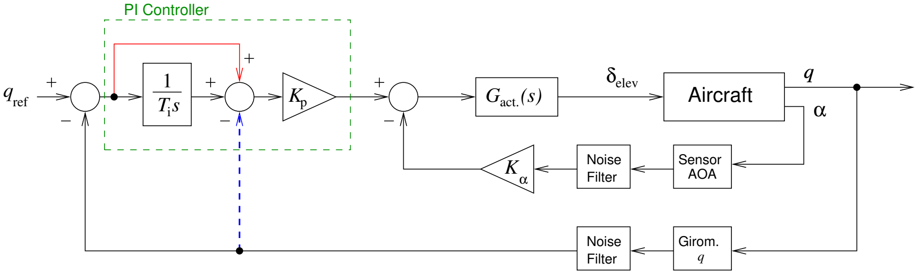
SAS: Yaw Damper II

- Goal: increase the damping in the Dutch-Roll mode.
- A high-pass filter is used in feeding back r so as not to affect coordinated turns.
- There is another filter to attenuate any possible oscillations picked up from a structural torsion mode during the measurement of p.
- Be careful: a negative sign must be included, if necessary, to account for the sign convention adopted for the aileron and rudder deflections.

Section 4

Control Augmentation System Examples

CAS: Pitch-Rate I



Notice that, by using Laplace Tranform properties, the Proportional-Integral (PI) controller is represented by $G_c(s) = K_p(1 + 1/T_i s)$, because

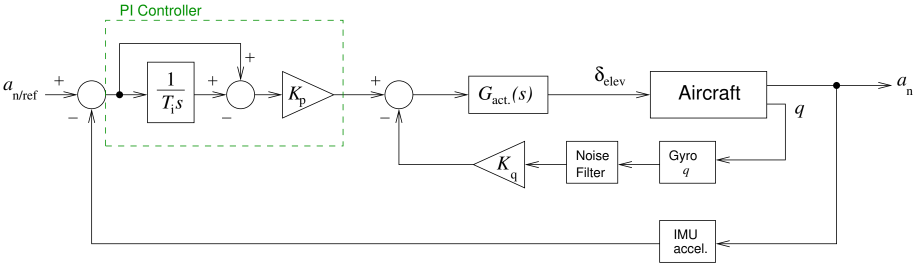
$$U(s) = K_p \left[1 + \frac{1}{T_i s}\right] E(s) \Rightarrow u(t) = K_p \left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right],$$

where $e(t) = q_{\text{ref}}(t) - q(t)$ is the error, and $u(t)$ is the output of the PI controller.

CAS: Pitch-Rate II

- Goal: precise pitch-rate control to improve target tracking and to facilitate pitch control during Approach (Cat. C flight phase). Usually one is interested in minimizing simultaneously the rise time and the settling time, while avoiding overshoot.
- In the alternative configuration (blue dashed line), the proportional action on the error (red continuous trace) is eliminated in favor of feeding back q direcly. This case resembles the use of integral control cascaded with the pitch damper SAS previously discussed. This usually results in smaller overshoots for step changes in q_{ref}.

CAS: Normal Acceleration I



CAS: Normal Acceleration II

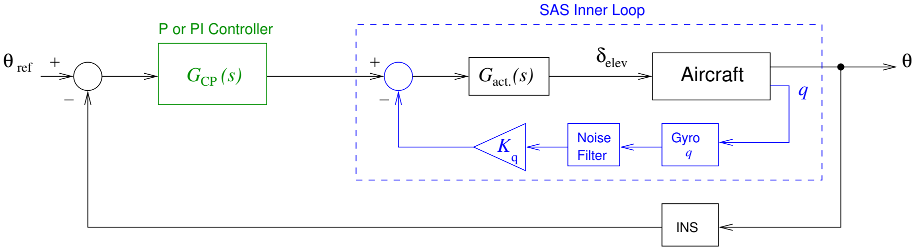
- Goal: to directly control the g-force (normal acceleration) at the pilot station. This allow the maximization of the turn-ratio in fast maneuvers without jeopardizing the pilot's physical integrity or the aircraft structure[1].
- Usually gain scheduling is necessary in this case, since a_{n} response to elevator deflections is highly dependent on dynamic pressure.
- Sometimes it is difficult to establish a proper balance between controlling q or a_{n}. A good option is to use the fictitious signal $C^{*}(t) = 12.4q(t) + a_{\mathrm{n}}(t)$ (see the C* criterion for Flying Qualities).

[1] It is interesting to carefully consider the location of the accelerometer such that it coincides with a node of the fuselage structural bending mode to not pick up this oscillatory movement.

Section 5

Autopilot Examples

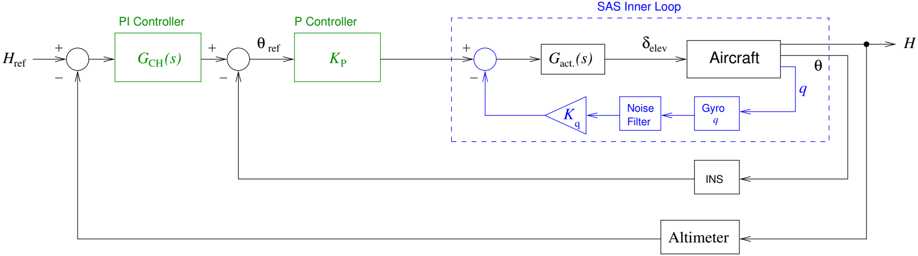
Pitch Hold I


SAS Inner Loop
P or PI Controller
θ ref
+
−
$G_{CP}(s)$
+
−
$G_{act.}(s)$
δ_{elev}
Aircraft
θ
q
K_q
Noise
Filter
Gyro
q
INS

Pitch Hold II

- Goal: pitch angle θ regulation, i.e. to keep it constant by automaticcaly rejecting disturbances that could change it.
- The inner loop is used to improve the pitch-axis stability in order to facilitate the regulation by the outer loop.
- To reduce the error, a simple proportional controller $G_{\mathrm{C}}(s) = k_{\mathrm{p}}$ can be employed. However, a PI controller will guarantee zero error in steady-state and makes the design more flexible: $G_{\mathrm{CP}}(s) = k_{\mathrm{p}} \left[1 + \frac{1}{T_{\mathrm{i}} s}\right]$.

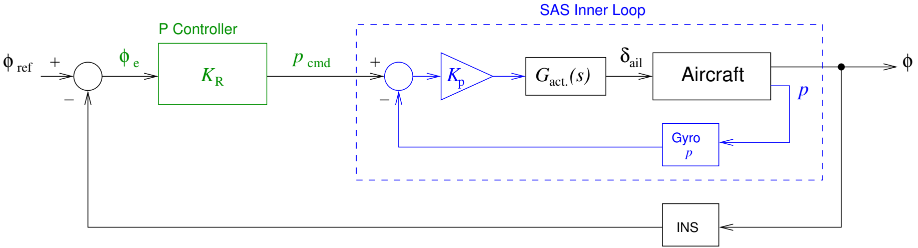
Altitude Hold I



Altitude Hold II

- Goal: altitude H regulation.
- The previous controller is used as an inner loop control system to track desired pitch angles determined by the altitude controller (in the outer loop).
- A PI controller will guarantee zero error in steady-state: $G_{\mathrm{CH}}(s) = k_{\mathrm{p}} \left[1 + \frac{1}{T_{\mathrm{i}} s}\right]$.
- Change in H_{ref} over time can be adjusted (i.e. slowed down) to avoid abrupt changes in the pitch angle with possibly undesirable consequences (stall).

Roll Angle Hold I



Roll Angle Hold II

- Goal: wings leveller that is useful to avoid the slowly divergence from the spiral mode, and part of the AFCS for turn coordination.
- Notice that the inner loop can be seen as a SAS function to increase the damping of the roll subsidence mode.
- Because there is a natural integrative process between roll rate p and roll angle ϕ, a simple proportional controller could be used in the outer loop: as long as the error ϕ_{e} is different from zero, there will be a commanded p_{cmd} that will make the aircraft rolls its wings.

Section 6

UAV Project Example

Project Example: Ground Target Circulation over the Amazon Forest

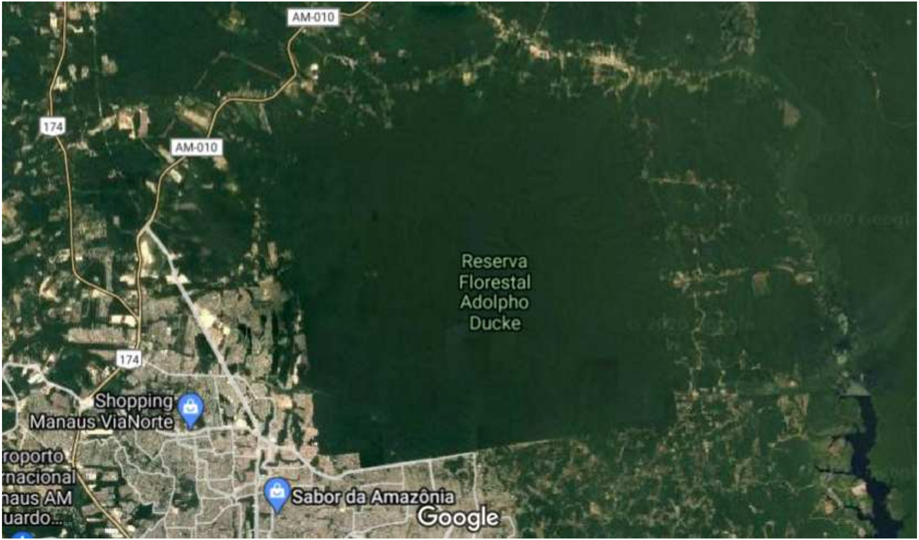
Vast areas in the Amazon Forest have to be surveilled by a small number of Environmental Protection Agents:

Important ground targets:

- Wildfires.
- Illegal deforestation.
- Unauthorized airstrips.

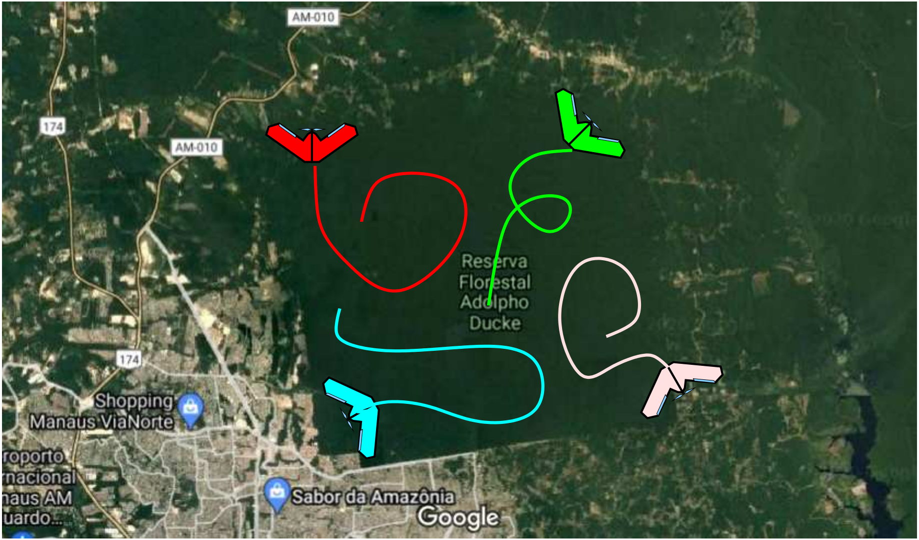
Project Example: Ground Target Circulation over the Amazon Forest

The mission:



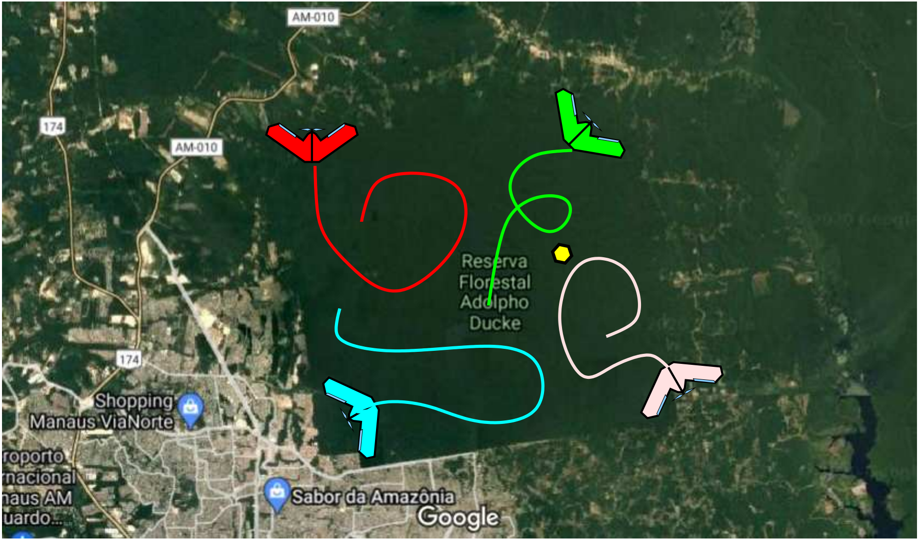
Project Example: Ground Target Circulation over the Amazon Forest

The mission:



Project Example: Ground Target Circulation over the Amazon Forest

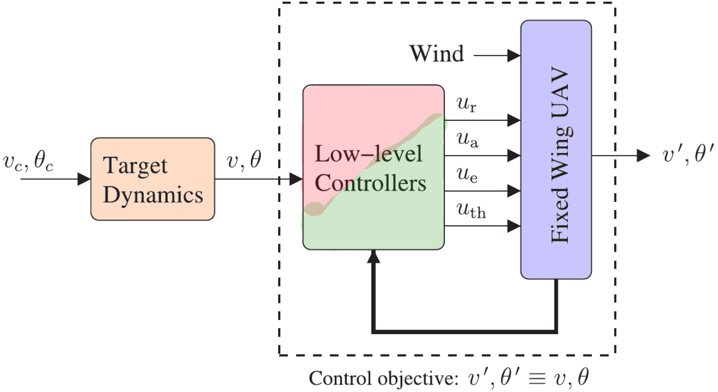
The mission:



Project Example: Ground Target Circulation over the Amazon Forest

The mission:

UAV Reference Model



δ_v, δ_ω and δ_a are norm bounded unknown disturbances.

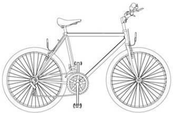
Warning: change in notation. θ, θ_c are heading angles here.

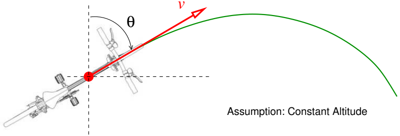
UAV Reference Model: interpretation

The target dynamics of the reference model is nonlinear and non-holonomic, but captures some essential features:

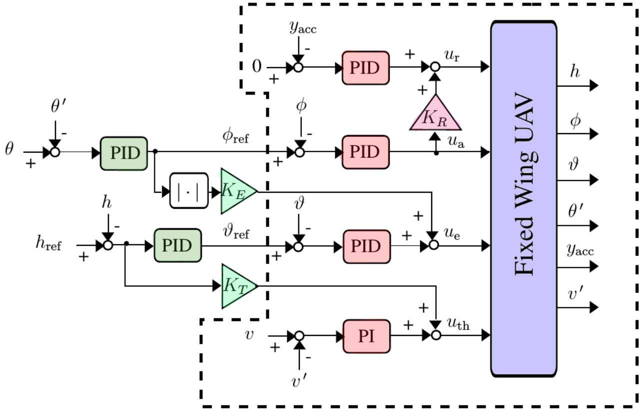
Reference Model:

$$\dot{x} = v\cos(\theta),$$
$$\dot{y} = v\sin(\theta),$$
$$\dot{z} = \frac{1}{\tau_z}(-z + z_c) + \delta_v,$$
$$\dot{\theta} = \frac{1}{\tau_\theta}(-\theta + \theta_c) + \delta_\omega,$$
$$\dot{v} = \frac{1}{\tau_v}(-v + v_c) + \delta_a.$$

Aircraft $\overset{?}{\simeq}$ 

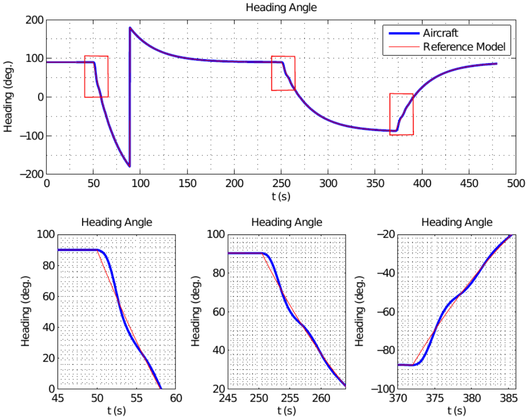


UAV Low-level Control Design



UAV Project: Simulation Results

Simulation results showing that the low-level controllers can make the UAV behaves as dictated by the "bicycle-like" reference model:



Bibliography I

Chi-Tsong Chen.
Linear System Theory and Design.
Oxford University Press, 3rd edition, 1999.

Richard C. Dorf and Robert H. Bishop.
Modern Control Systems.
McGraw-Hill, 13rd edition, 2017.

Gene F. Franklin, J. David Powell, and Abbas Emami-Naeini.
Feedback Control of Dynamic Systems.
Addison-Wesley Publishing Company, Inc., 1986.

Brian L. Stevens, Frank L. Lewis, and Eric N. Johnson.
Aircraft Control and Simulation: Dynamics, Controls Design, and Autonomous Systems.
John Wiley & sons, Inc., 2016.