



ON Semiconductor®

Dc-dc converters feedback and control

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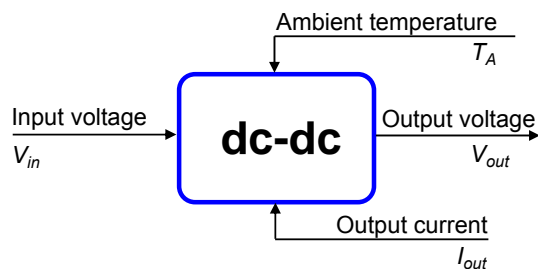
Course agenda

- Feedback generalities
- Building an oscillator
- Poles and zeros
- Phase margin and quality coefficient
- Undershoot and crossover frequency
- Compensating the converter
- Current-mode converters
- Automated pole-zero placement
- Manual pole-zero placement
- Compensating with a TL431
- Watch the optocoupler!
- Input filter
- A real case example
- Conclusion

What do we expect from a dc-dc?

- ❑ A stable output voltage, whatever loading, input, temperature and aging conditions.
- ❑ A fast reaction to a incoming perturbation such as a load transient or an input voltage change.
- ❑ A quick settling time when starting-up or recovering from a transient state.

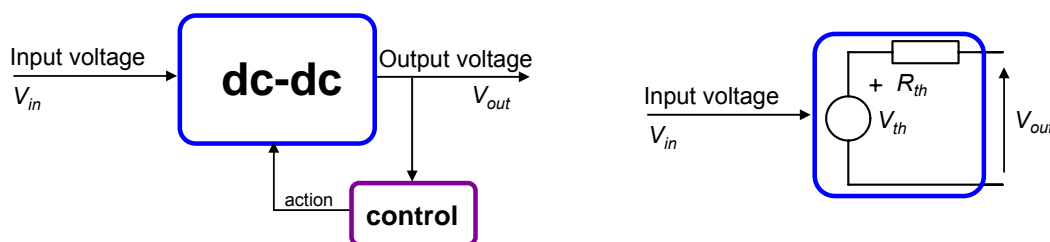
➔ A stable and noiseless dc source we can trust!



What is feedback?

- ❑ A target is assigned to one or several state-variables, e.g. $V_{out} = 12\text{ V}$.
- ❑ A dedicated circuit monitors V_{out} deviations.
- ❑ If V_{out} deviates from its target, an error is created and fed-back to the power stage for action.
- ❑ The action is a change in the control variable: duty ratio (VM), peak current (CM) or switching frequency.

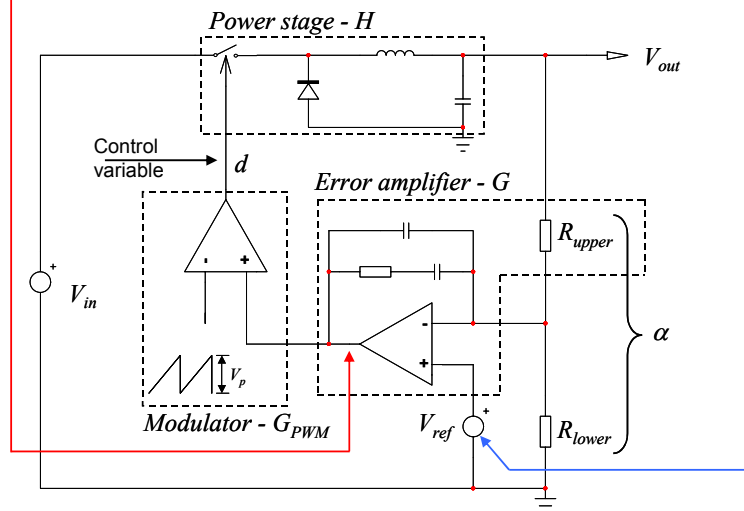
➔ Compensating for the converter shortcomings!





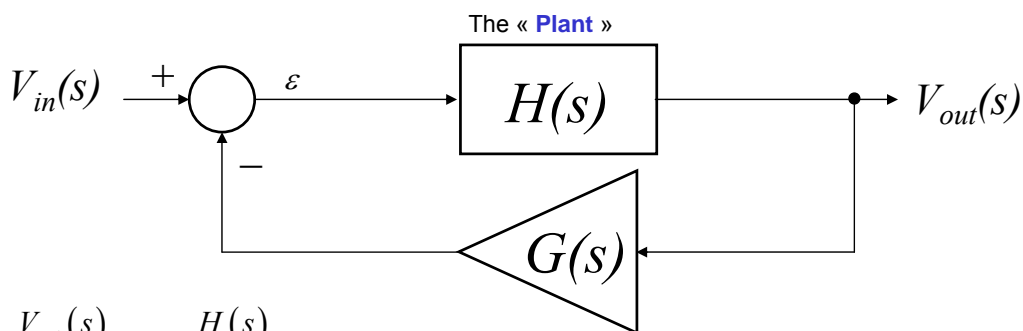
The feedback implementation

- ❑ V_{out} is permanently compared to a reference voltage V_{ref}
- ❑ The reference voltage V_{ref} is precise and stable over temperature.
- ❑ The error $\varepsilon = V_{ref} - \alpha V_{out}$ is amplified and sent to the control input.
- ❑ The power stage reacts to reduce ε as much as it can.



Positive or negative feedback?

- ❑ Do we want to build an oscillator?



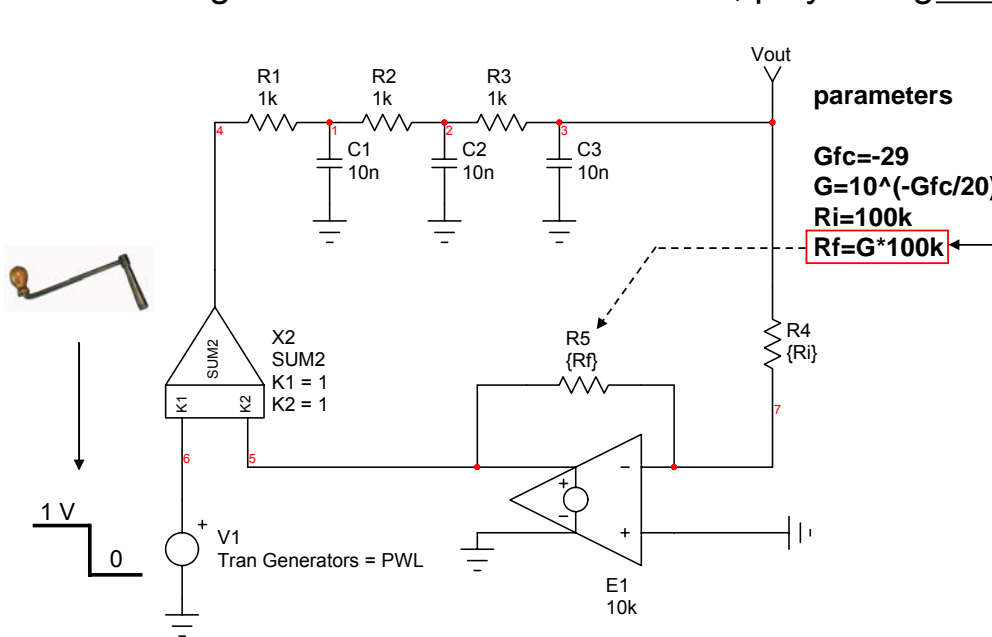
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{H(s)}{1 + H(s)G(s)} \rightarrow \text{Open-loop gain } T(s)$$

$$V_{out}(s) = \lim_{V_{in}(s) \rightarrow 0} \left[\frac{H(s)}{1 + G(s)H(s)} V_{in}(s) \right] \quad \text{To sustain self-oscillations, as } V_{in}(s) \text{ goes to zero, quotient must go infinite}$$

$$1 + G(s)H(s) = 0 \rightarrow \begin{cases} |G(s)H(s)| = 1 \\ \angle G(s)H(s) = -180^\circ \end{cases} \rightarrow \begin{matrix} \text{Nyquist} \\ -1, j0 \end{matrix}$$

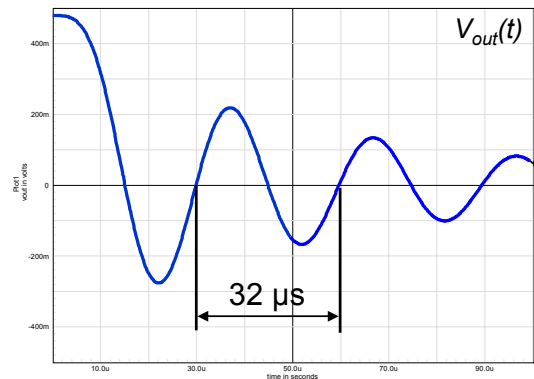
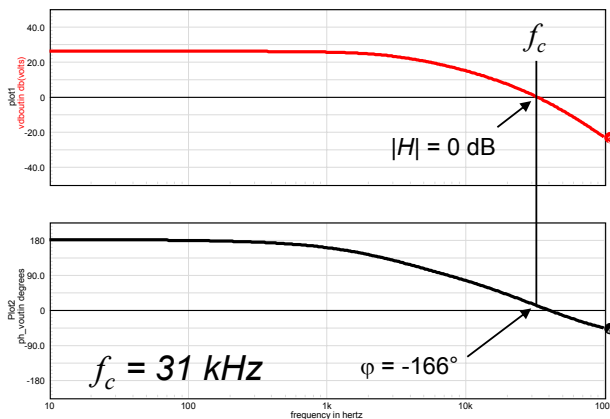
A constant gain with a 180° rotation

- Starting the oscillator with a « crank », play with gain



A simple oscillator

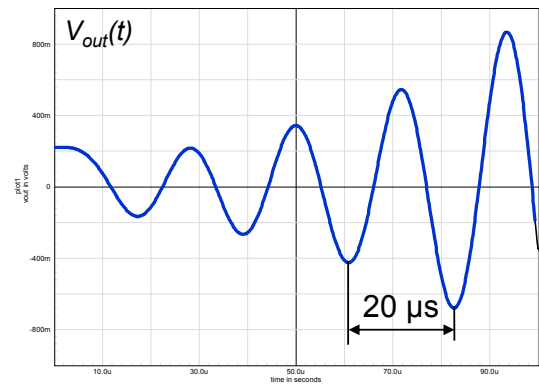
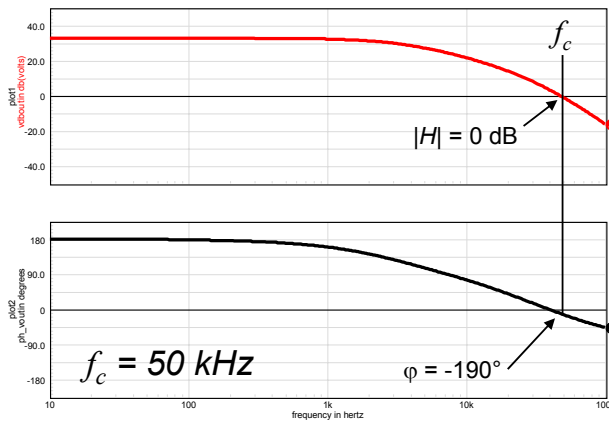
- Case n°1, at 0-dB gain crossover, phase φ is below -180°
- Oscillations are damped, system is asymptotically stable



- The oscillation frequency is the crossover frequency f_c
- The poles have an imaginary and a negative real part (LHPP)

A simple oscillator

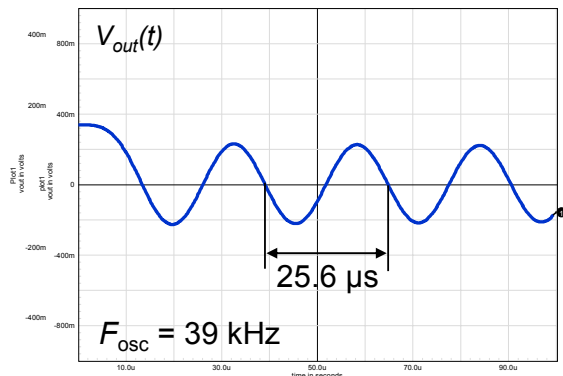
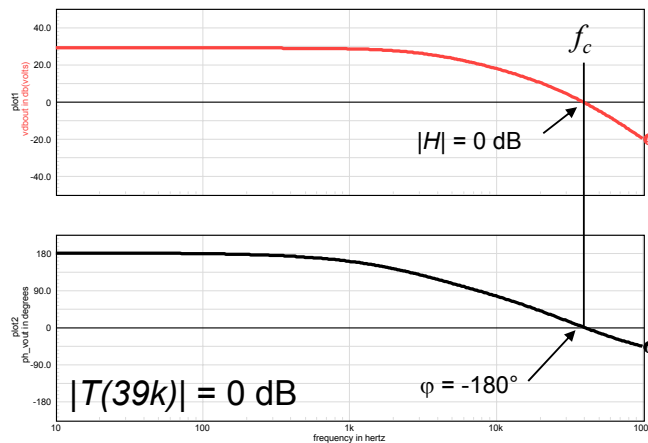
- ❑ Case n°2, at 0-dB gain crossover, ϕ is beyond -180°
- ❑ Oscillations are not damped, system diverges



- ❑ The poles have an imaginary and a positive real part (RHPP)

A simple oscillator

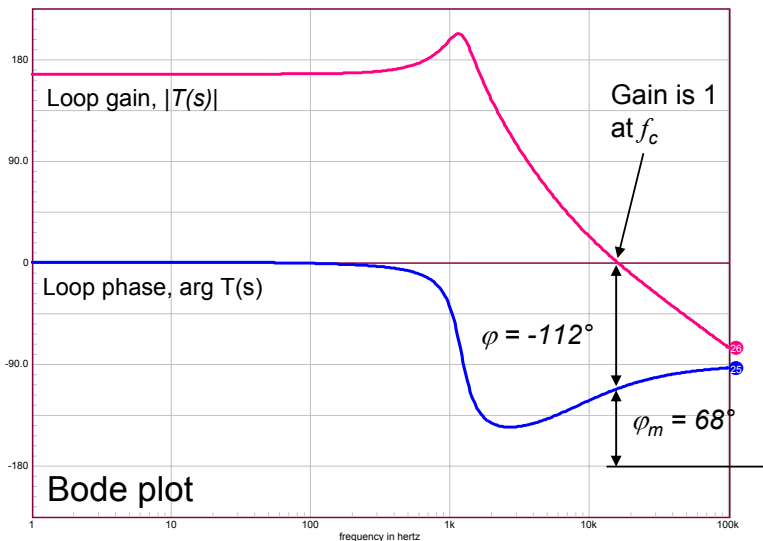
- ❑ Case n°3, at 0-dB gain crossover, ϕ is exactly -180°
- ❑ Oscillations are sustained, we have an oscillator!



- ❑ The poles are pure imaginary: no damping

Conditions for steady-state stability

- ❑ We do not want to create an oscillator!
- ❑ Conditions for non-permanent oscillations are:
 - total phase rotation **less** than -360° at the crossover point



Total phase delay at f_c :

- 112° $H(s)$ power stage
- 180° $G(s)$ opamp

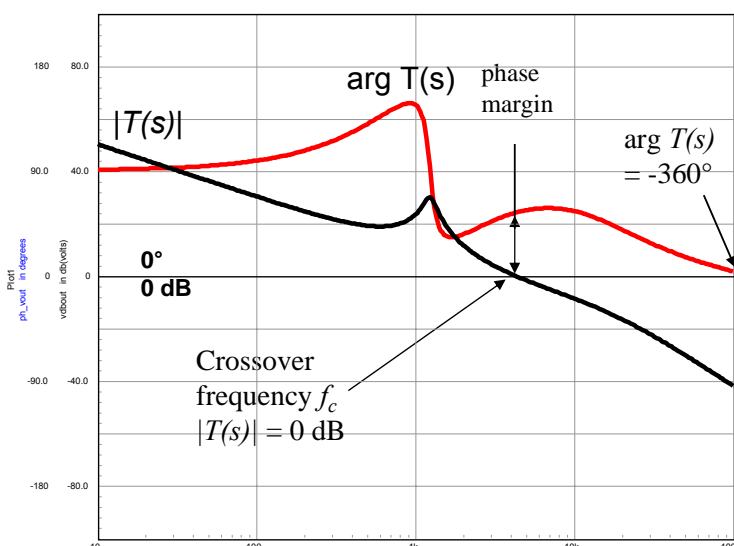
total = -292°

Stable!

H.W. Bode – 1905-1982

The need for phase margin

- ❑ we need **phase** margin when $|T(s)| = 0$ dB
- ❑ we need **gain** margin when $\arg T(s) = -360^\circ$



Phase margin:

The margin before the loop phase rotation $\arg T(s)$ reaches -360° at $|T(s)| = 0$ dB

Gain margin:

The margin before the loop gain $|T(s)|$ reaches 0 dB at a freq. where $\arg T(s) = -360^\circ$



Poles, zeros and s-plane

□ A plant loop gain is defined by:

$$H(s) = \frac{N(s)}{D(s)} \begin{matrix} \longrightarrow \text{numerator} \\ \longrightarrow \text{denominator} \end{matrix}$$

□ solving for $N(s) = 0$, the roots are called the **zeros**

□ solving for $D(s) = 0$, the roots are called the **poles**

$$H(s) = \frac{(s + 5k)(s + 30k)}{s + 1k}$$

$s_{z_1} = -5k$
 $s_{z_2} = -30k$

\longrightarrow

$f_{z_1} = \frac{5k}{2\pi} = 796 \text{ Hz}$
 $f_{z_2} = \frac{30k}{2\pi} = 4.77 \text{ kHz}$

$s_{p_1} = -1k$

\longrightarrow
 $f_{p_1} = \frac{1k}{2\pi} = 159 \text{ Hz}$

Numerator roots
Denominator root

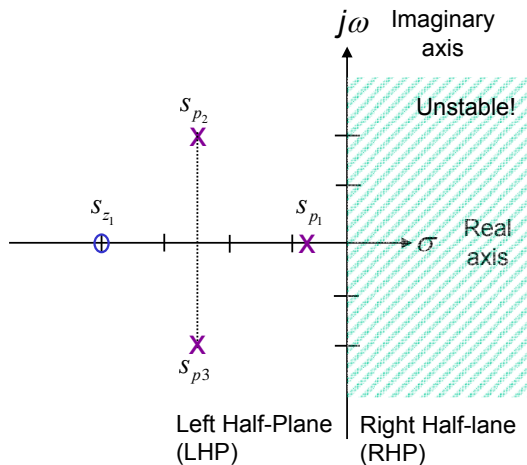


Poles, zeros and s-plane

□ The roots can either be *real* or *imaginary*:

$$H(s) = \frac{s + 4}{(s + 0.8) \underbrace{[(s + 2.5)^2 + 4]}_{\pm 2j}}$$

$$\begin{matrix} s_{z_1} = -4 \\ s_{p_1} = -0.8 \\ s_{p_2} = -2.5 + 2j \\ s_{p_3} = -2.5 - 2j \end{matrix} \left. \vphantom{\begin{matrix} s_{z_1} \\ s_{p_1} \\ s_{p_2} \\ s_{p_3} \end{matrix}} \right\} \begin{matrix} \\ \\ \text{Conjugated} \\ \text{roots} \end{matrix}$$



- We can place these roots in the imaginary plane
 - root-locus analysis in the s-plane
 - their positions in the s-plane affect the time domain response



Poles, zeros and s-plane

- ❑ How the poles do influence the time domain response of the plant?
 - assume an input-step response is wanted:
 - ✓ multiply $H(s)$ by $1/s$
 - ✓ take the inverse Laplace transform
 - ✓ plot the response

$$H(s) = \frac{2}{(s+1)(s+2)} \xrightarrow{\times \frac{1}{s}} H(s) \frac{1}{s} = \frac{1}{s} \frac{2}{(s+1)(s+2)}$$

$s_{p1} = -1$ $s_{p2} = -2$

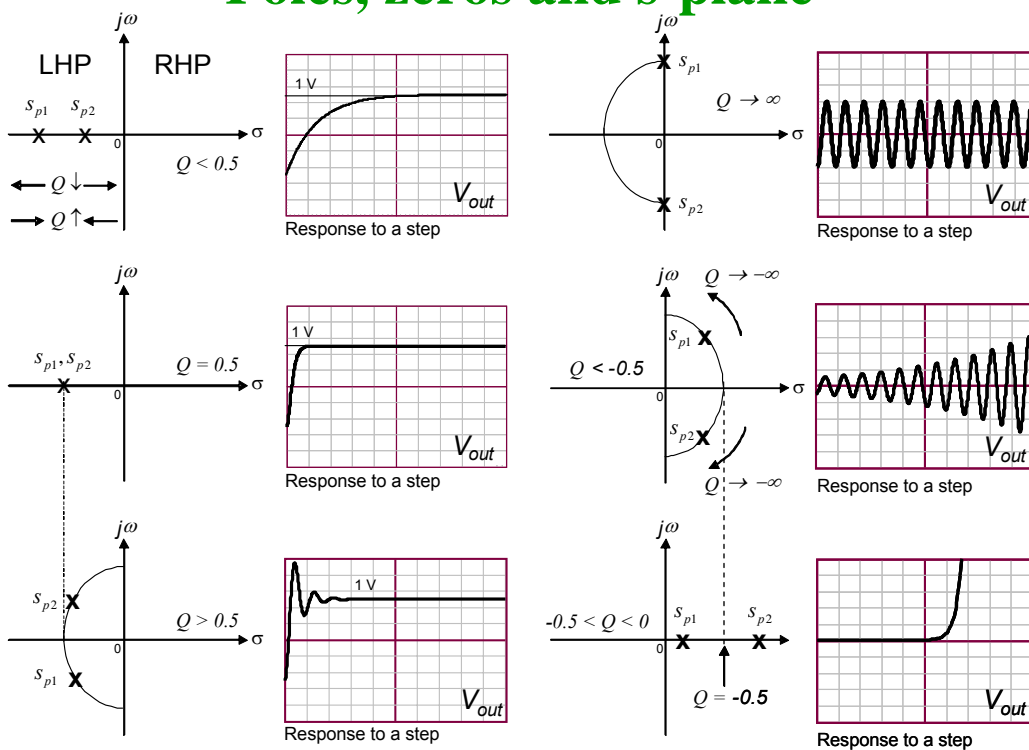
Take inverse Laplace transform

$$\mathcal{L}^{-1}\left\{H(s) \frac{1}{s}\right\} = 0.5 - e^{-1t} + 0.5e^{-2t}$$

- ❑ The roots are the exponentials exponents: -1 and -2
- ❑ If the denominator roots are negative, the signal is **decaying**
- ❑ If the denominator roots are positive, the response **diverges**



Poles, zeros and s-plane



Poles, zeros and s-plane

- ❑ the **pole** magnitude at the cutoff frequency is **-3 dB**
- ❑ Its asymptotic phase at $f = \infty$ is -90°
- ❑ The pole "lags" the phase

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1+sRC} = \frac{1}{1+s/\omega_0} \rightarrow \left| \frac{V_{out}(\omega_0)}{V_{in}(\omega_0)} \right| = \frac{1}{\sqrt{1+[\omega_0/\omega_0]^2}} = \frac{1}{\sqrt{2}}$$

$$\arg \frac{V_{out}(\infty)}{V_{in}(\infty)} = \arg(1) - \arg\left(1 + \frac{\infty}{s_{p1}}\right) = -\arctan(\infty) = -\frac{\pi}{2} = -90^\circ \quad \text{At } f = \infty$$

- ❑ the **zero** magnitude at the cutoff frequency is **+3 dB**
- ❑ Its asymptotic phase at $f = \infty$ is $+90^\circ$
- ❑ The zero "boosts" the phase

$$\frac{V_{out}(s)}{V_{in}(s)} = 1 + \frac{s}{\omega_0} \rightarrow \left| \frac{V_{out}(\omega_0)}{V_{in}(\omega_0)} \right| = \sqrt{1+[\omega_0/\omega_0]^2} = \sqrt{2}$$

$$\arg \frac{V_{out}(\infty)}{V_{in}(\infty)} = \arg\left(1 + \frac{\infty}{s_{p1}}\right) = \arctan(\infty) = +\frac{\pi}{2} = 90^\circ \quad \text{At } f = \infty$$

Poles, zeros and s-plane

- ❑ Poles and zeros can sometimes appear "at the origin"

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{s}{D(s)} \quad \text{Zero for } s = 0: \text{ zero at the origin} \rightarrow \text{As } f \text{ increases the gain increases with a +1 slope (+20 dB/decade)}$$

$$\arg \frac{V_{out}(s)}{V_{in}(s)} = \arg\left(\frac{s}{0}\right) = \arctan(\infty) = +\frac{\pi}{2} \quad \text{For } f > f_{zo}$$

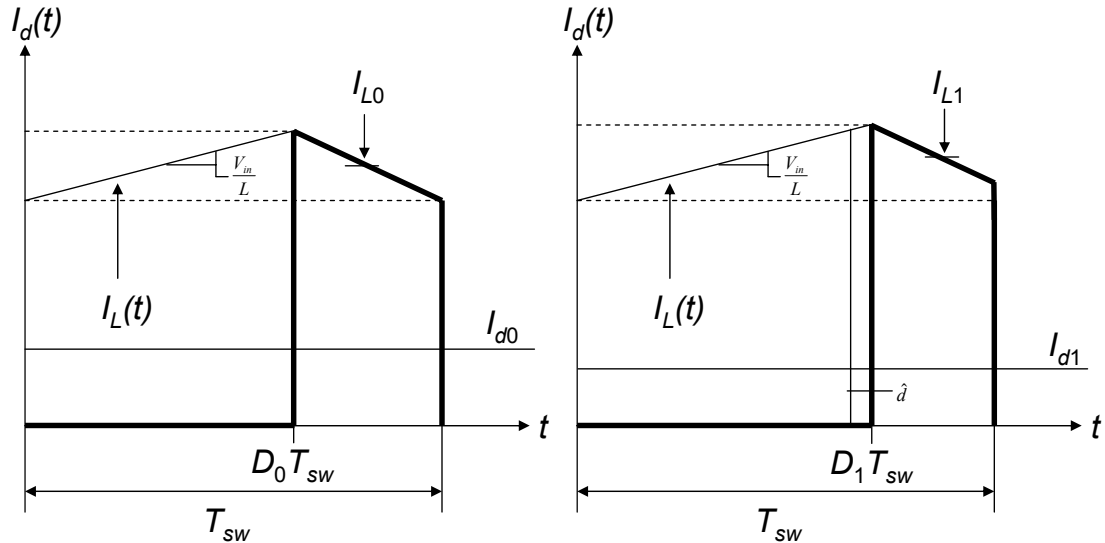
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{N(s)}{s} \quad \text{Pole for } s = 0: \text{ pole at the origin} \rightarrow \text{As } f \text{ increases the gain decreases with a -1 slope (-20 dB/decade)}$$

$$\arg \frac{V_{out}(s)}{V_{in}(s)} = \arg(1) - \arg\left(\frac{s}{0}\right) = -\arctan(\infty) = -\frac{\pi}{2} \quad \text{For } f > f_{po}$$

A pole at the origin introduces a fixed phase rotation of -90°

The Right Half-Plane Zero

□ In a CCM boost, I_{out} is delivered during the off time: $I_{out} = I_d = I_L(1-D)$

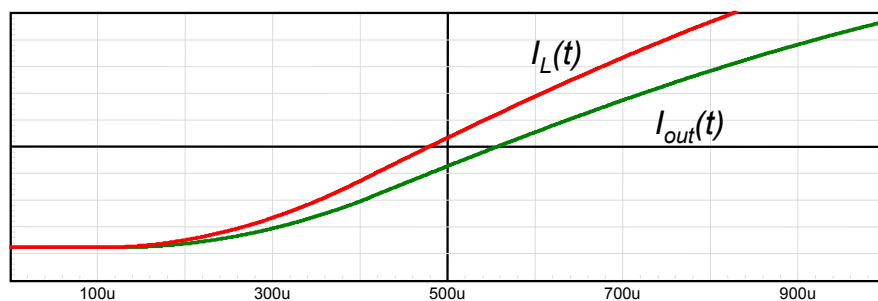
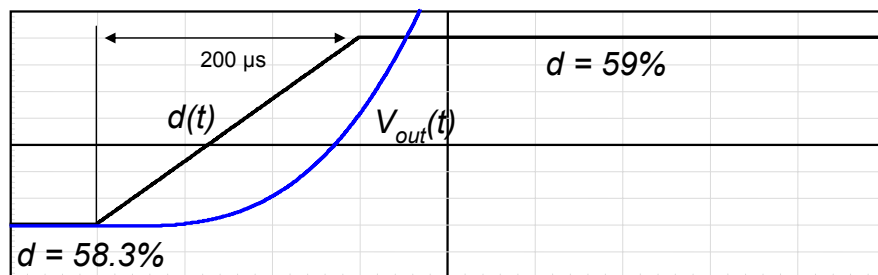


□ If D brutally increases, D' reduces and I_{out} drops!

□ What matters is the inductor current slew-rate $\rightarrow \frac{d\langle V_L \rangle(t)}{dt}$

The Right Half-Plane Zero

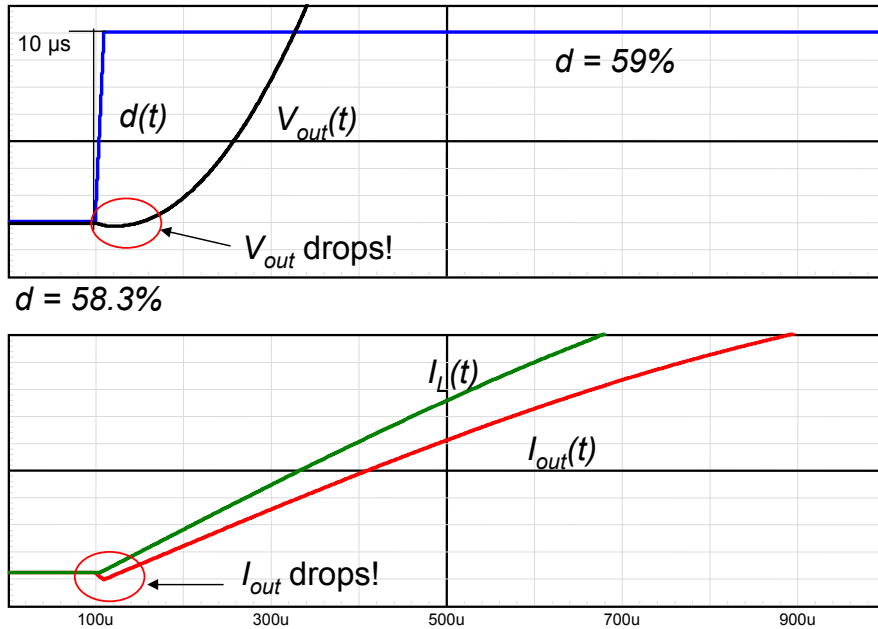
□ If $I_L(t)$ can rapidly change, I_{out} increases when D goes up





The Right Half-Plane Zero

- If $I_L(t)$ is limited because of a big L , I_{out} drops when D increases



The Right Half-Plane Zero

- Small-signal equations can help us to formalize it

$$I_{out} = I_L(1-D)$$

$$\hat{i}_{out} = \left(\frac{\partial I_{out}}{\partial I_L} \hat{i}_L \right)_D + \left(\frac{\partial I_{out}}{\partial D} \hat{d} \right)_{I_L} = \hat{i}_L(1-D) - \hat{d}I_L$$

The negative sign indicates a positive root!

$$\frac{\hat{i}_{out}(s)}{\hat{d}(s)} = \frac{V_{out}D'}{sL} \left(1 - \frac{sL}{D'^2 R_{load}} \right) = \frac{1 - \frac{s}{\omega_{z_2}}}{\frac{s}{\omega_0}}$$

$$\omega_0 = \frac{V_{out}D'}{L} \quad \omega_{z_2} = \frac{R_{load}D'^2}{L}$$

Voltage mode

$$\frac{\hat{i}_{out}(s)}{\hat{v}_c(s)} = \frac{D'}{R_{sense}} - \frac{sL}{D'R_{load}R_{sense}} = G_0 \left(1 - \frac{s}{\omega_{z_2}} \right)$$

$$\omega_{z_2} = \frac{R_{load}D'^2}{L}$$

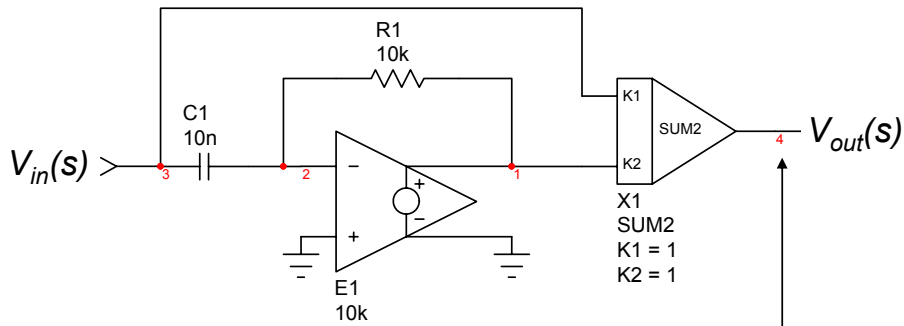
Current mode

- Voltage mode or current mode, the RHPZ remains the same



The Right Half-Plane Zero

- ❑ To limit the effects of the RHPZ, limit the duty ratio slew-rate
- ❑ Chose a cross over frequency equal to 20-30% of RHPZ position
- A simple RHPZ can be easily simulated:

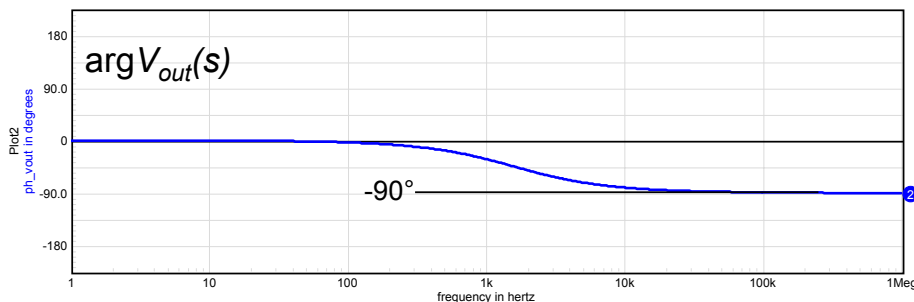
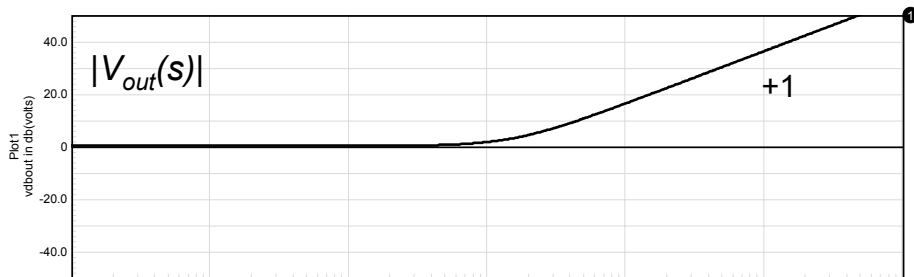


$$V(s) = V_{in}(s) - V_{in}(s) \frac{R_1}{1 + sC_1R_1} = V_{in}(s) \left(1 - \frac{s}{\omega_0} \right)$$



The Right-Half-Plane-Zero

- ❑ With a RHPZ we have a boost in gain but a lag in phase!



LHPZ

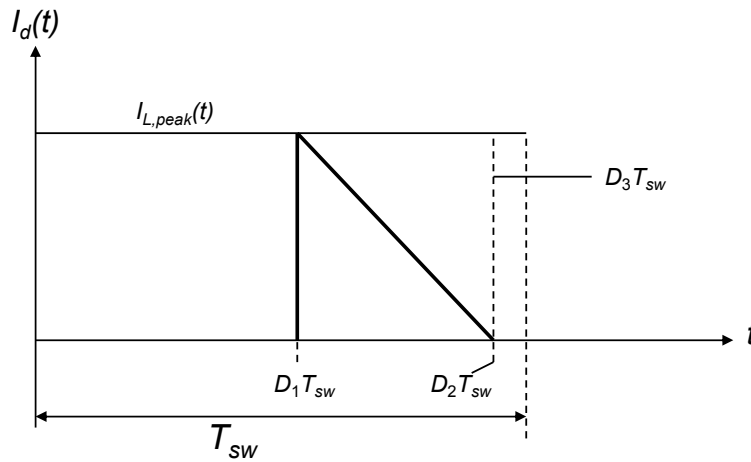
$$G(s) = 1 + \frac{s}{\omega_0}$$

RHPZ

$$G(s) = 1 - \frac{s}{\omega_0}$$

The Right-Half-Plane-Zero

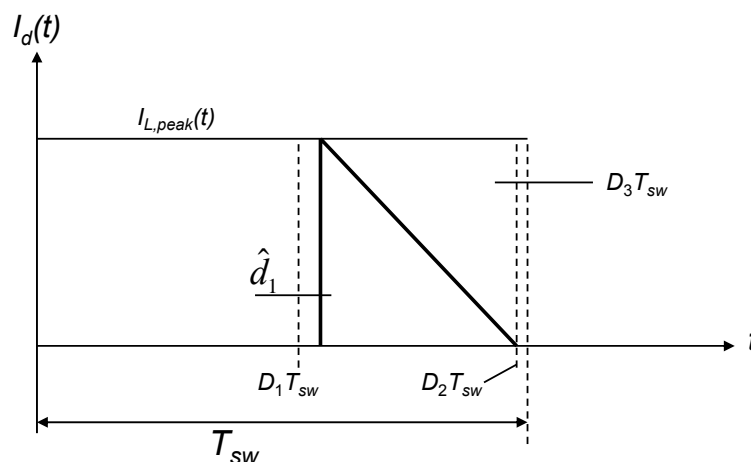
- A RHPZ also exists in DCM boost, buck-boost converters...



- When D_1 increases, $[D_1, D_2]$ stays constant but D_3 shrinks

The Right-Half-Plane-Zero

- The triangle is simply shifted to the right by \hat{d}_1

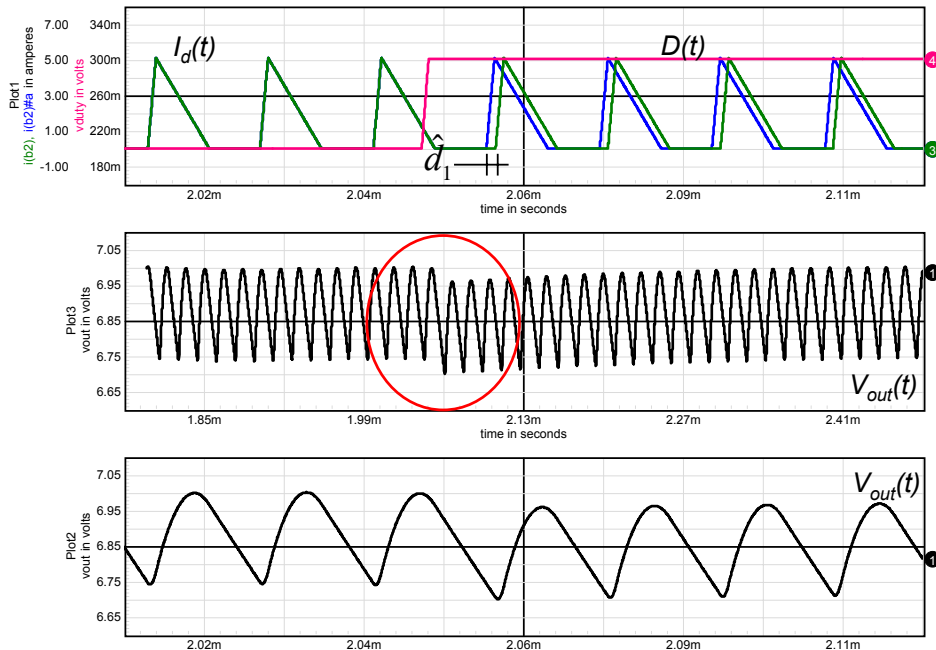


- The refueling time of the capacitor is delayed and a drop occurs



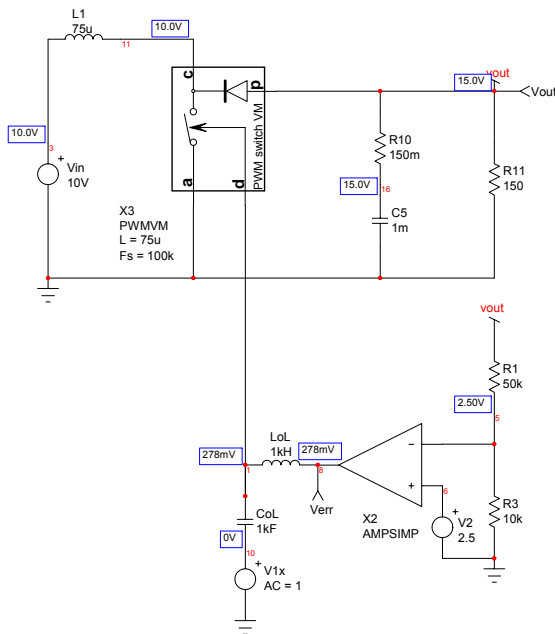
The Right-Half-Plane-Zero

□ If D increases, the diode current is delayed by \hat{d}_1



The Right-Half-Plane-Zero

□ Averaged models can predict the DCM RHPZ



$$\frac{\hat{v}_{out}(s)}{\hat{d}(s)} = H_d \frac{(1 + s/s_{z_1})(1 - s/s_{z_2})}{(1 + s/s_{p_1})(1 + s/s_{p_2})}$$

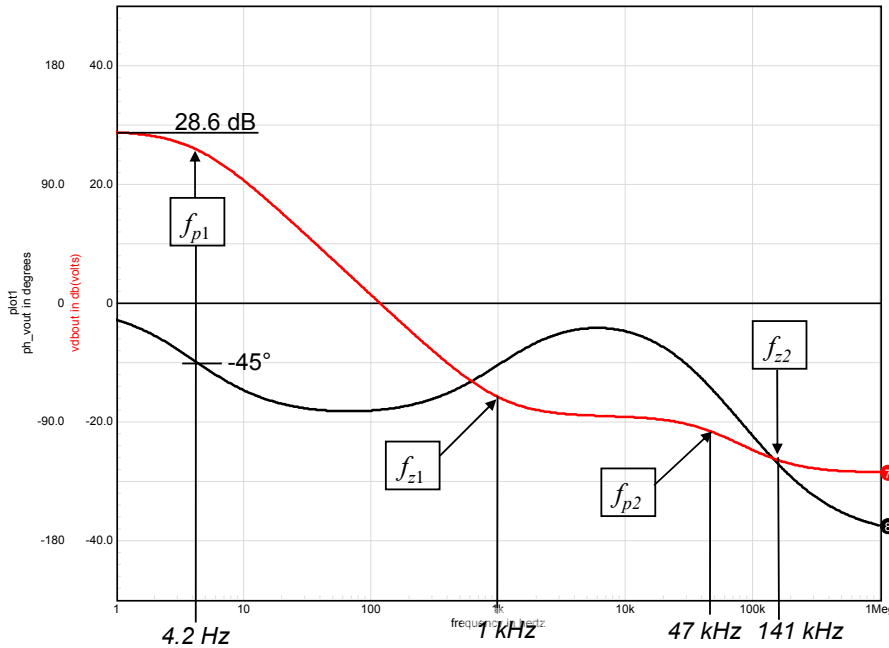
$$s_{z_1} = \frac{1}{C_{out} R_{ESR}} \quad s_{z_2} = \frac{R_{load}}{M^2 L}$$

$$s_{p_1} = \frac{2M-1}{M-1} \frac{1}{C_{out} R_{ESR}} \quad s_{p_2} = 2F_{sw} \left(\frac{1-1/M}{D} \right)^2$$

$$H_d = \frac{2V_{out}}{D} \frac{M-1}{2M-1}$$

The Right-Half-Plane-Zero

☐ Averaged models can predict the DCM RHPZ



$$H_d = 28.75 \text{ dB}$$

$$f_{z1} = 1.06 \text{ kHz}$$

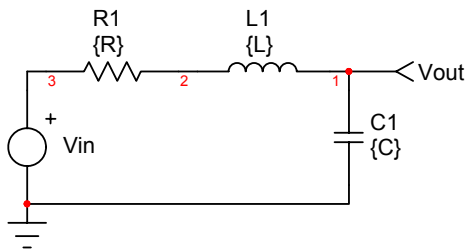
$$f_{z2} = 141 \text{ kHz}$$

$$f_{p1} = 4.2 \text{ Hz}$$

$$f_{p2} = 47.1 \text{ kHz}$$

How much margin? The RLC filter

☐ let us study an RLC low-pass filter, a 2nd order system



$$T(s) = \frac{1}{LCs^2 + RCs + 1}$$

$$T(s) = \frac{1}{\frac{s^2}{\omega_r^2} + 2\zeta \frac{s}{\omega_r} + 1} = \frac{1}{\frac{s^2}{\omega_r^2} + \frac{s}{\omega_r Q} + 1}$$

parameters

$$f_0 = 235 \text{ k}$$

$$L = 10 \mu$$

$$C = 1 / (4 * 3.14159^2 * f_0^2 * L)$$

$$\omega_0 = (\{L\} * \{C\})^{-0.5}$$

$$Q = 10$$

$$R = 1 / (((\{C\}) / (4 * \{L\}))^{0.5} * 2 * \{Q\})$$

Change Q and run the simulation

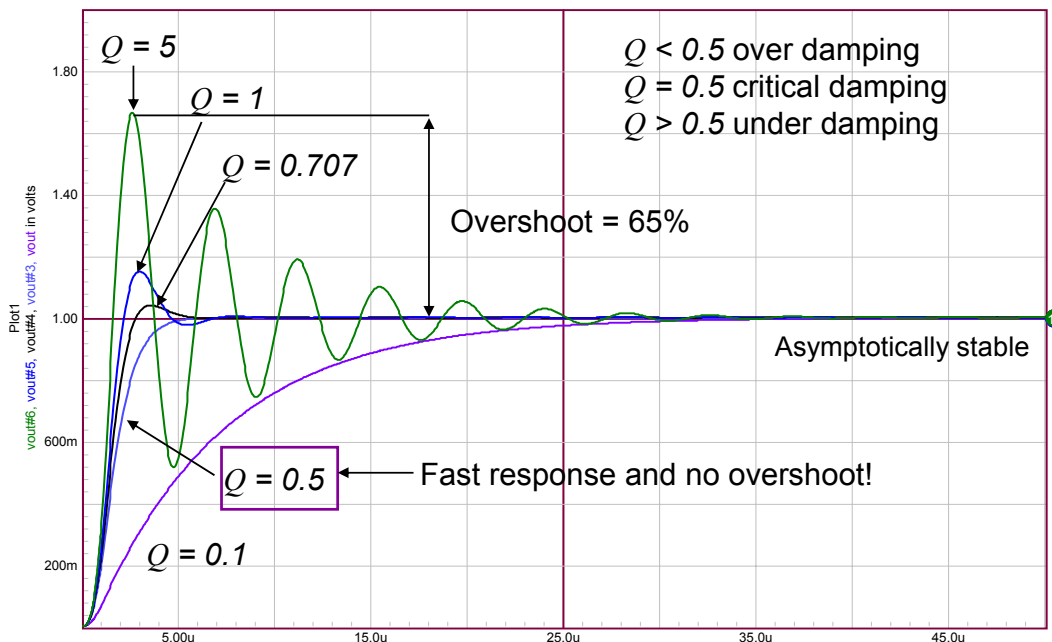
$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\zeta = R \sqrt{\frac{C}{4L}} \quad Q = \frac{1}{2\zeta}$$

↓
zeta

The RLC response to an input step

changing Q affects the transient response



The RLC response to an input step

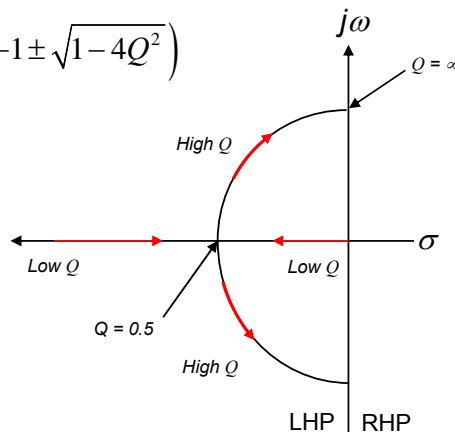
Q affects the poles position

$$T(s) = \frac{1}{\frac{s^2}{\omega_r^2} + \frac{s}{\omega_r Q} + 1} = \frac{N(s)}{D(s)}$$

\longrightarrow Solve $N(s) = 0$ to obtain zeros
 \longrightarrow Solve $D(s) = 0$ to obtain poles

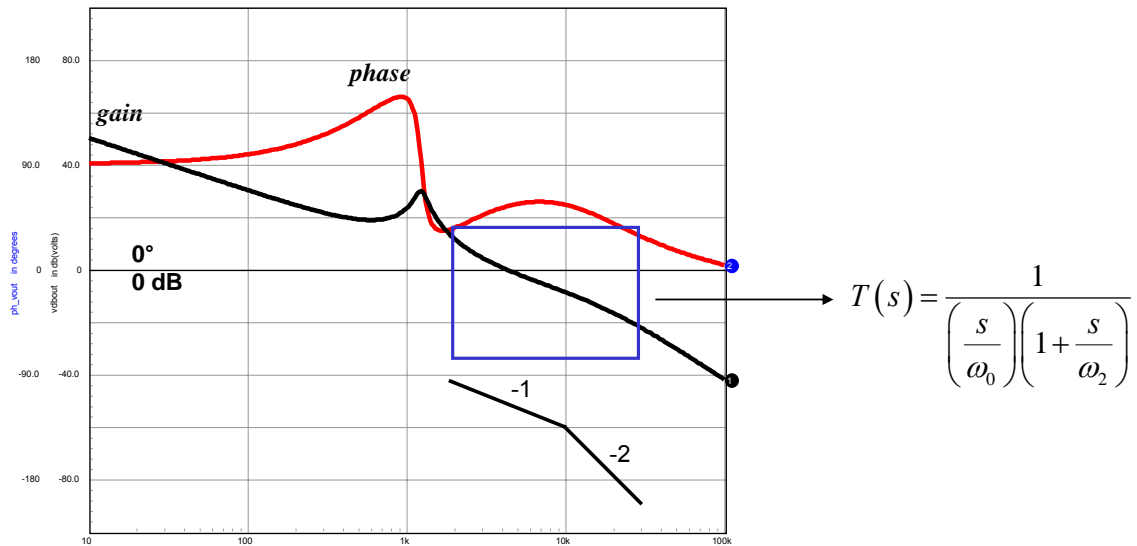
$$\frac{s^2}{\omega_r^2} + \frac{s}{\omega_r Q} + 1 = 0 \longrightarrow 2 \text{ roots, } s_1 \text{ and } s_2 = \frac{\omega_r}{2Q} \left(-1 \pm \sqrt{1 - 4Q^2} \right)$$

- $Q < 0.5$, two real negatives roots
- $Q = 0.5$, two real coincident negative roots
- $Q > 0.5$, two complex roots
- $Q = \infty$, two imaginary conjugate roots



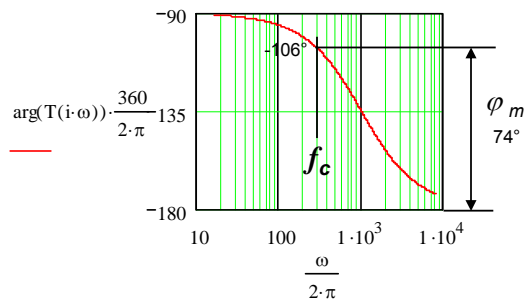
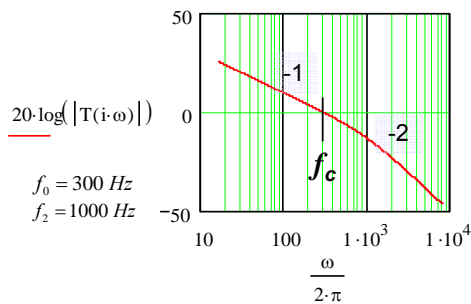
Where is the analogy with $T(s)$?

- in the vicinity of the crossover point, $T(s)$ combines:
 - one pole at the origin, ω_0
 - one high frequency pole, ω_2



Closed-loop gain study

- Linking the open-loop phase margin to the closed-loop Q



$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

Closed loop →

$$\frac{T(s)}{1 + T(s)} = \frac{1}{\frac{s^2}{\omega_0\omega_2} + \frac{s}{\omega_0} + 1}$$

$$\frac{1}{\frac{s^2}{\omega_0\omega_2} + \frac{s}{\omega_0} + 1} = \frac{1}{\frac{s^2}{\omega_r^2} + \frac{s}{\omega_r Q} + 1}$$

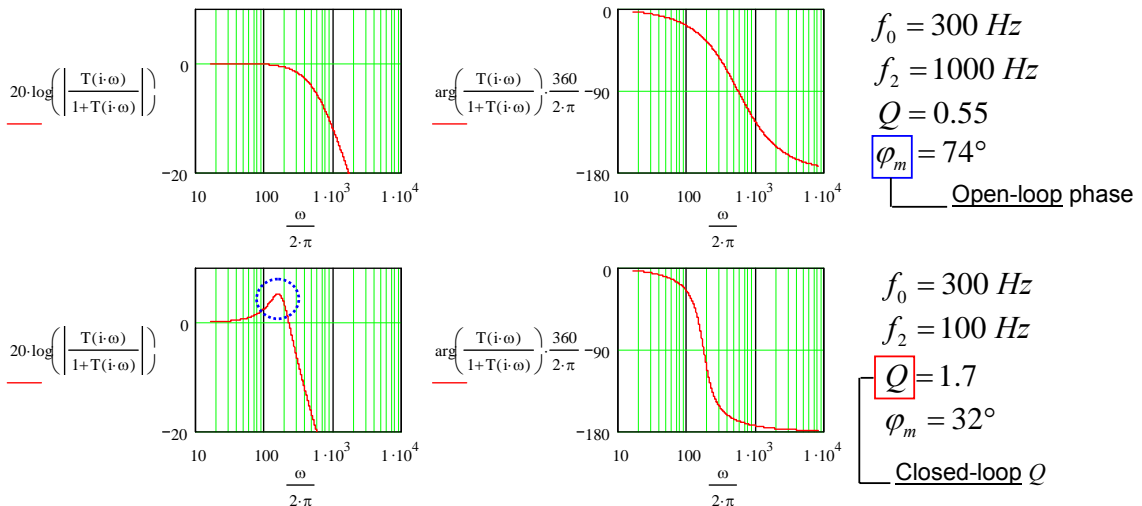
solve → $Q = \sqrt{\frac{\omega_0}{\omega_2}}$ $\omega_r = \sqrt{\omega_0\omega_2}$

Closed-loop gain study

□ if we plot the closed-loop expression:

- $\omega_2 > \omega_0$, low Q , no peaking
- $\omega_2 < \omega_0$, high Q , peaking

$$\frac{T(s)}{1+T(s)} = \frac{1}{\frac{s^2}{\omega_r^2} + \frac{s}{\omega_r Q} + 1}$$



Linking φ_m and Q

- an open-loop phase margin leads to a closed-loop quality coeff. Q
- we have seen that Q affects the transient response (RLC filter)
- let us link the phase margin to the quality coefficient:

1. calculate the crossover frequency for which $|T(s)| = 1$

$$\left| \frac{1}{\left(\frac{j\omega_c}{\omega_0}\right)\left(1 + \frac{j\omega_c}{\omega_2}\right)} \right| = 1 \longrightarrow \omega_c = \frac{\omega_2 \sqrt{\left(\sqrt{1+4Q^4} - 1\right)}}{\sqrt{2}}$$

2. substitute ω_c into $T(s)$, calculate its argument (phase margin)

$$\arg T(s)@f_c = \arctan\left(\frac{2}{\sqrt{\sqrt{1+4Q^4} - 1}}\right)$$

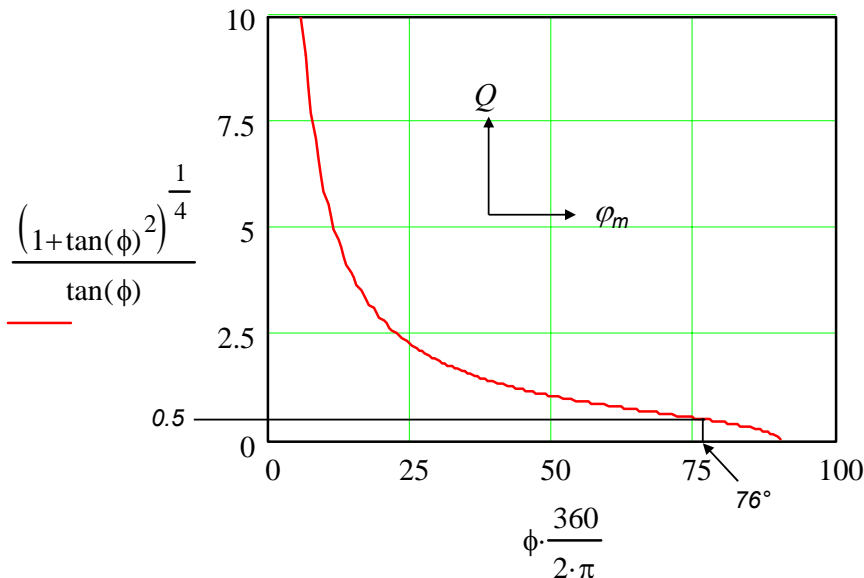
3. extract the quality coefficient Q :

$$Q = \frac{\sqrt[4]{1 + \tan^2(\varphi_m)}}{\tan(\varphi_m)} = \frac{\sqrt{\cos(\varphi_m)}}{\sin(\varphi_m)}$$



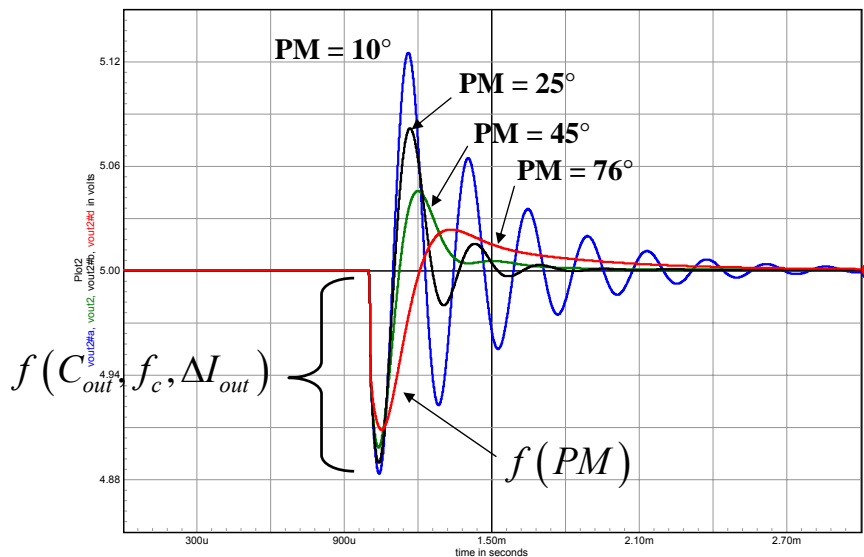
We can now plot Q versus φ_m

- ❑ a Q factor of 0.5 (critical response) implies a φ_m of 76°
- ❑ a 45° φ_m corresponds to a Q of 1.2: oscillatory response!



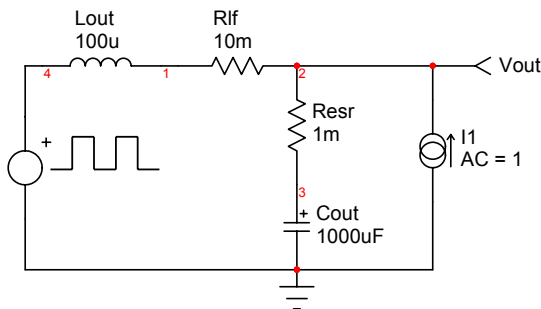
Summary on the design criteria

- ❑ compensate the open-loop gain for a phase margin of 70°
- ❑ make sure the open-loop gain margin is better than 15 dB
- ❑ do not accept a phase margin lower than 45° in worst case



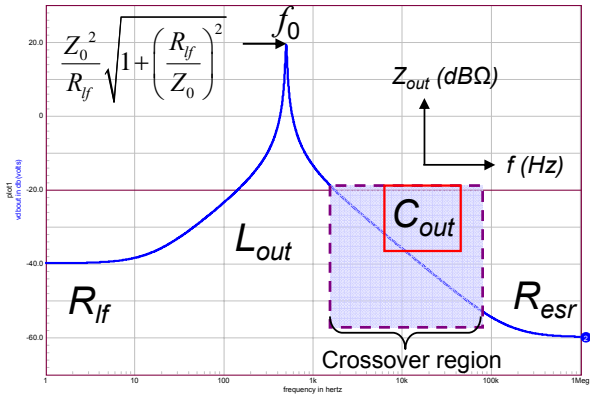
Dc-dc output impedance

- ❑ A dc-dc conv. combines an inductor and a capacitor
- ❑ As f is swept, different elements dominate Z_{out}



A buck equivalent circuit

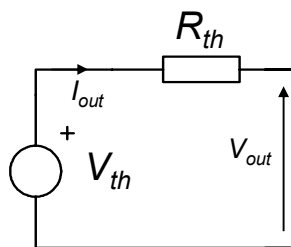
$$Z_{out} = (sL_{out} + r_{Lf}) \parallel \left(R_{esr} + \frac{1}{sC_{out}} \right)$$



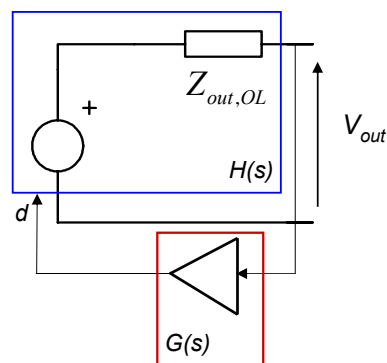
To avoid stability issues,
 $f_c \gg f_0$

Closing the loop...

- ❑ Any circuit can be represented by its Thévenin model
- ❑ At high frequency, C_{out} impedance dominates
- ❑ Once in closed-loop, Z_{out} goes down as $T(s)$ is high



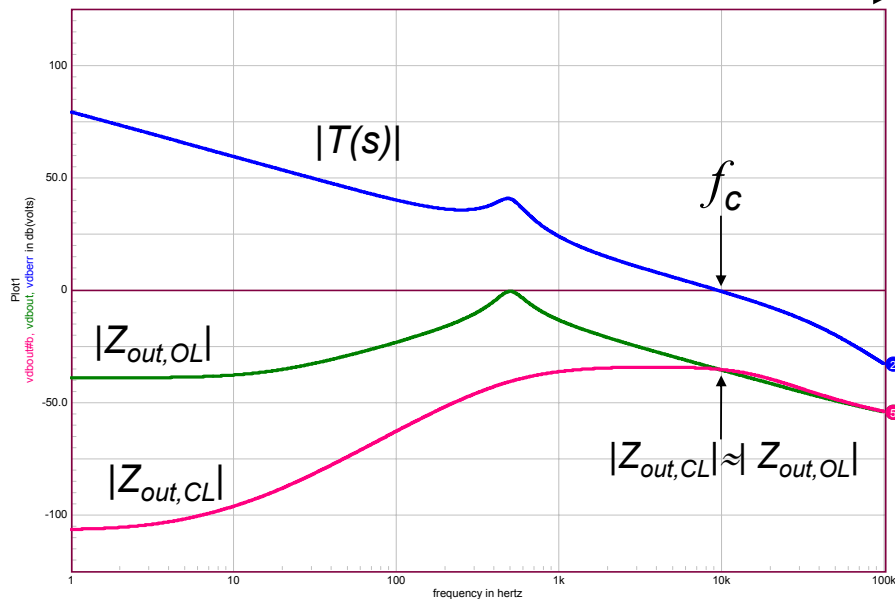
$$Z_{out,OL} = \frac{1}{2\pi C_{out} f}$$



$$|Z_{out,CL}| = |Z_{out,OL}| \left| \frac{1}{1+T(s)} \right|$$

Closing the loop...

- At the crossover frequency $Z_{out,CL} \approx Z_{out,OL}$ Let's assess « almost » :-)



Calculating the output impedance

- closed-loop output impedance is dominated by C_{out}

$$|Z_{out,CL}| \approx \frac{1}{2\pi f_c C_{out}} \left| \frac{1}{1+T(s)} \right|$$

- we have calculated the crossover value, $|T(s)| = 1$

$$\omega_c = \frac{\omega_2 \sqrt{(\sqrt{1+4Q^4} - 1)}}{\sqrt{2}}$$

- substitute into $1/(1+T(s))$ and extract module

$$\left| \frac{1}{1+T(s)} \right| = \frac{1}{\sqrt{\left(\frac{-\omega_0}{\omega_2 \left(1 + \frac{\omega^2}{\omega_2^2} \right)} + 1 \right)^2 + \frac{\omega_0^2}{\omega^2 \left(1 + \frac{\omega^2}{\omega_2^2} \right)^2}}$$

Calculating the output impedance

- Introduce the quality factor coefficient

$$Q = \sqrt{\frac{\omega_0}{\omega_2}} \quad \left| \frac{1}{1+T(s)} \right| = \frac{1}{2Q \sqrt{\frac{2Q^2 + 1 - \sqrt{1+4Q^4}}{(1+\sqrt{1+4Q^4})(\sqrt{1+4Q^4}-1)\omega_2^2}}}$$

- Now replace Q by its definition $Q = \frac{\sqrt{\cos(\varphi_m)}}{\sin(\varphi_m)}$

$$\left| \frac{1}{1+T(s)} \right| = \frac{1}{\sqrt{\frac{1}{\cos(\varphi_m)} \left(2\cos(\varphi_m) + \left[\frac{1+\cos^2(\varphi_m)}{\sin^2(\varphi_m)} \right] [\cos^2(\varphi_m)-1] + 1 - \cos^2(\varphi_m) \right)}}$$

- Simplify $\longrightarrow \left| \frac{1}{1+T(s)} \right| = \frac{1}{\sqrt{2-2\cos(\varphi_m)}}$

An example with a buck

- Let's assume an output capacitor of 1 mF
- The spec states a 80-mV undershoot for a 2-A step
- How to select the crossover frequency?

$$\Delta V_{out} \approx \frac{\Delta I_{out}}{2\pi f_c C_{out}} \longrightarrow f_c \approx \frac{\Delta I_{out}}{\Delta V_{out} C_{out} 2\pi}$$

$$f_c \approx \frac{2}{80m \times 1m \times 2\pi} = 4 \text{ kHz} \quad Z_{C_{out}} @ 4 \text{ kHz} = \frac{1}{2\pi \times 4k \times 1m} = 40 \text{ m}\Omega$$

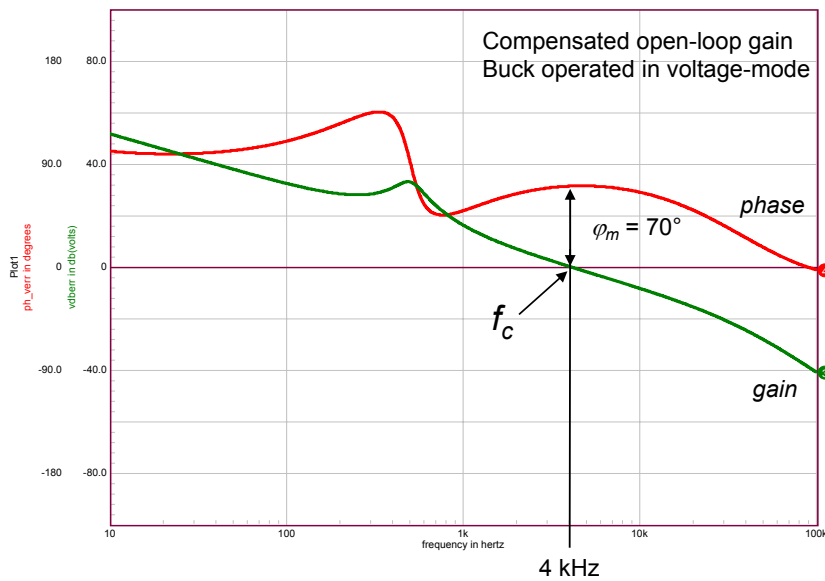


Select a 1000-μF capacitor featuring less than 40-mΩ ESR

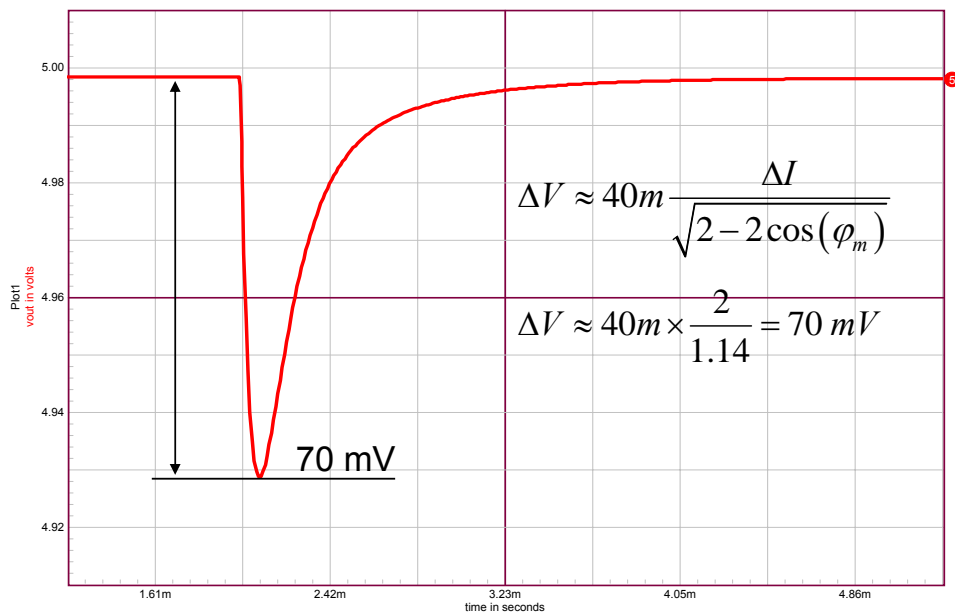


Setting the right crossover frequency

- Compensate the converter for a 4-kHz f_c

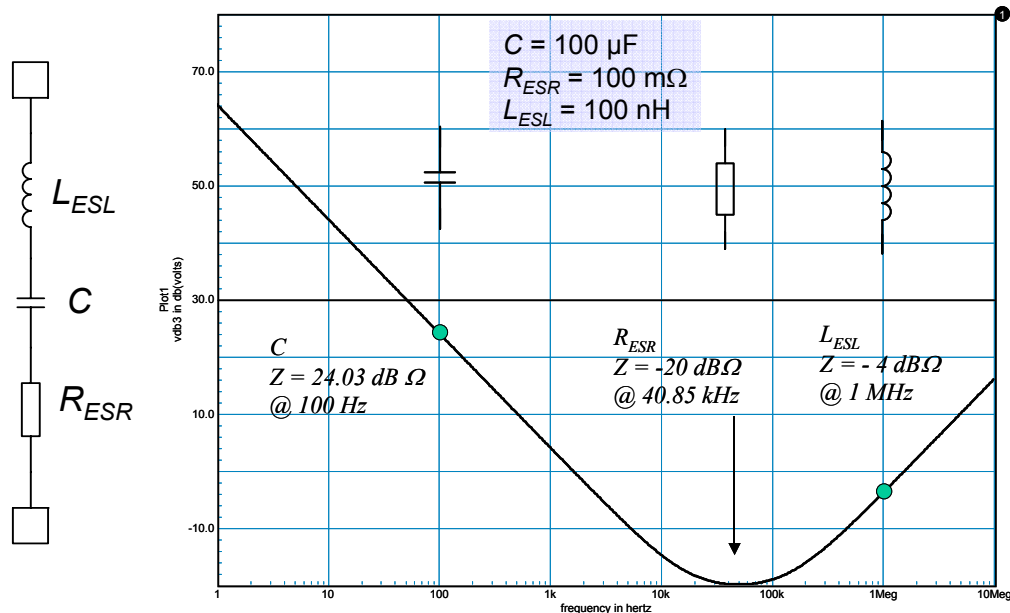


Measure the obtained undershoot



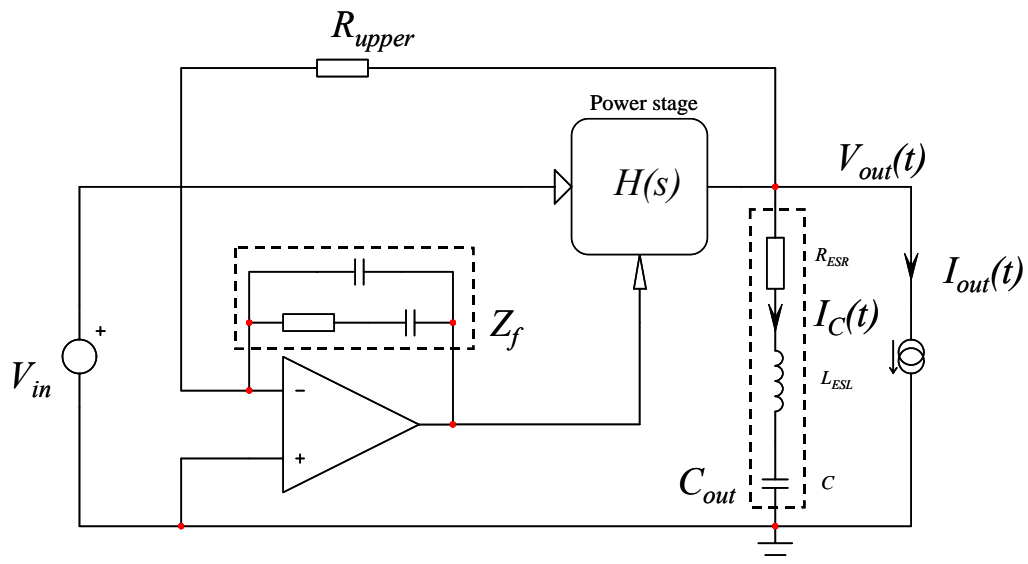
Is my capacitor a real capacitor?

- ❑ A capacitor is made of parasitic elements



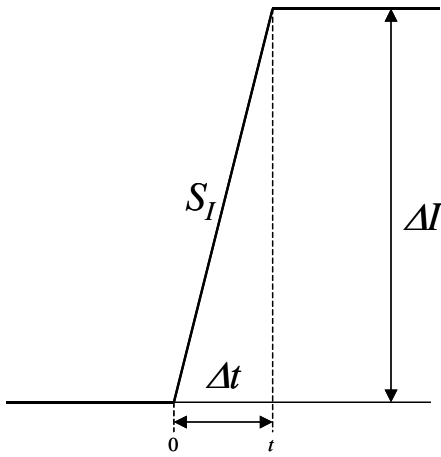
How these elements affect the undershoot?

- ❑ The output current slope affects the undershoot
- ❑ If slope is steep, stray elements dominate the answer

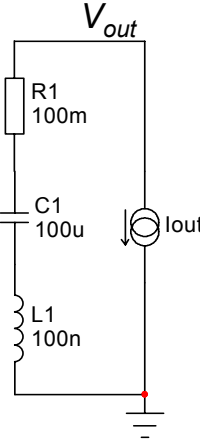


How these elements affect the undershoot?

□ Because of bandwidth limits, R_{ESR} and L_{ESL} play alone



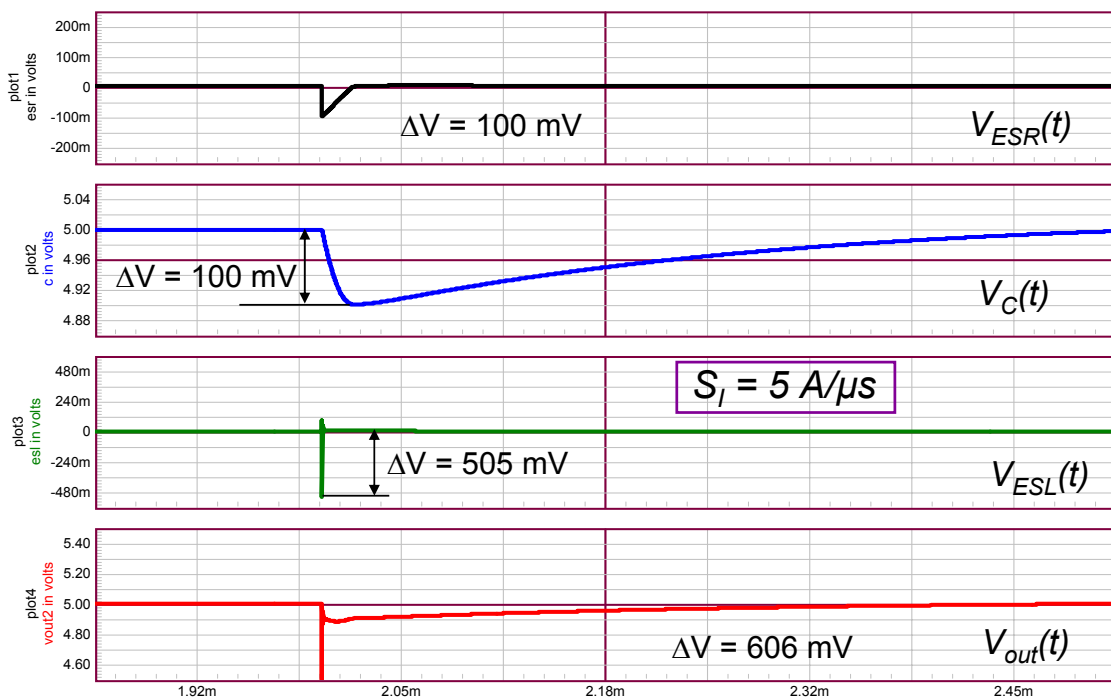
$$I_{out}(t) = \Delta I \frac{t}{\Delta t} = S_I t$$



$$\Delta V_{out} = R_{ESR} S_I t + L_{ESL} S_I + \frac{1}{C} \int_0^t S_I t \cdot dt$$

$$\Delta V_{out} = R_{ESR} S_I t + L_{ESL} S_I + \frac{S_I t^2}{2C}$$

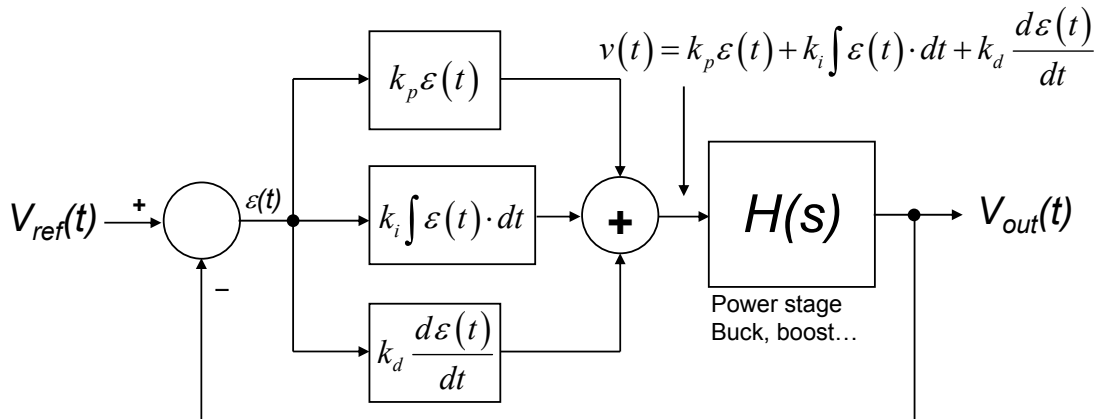
The capacitor contribution is small...





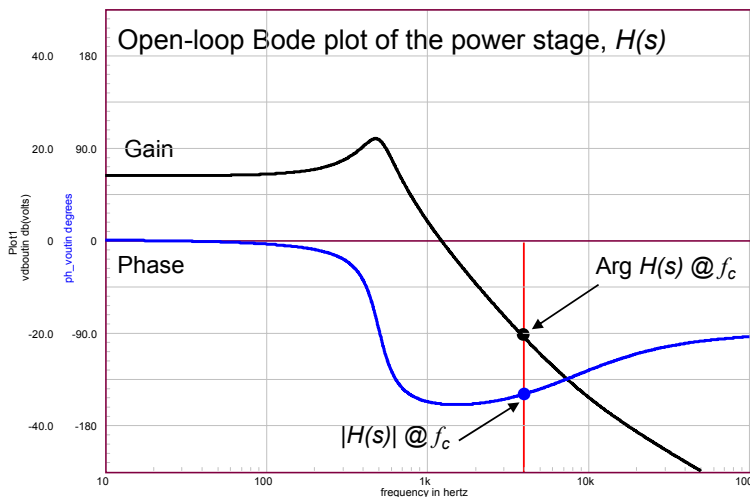
Compensating the converter

- ❑ Fix the current error with a **proportional** term (P)
 - The proportional gain gives fast reaction time but also overshoot
- ❑ Fix the long-term static error with an **integral** term (I)
 - The integral term cancels the static error but slows down the response
- ❑ Fix the immediate error by observing the slope with a **derivative** term (D)
 - The derivative term decreases overshoot but slows down the response



How do we stabilize the converter?

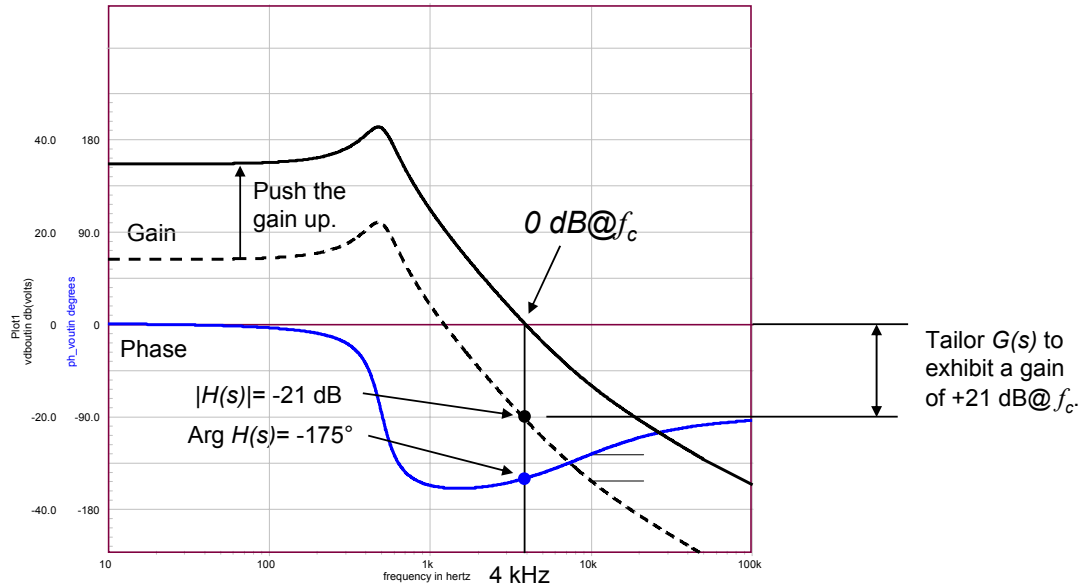
1. Select the crossover frequency f_c (assume 4 kHz)
2. Provide a high dc gain for a low static error and good **input rejection**
3. Shoot for a 70° phase margin at f_c
4. Evaluate the needed phase boost at f_c to meet (3)
5. Shape the $G(s)$ path to comply with 1, 2 and 3



$$A_{sc,CL}(s) = \frac{A_{sc,OL}(s)}{1 + T(s)}$$

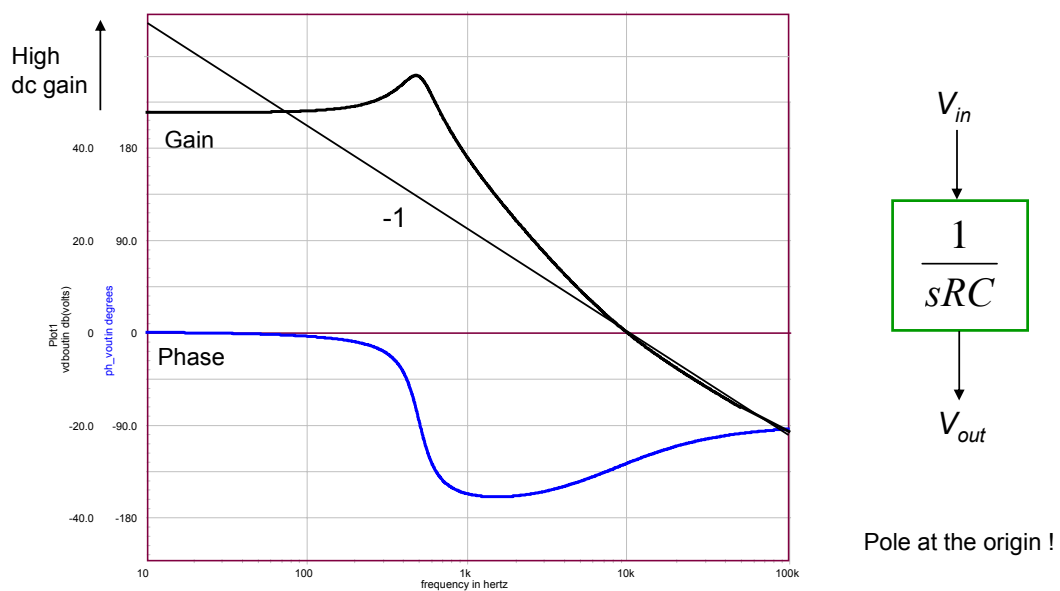
First, provide mid-band gain at crossover

1. Adjust $G(s)$ to boost the gain by +21 dB at crossover



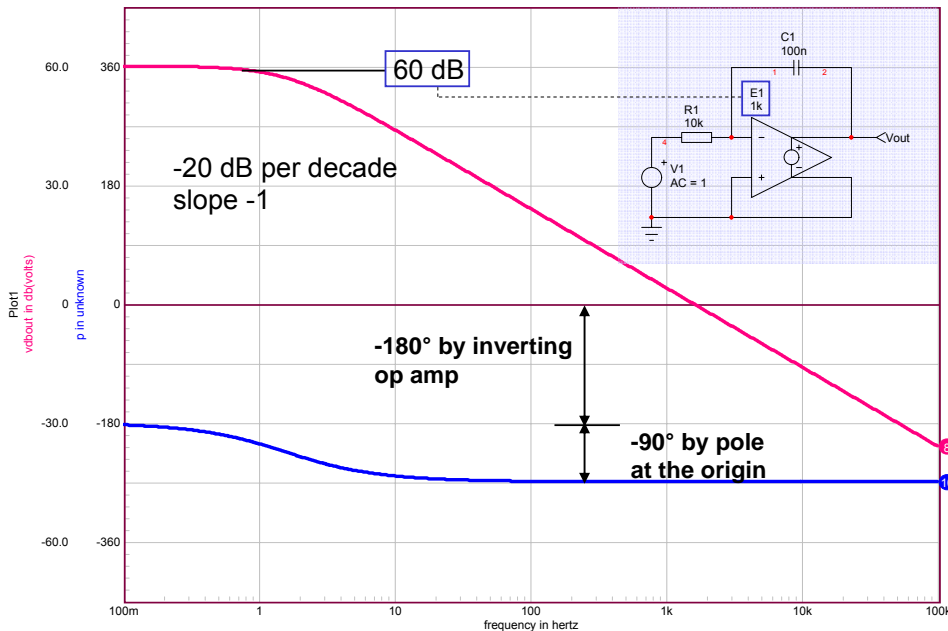
Second, provide high gain in dc

2. High dc gain lowers static error and brings good input rejection



Second, provide high gain in dc

2. An integrator provides a high dc gain but rotates by -270°



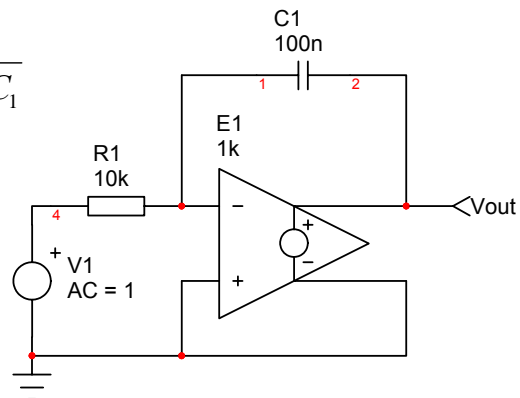
Second, provide high gain in dc

2. An integrator provides a high dc gain but rotates by -270°

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{sR_1C_1} = \frac{1}{s} \quad \omega_{po} = \frac{1}{R_1C_1}$$

$$\arg G(j\omega) = -\arg\left(\frac{j\omega}{\omega_{po}}\right)$$

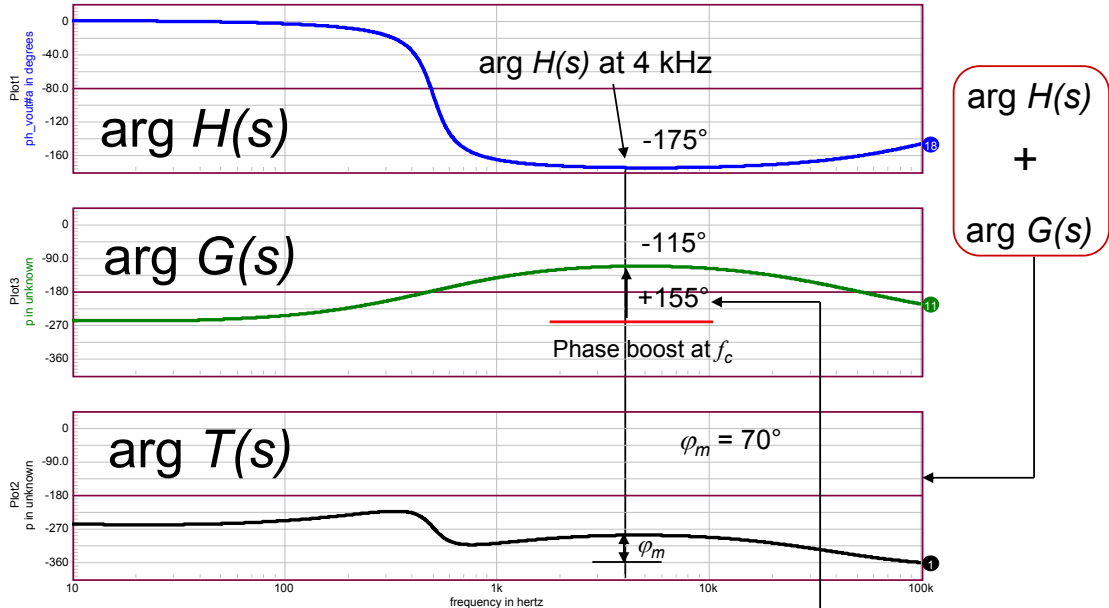
$$\lim_{s \rightarrow \infty} \arg G(s) = \lim_{\omega \rightarrow \infty} -\arctan\left(\frac{\omega}{0}\right) = -\frac{\pi}{2}$$



→ Total phase lag brought by an origin pole is $-3\pi/2$ or -270°

All compensators (1, 2 or 3) feature an origin pole

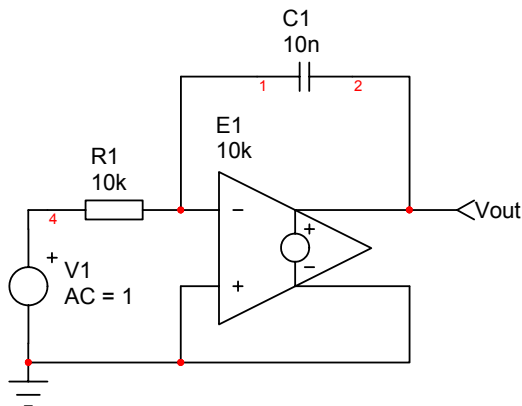
Third, evaluate the phase boost at f_c



$$\begin{aligned} \arg H(f_c) - 270^\circ + \text{BOOST} - \phi_m &= -360^\circ \\ \text{BOOST} &= \phi_m - \arg H(f_c) - 90^\circ = 70^\circ + 175^\circ - 90^\circ = 155^\circ \end{aligned}$$

How do we boost the phase at f_c ?

- The type 1 configuration
- No phase boost, pure integral term
- Permanent phase lag of -270°
- Ok if $\arg H(f_c) < -45^\circ$ for a ϕ_m of 45°



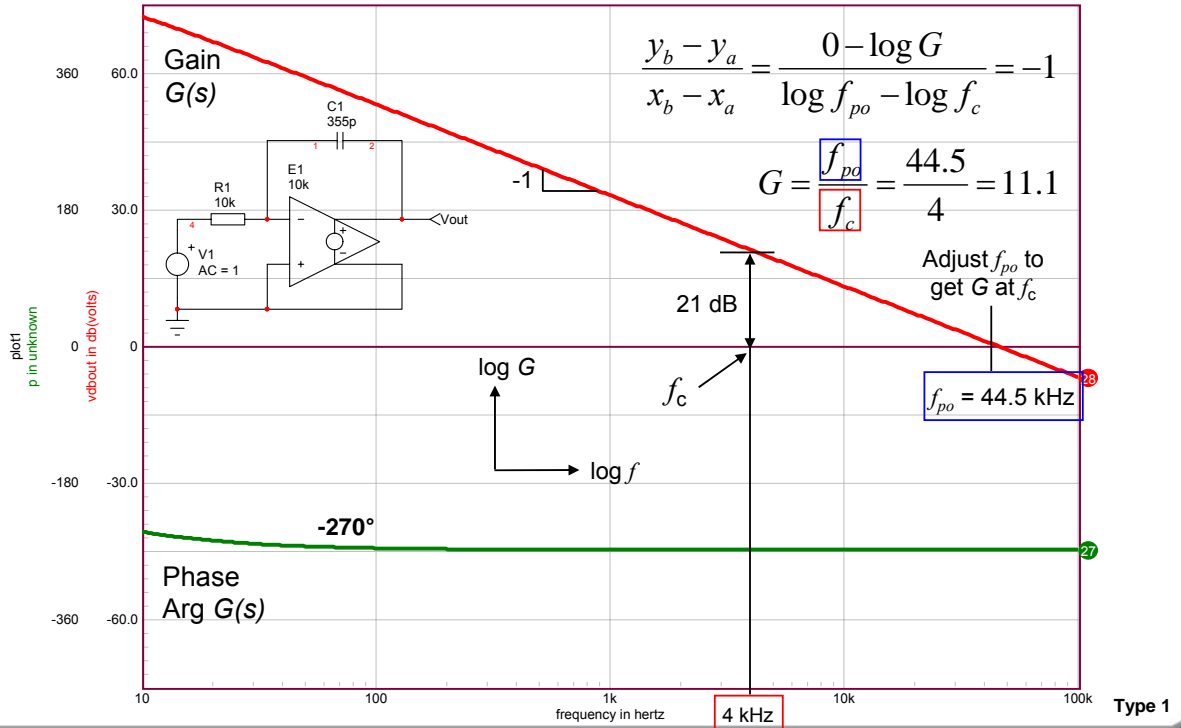
1 pole at the origin

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{sR_1C_1} = \frac{1}{s} \frac{1}{\omega_{po}}$$

$$\omega_{po} = \frac{1}{R_1C_1}$$

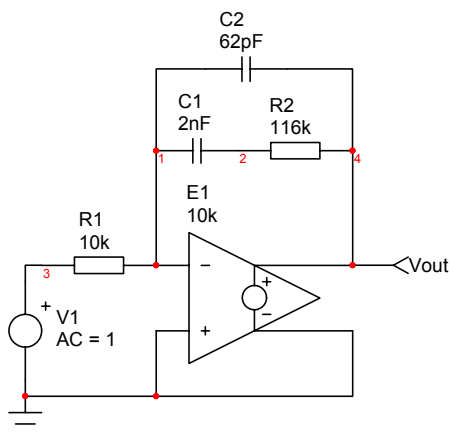
$$G = \frac{f_{po}}{f_c} \quad \text{Select } f_{po} \text{ to have } G \text{ at } f_c$$

How do we boost the phase at f_c ?



How do we boost the phase at f_c ?

- The type 2 configuration
- Phase boost up to 90°
- Ok if $\arg H(f_c) < -90^\circ$



$$G(s) = - \frac{1 + sR_2C_1}{sR_1(C_1 + C_2) \left(1 + sR_2 \left[\frac{C_1C_2}{C_1 + C_2} \right] \right)}$$

If $C_2 \ll C_1$

$$\omega_{po} = \frac{1}{R_1C_1} \quad \omega_{p1} = \frac{1}{R_2C_2} \quad \omega_{z1} = \frac{1}{R_2C_1}$$

1 pole at the origin

1 zero

1 pole



Pole/zero placement and boost at f_c

□ Phase boost appears between the zero and the pole

$$G(j\omega) = \frac{\left(1 + j \frac{\omega}{\omega_{z1}}\right)}{\left(1 + j \frac{\omega}{\omega_{p1}}\right)} \quad \arg G(j\omega) = \text{boost} = \arg \left(\frac{1 + j \frac{\omega}{\omega_{z1}}}{1 + j \frac{\omega}{\omega_{p1}}} \right) \quad G = a + jb$$

$$\arg G = \arctan\left(\frac{b}{a}\right)$$

$$\arg G(f) = \arctan\left(\frac{f}{f_{z1}}\right) - \arctan\left(\frac{f}{f_{p1}}\right) \xrightarrow{\text{Peaks to a max at}} f = \sqrt{f_{z1}f_{p1}}$$

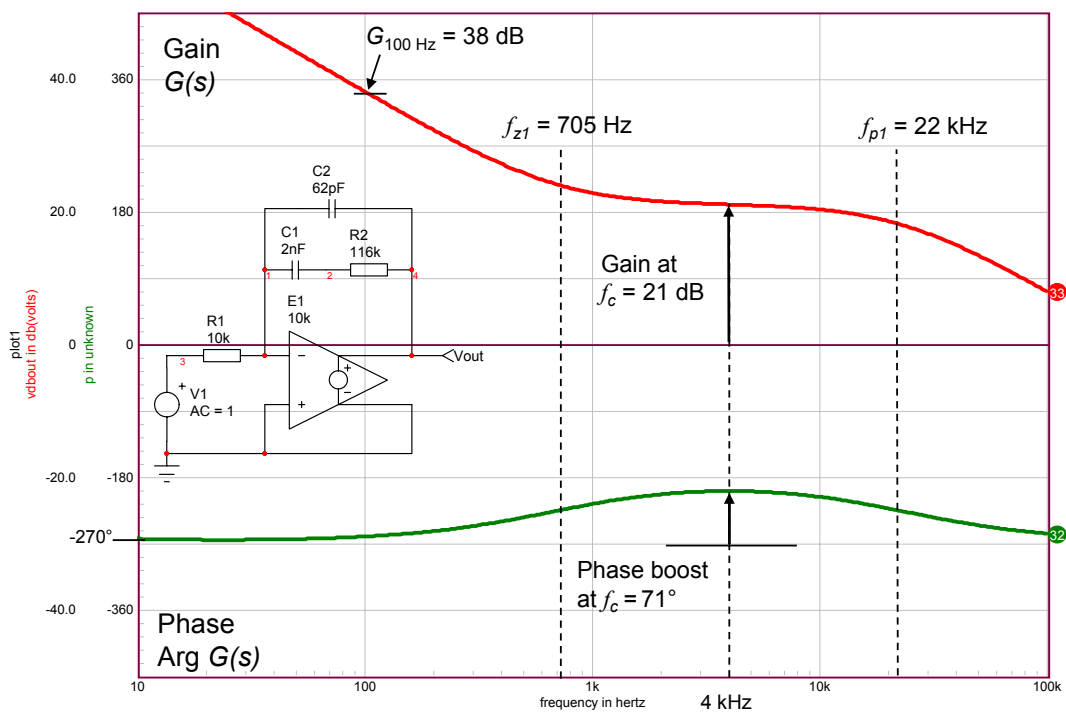
Assume 1 zero placed at 705 Hz, 1 pole at 22 kHz and a 4-kHz crossover:

$$\arg G(4 \text{ kHz}) = \arctan\left(\frac{4k}{705}\right) - \arctan\left(\frac{4k}{22k}\right) = 80 - 10.3 \approx 70^\circ$$

Type 2



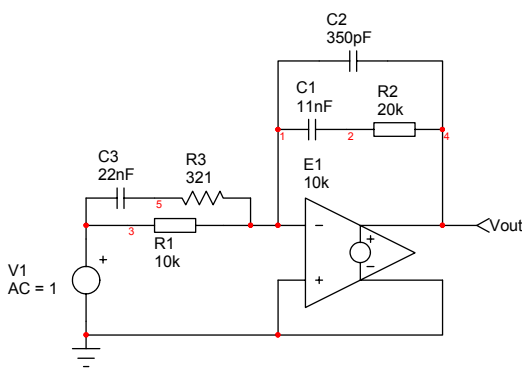
How do we boost the phase at f_c ?



Type 2

How do we boost the phase at f_c ?

- The type 3 configuration
- Phase boost up to 180°
- Ok if $\arg H(f_c) < -180^\circ$



$$G(s) = - \frac{sR_2C_1 + 1}{sR_1(C_1 + C_2)} \frac{sC_3(R_1 + R_3) + 1}{\left(1 + sR_2 \frac{C_1C_2}{C_1 + C_2}\right) (sR_3C_3 + 1)}$$

If $C_2 \ll C_1$ and $R_3 \ll R_1$

$$\omega_{z1} = \frac{1}{R_2C_1} \quad \omega_{z2} = \frac{1}{R_1C_3} \quad \omega_{po} = \frac{1}{R_1C_1}$$

$$\omega_{p1} = \frac{1}{R_3C_3} \quad \omega_{p2} = \frac{1}{R_2C_2}$$

1 pole at the origin

2 zeros

2 poles

Type 3

Pole/zero placement and boost at f_c

- Phase boost appears between the zero and the pole

$$G(j\omega) = \frac{\left(1 + j \frac{\omega}{\omega_{z1}}\right) \left(1 + j \frac{\omega}{\omega_{z2}}\right)}{\left(1 + j \frac{\omega}{\omega_{p1}}\right) \left(1 + j \frac{\omega}{\omega_{p2}}\right)} \quad \arg G(j\omega) = \text{boost} = \arg \frac{\left(1 + j \frac{\omega}{\omega_{z1}}\right) \left(1 + j \frac{\omega}{\omega_{z2}}\right)}{\left(1 + j \frac{\omega}{\omega_{p1}}\right) \left(1 + j \frac{\omega}{\omega_{p2}}\right)}$$

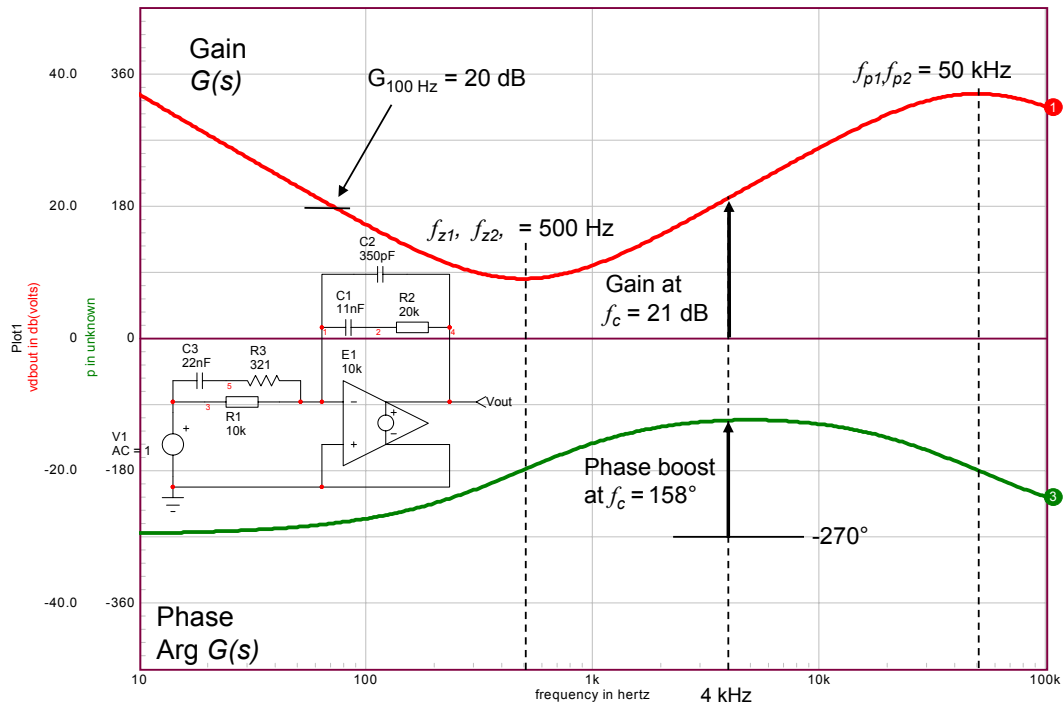
$$\arg G(f) = \arctan\left(\frac{f}{f_{z1}}\right) + \arctan\left(\frac{f}{f_{z2}}\right) - \arctan\left(\frac{f}{f_{p1}}\right) - \arctan\left(\frac{f}{f_{p2}}\right)$$

Assume 2 zeros placed at 500 Hz, 2 poles at 50 kHz and a 4-kHz crossover:

$$\arg G(4 \text{ kHz}) = 2 \arctan\left(\frac{4k}{500}\right) - 2 \arctan\left(\frac{4k}{50k}\right) = 166 - 4.6 \approx 161^\circ$$

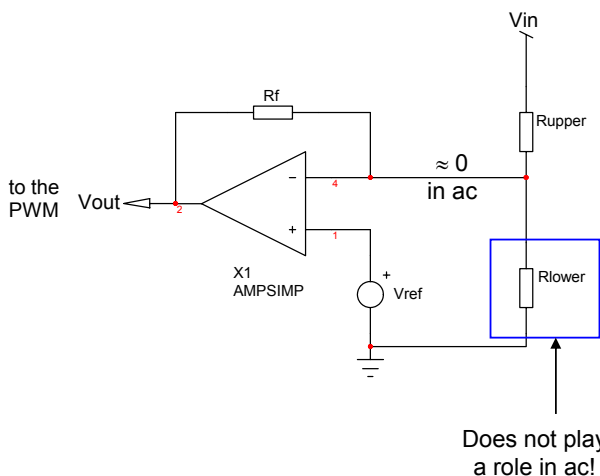
Type 3

How do we boost the phase at f_c ?



Why R_{lower} never appears in the ac functions?

- Because of the virtual ground, R_{lower} disappears in ac!



dc equation

$$V_{out} = V_{ref} \left(\frac{R_f}{R_{upper} \parallel R_{lower}} + 1 \right) - \frac{R_f}{R_{upper}} V_{in}$$

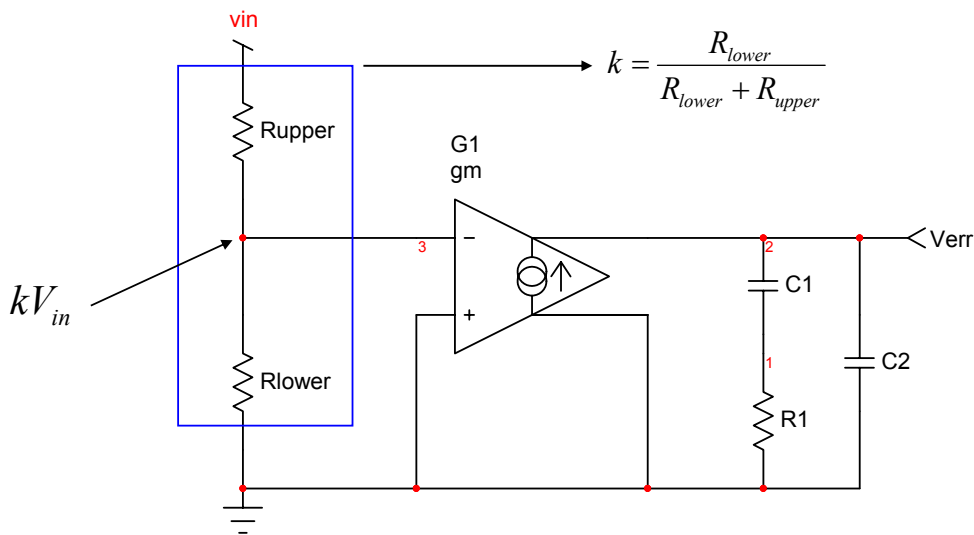
ac equation

$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_f}{R_{upper}}$$

➔ As long as a virtual ground exists, the division ratio plays no role!

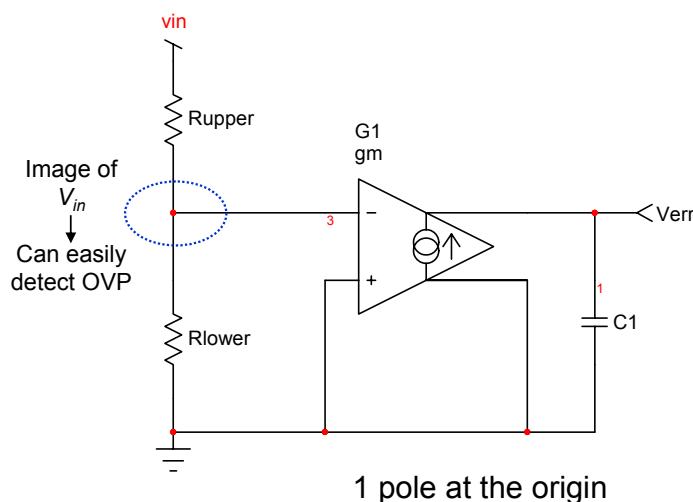
In absence of virtual ground, R_{lower} comes back

- In an OTA-based circuit, there is no virtual ground
- R_{lower} now affects $G(s)$



Boosting the phase at f_c with OTAs

- The type 1 configuration with an OTA
- OTA types are popular in PFC circuits
- No virtual grounds, the divider enters the picture!



$$G(s) = \frac{1}{s \frac{R_{lower} + R_{upper}}{gmR_{lower}} C_1}$$

$$\omega_{po} = \frac{1}{C_1 \frac{R_{lower} + R_{upper}}{gmR_{lower}}}$$

$$G = \frac{f_{po}}{f_c} \quad \text{Select } f_{po} \text{ to have } G \text{ at } f_c$$

1 pole at the origin

Type 1 - OTA

Boosting the phase at f_c with OTAs

□ The calculation is simple:

parameters

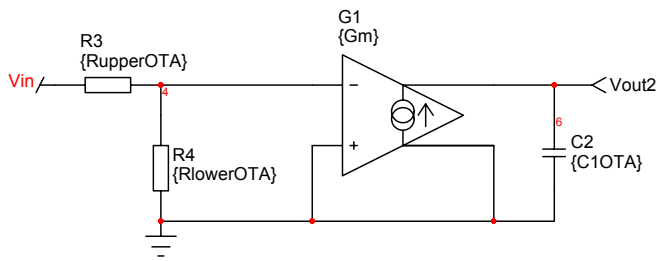
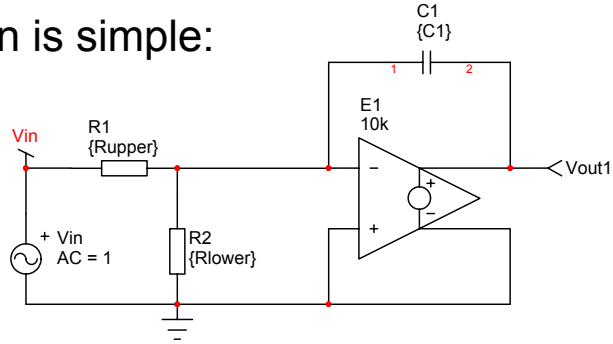
$V_{out}=385V$
 $I_b=250\mu$
 $V_{ref}=2.5$

$R_{upper}=(V_{out}-V_{ref})/I_b$
 $R_{lower}=2.5/I_b$

$f_c=20$
 $G_{fc}=46$

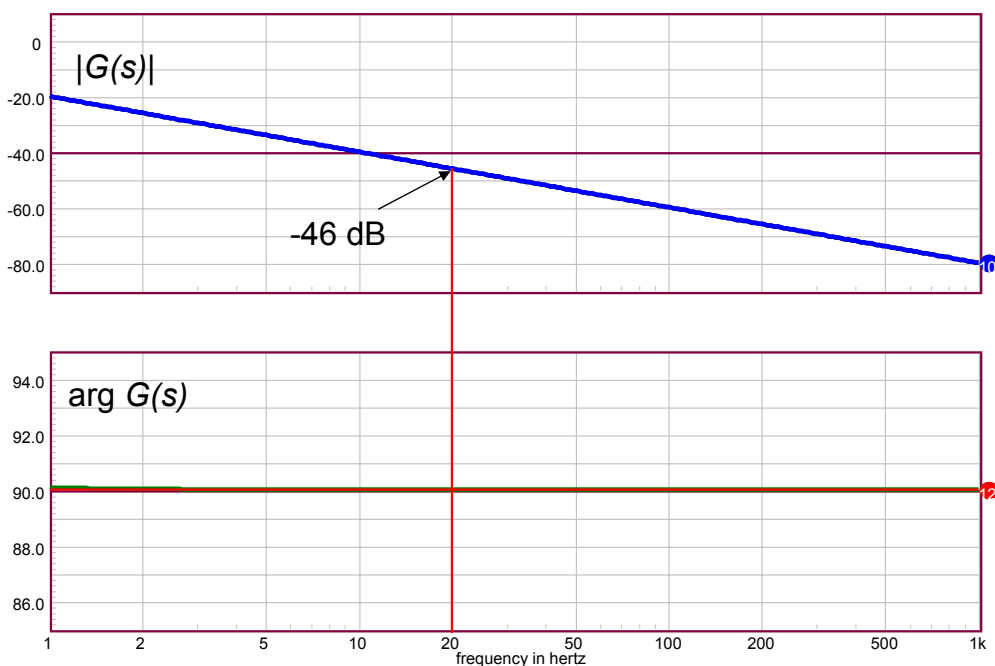
$g_m=200\mu$
 $f_{po}=G \cdot f_c$
 $R_{lowerOTA}=V_{ref}/I_b$
 $R_{upperOTA}=(V_{out}-V_{ref})/I_b$
 $R_{eq}=(R_{lower}+R_{upper})/(g_m \cdot R_{lower})$
 $C1_{OTA}=1/(2 \cdot \pi \cdot f_{po} \cdot R_{eq})$

$G=10^{(-G_{fc}/20)}$
 $\pi=3.14159$
 $C1=1/(2 \cdot \pi \cdot f_c \cdot G \cdot R_{upper})$



Type 1 - OTA

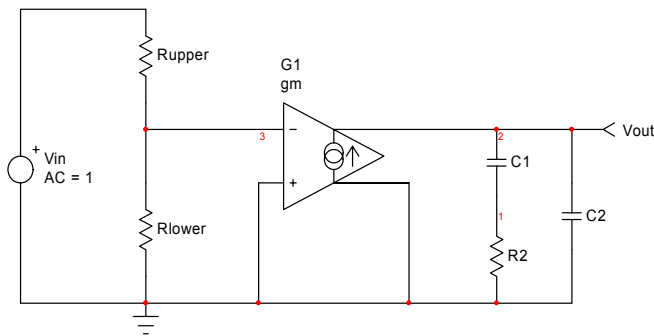
Boosting the phase at f_c with OTAs



Type 1 - OTA

Boosting the phase at f_c with OTAs

- ❑ The type 2 configuration with an OTA
- ❑ Type 2 improves transient response in BCM PFCs
- ❑ No virtual grounds, the divider enters the picture!



Mid-band gain

$$G(s) \approx \frac{R_{lower} gm R_2}{R_{lower} + R_{upper}} \frac{1 + sR_2 C_1}{sR_2 C_1 (1 + sR_2 C_2)}$$

If $C_2 \ll C_1$

$$\omega_{po} = \omega_z = \frac{1}{R_2 C_1} \quad \omega_p = \frac{1}{R_2 C_2}$$

1 pole at the origin
1 zero
1 pole

Type 2 - OTA

Boosting the phase at f_c with OTAs

parameters

Rupper=3.6Meg
Rlower=23k
gm=200u
fc=10
Gfc=-15

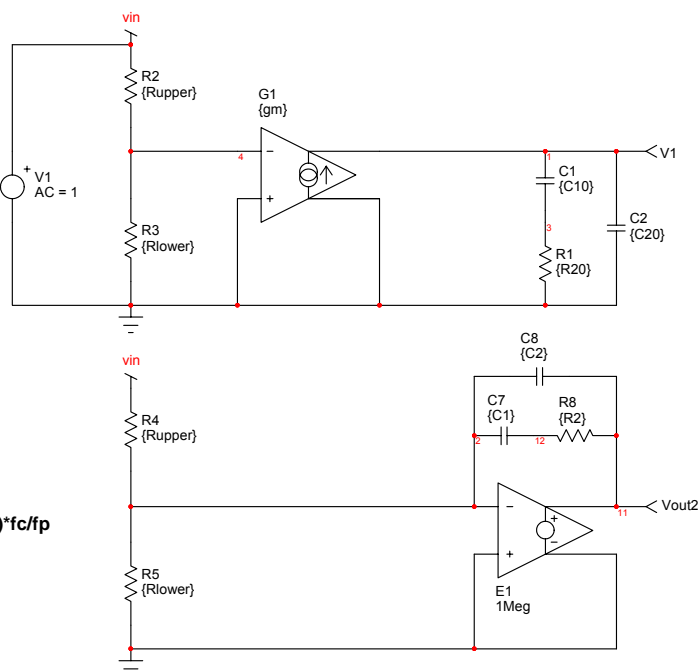
G=10⁻¹⁵*(Gfc/20)
pi=3.14159

fp=100
fz=1

a=sqrt((fc²+fp²)*(fc²+fz²))
c=(fc²+fz²)

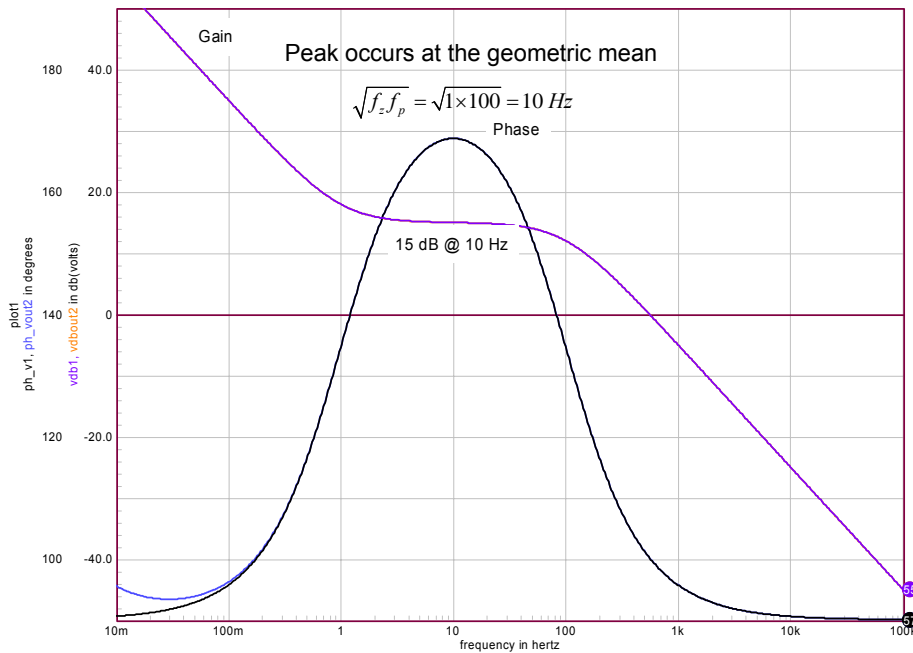
R2=(a/c)*Rupper*fc*G/fp
C2=1/(2*pi*fp*R2)
C1=1/(2*pi*fz*R2)

R20=G*(Rupper+Rlower)/(Rlower*gm)*(a/c)*fc/fp
C10=1/(2*pi*fz*R20)
C20=C10/(2*pi*fp*C10*R20-1)



Type 2 - OTA

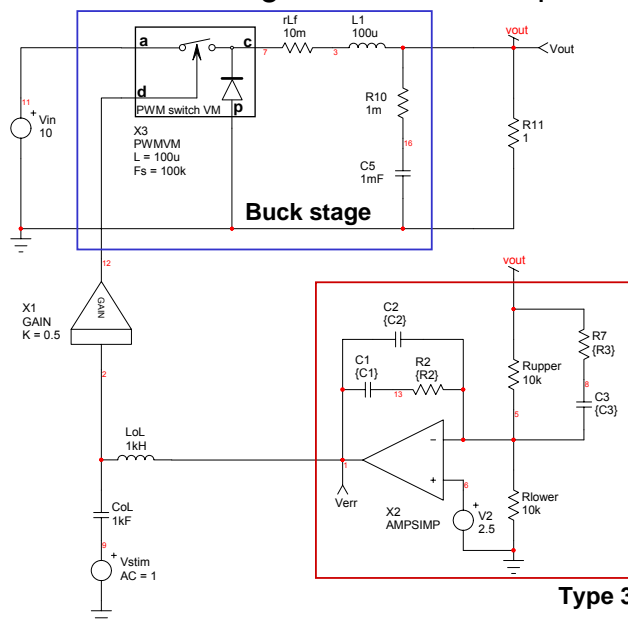
Boosting the phase at f_c with OTAs



Type 2 - OTA

Finally, we test the open-loop gain

5. Given the necessary boost of 155° , we select a type-3 amplifier
6. A SPICE simulation can give us the whole picture!



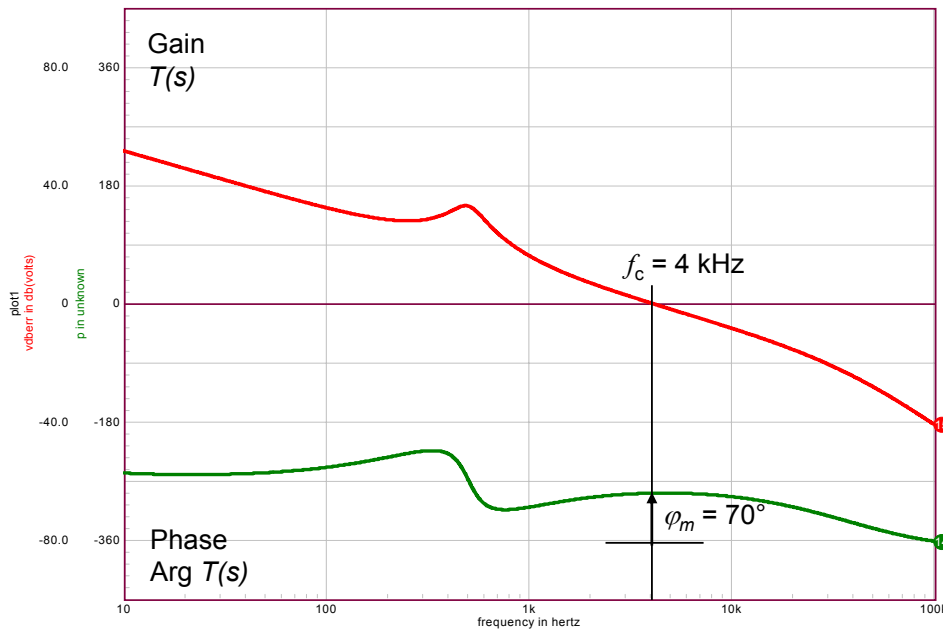
1 pole at the origin
2 zeros at 500 Hz
2 poles at 50 kHz

Type 3



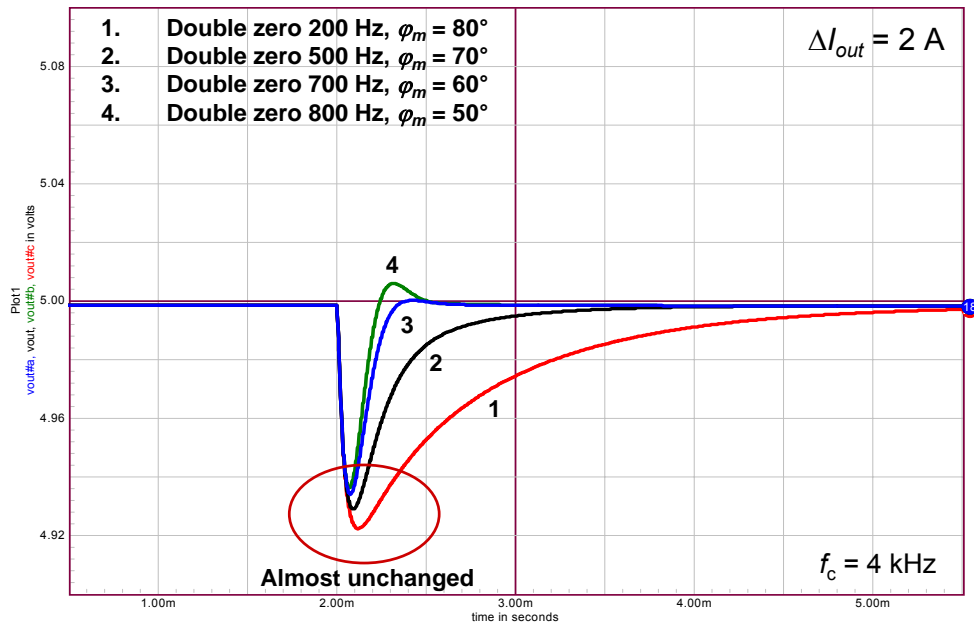
Finally, we test the open-loop gain

An ac simulation gives us the open-loop Bode plot



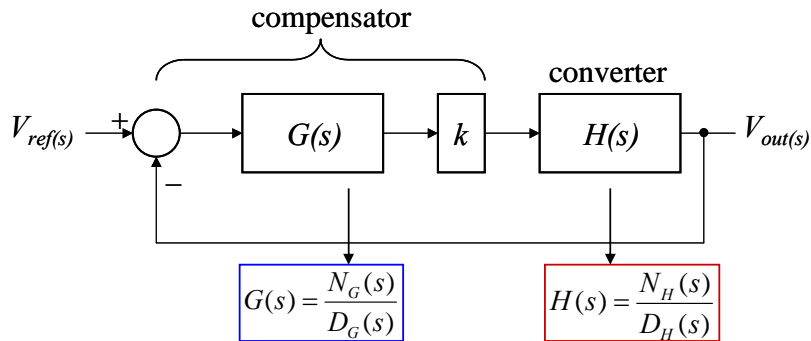
What transient response?

- The crossover frequency is constant, but zeros position is changed



Zeros become poles??

- ❑ $T(s)$ is shaped by introducing zeros/poles in $G(s)$
- ❑ In closed-loop conditions, the zeros of $G(s)$ become poles...



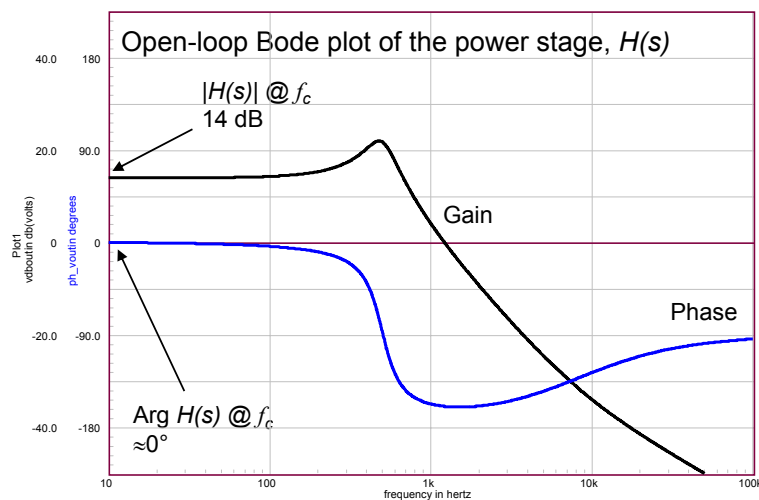
$$\frac{V_{out}(s)}{V_{ref}(s)} = \frac{kG(s)H(s)}{1+kG(s)H(s)} \rightarrow \frac{V_{out}(s)}{V_{ref}(s)} = \frac{k \frac{N_G(s)}{D_G(s)} \frac{N_H(s)}{D_H(s)}}{1+k \frac{N_G(s)}{D_G(s)} \frac{N_H(s)}{D_H(s)}} = \frac{kN_G(s)N_H(s)}{D_G(s)D_H(s) + kN_G(s)N_H(s)}$$

For $k \gg 1$
zeros of $G(s)$ appear
in denominator as poles

→ Pushing the zeros towards low frequency slows down the response!

If we roll-off the BW in the low frequencies?

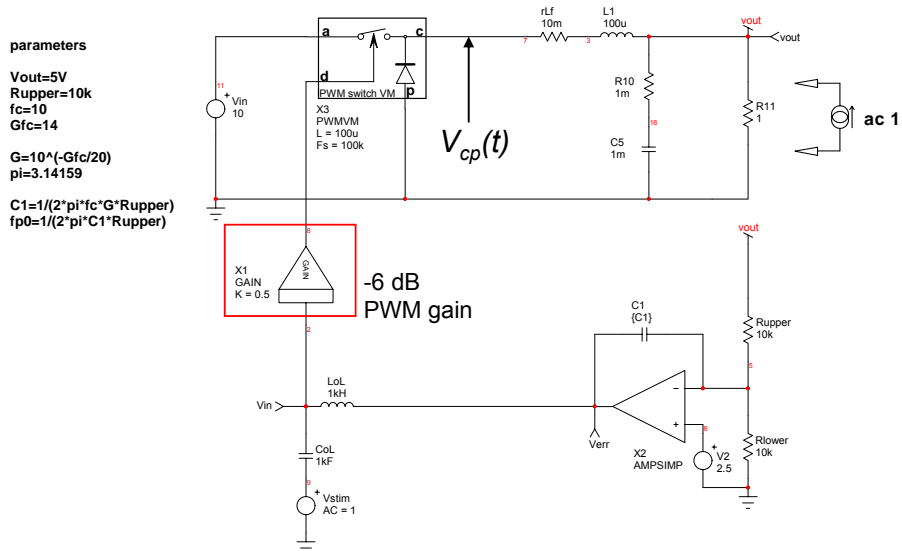
- ❑ Could we avoid the resonance with a 10 Hz crossover point?



- ❑ We have almost no phase rotation at 10 Hz, a type 1 could do??

If we roll-off the BW in the low frequencies?

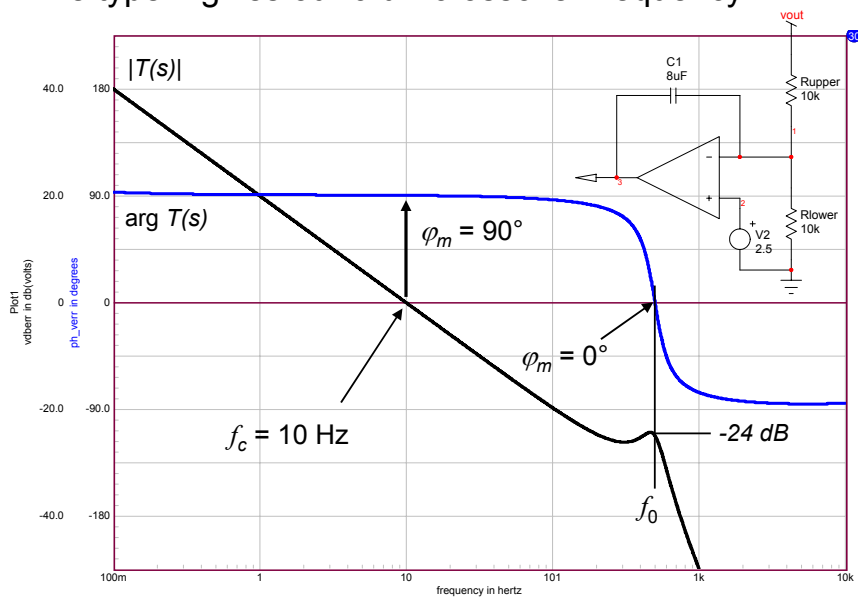
- A simple SPICE ac simulation gives us the open-loop gain



- An ac current source sweeps the output impedance

The open-loop gain looks good...

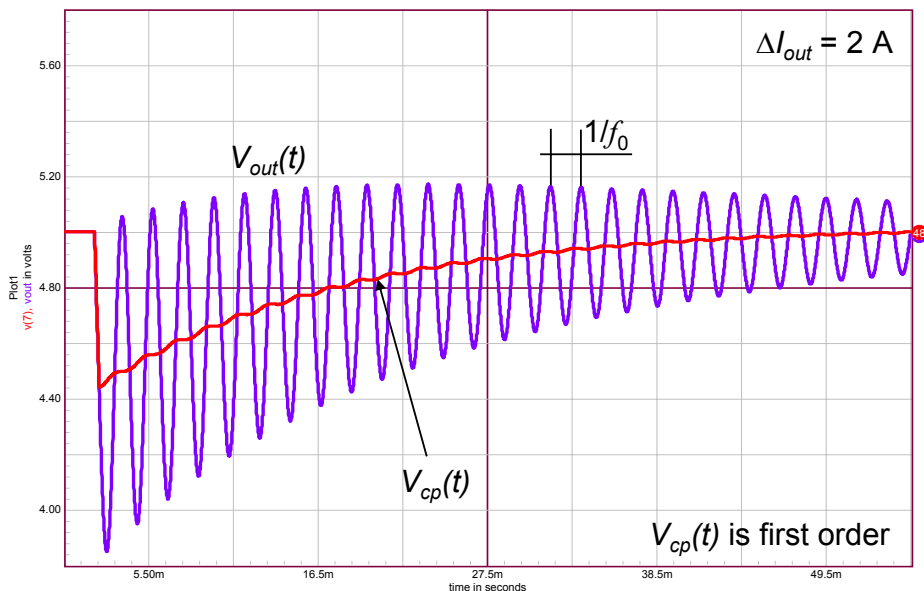
- The type 1 gives our 0 dB crossover frequency



- We have plenty of phase margin, a slow system, ok...

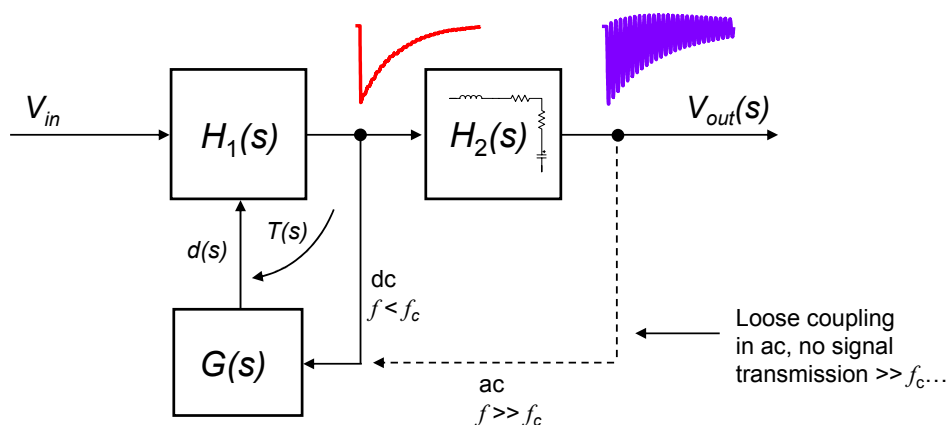
Oh no, it is ringing!

- The load step reveals a ringing ac output



The RLC network rings alone...

- H_1 is stable per Bode analysis, but H_2 is out of the loop...



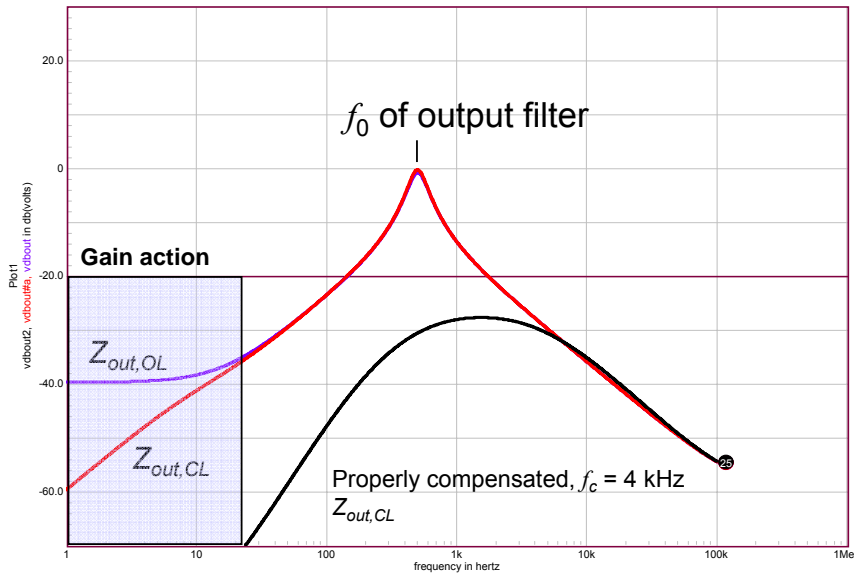
- The dc is fed back via the loop but not the ac...
- Oscillations are NOT due to the loop!



"Fast Analytical Techniques for Electrical and Electronic Circuits"
Vatché Vorpérian, Cambridge Press, 2002

There is no gain to compensate the peaking!

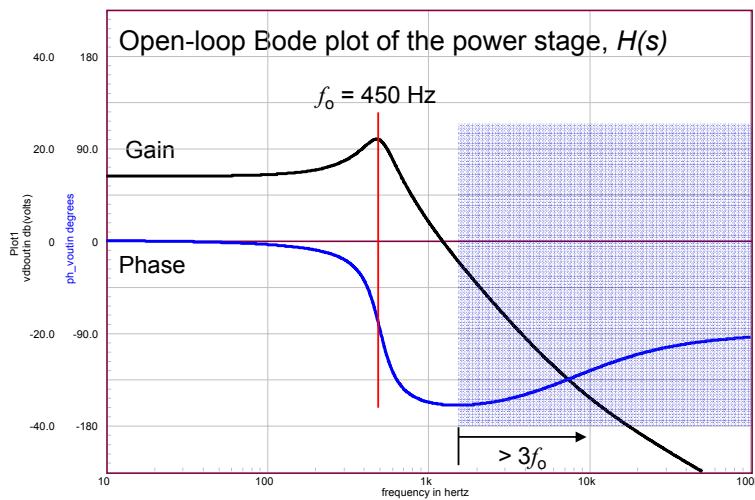
- No gain when the resonance occurs: the *RLC* network runs open loop



- The system cannot reduce the Q at the resonant frequency

The crossover must be above f_0

- There still must be gain at the resonance to damp the filter
- f_c should be far from f_0 to reduce phase stress at resonance

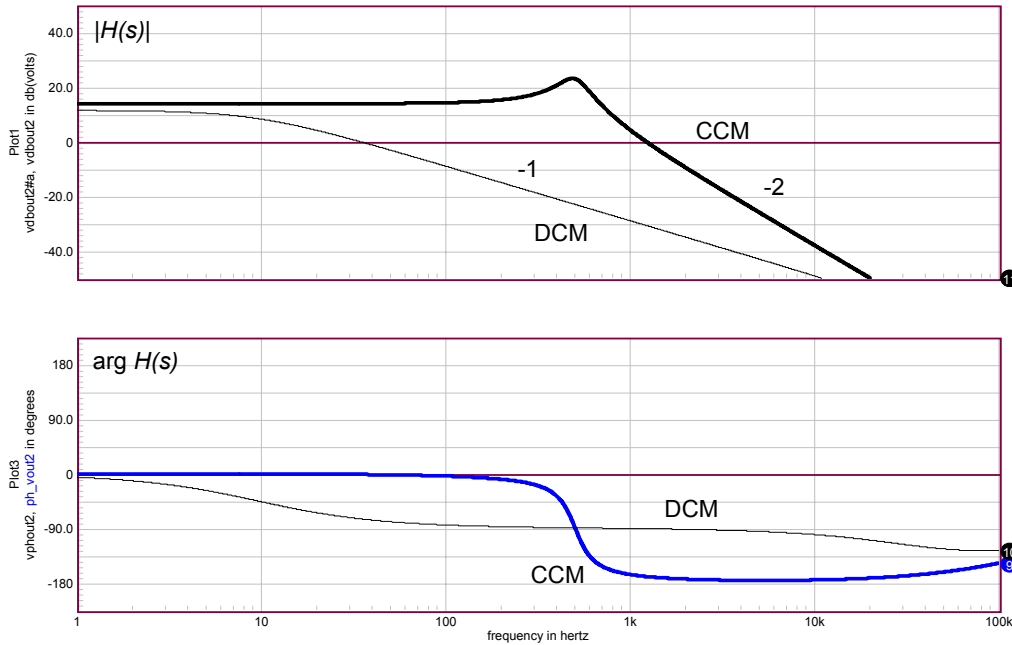


Place f_c at least three times above resonance!



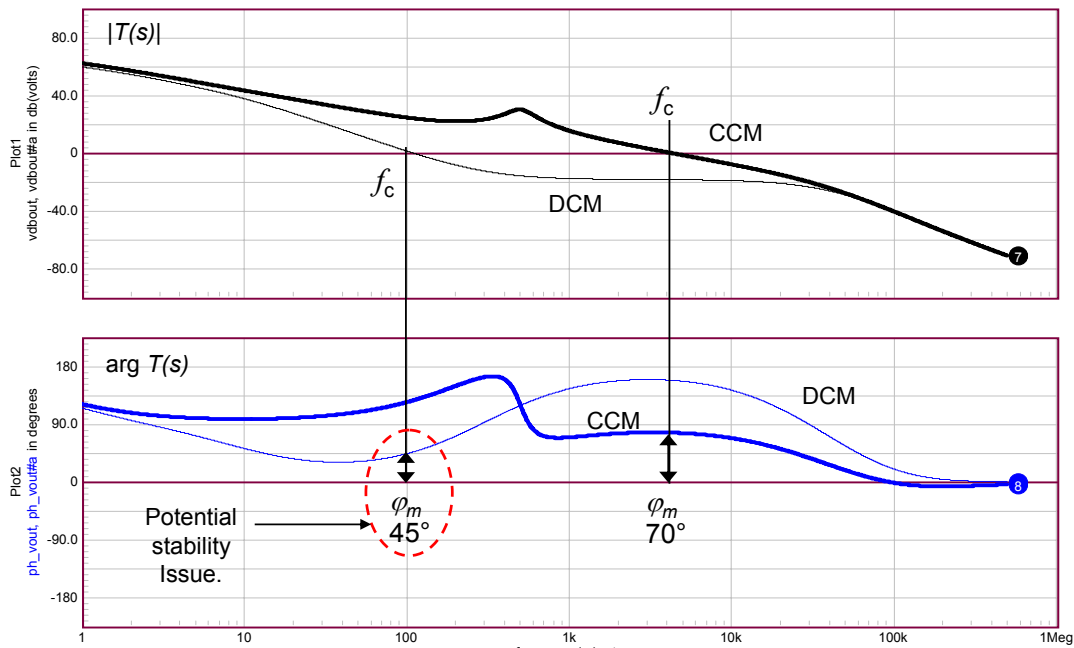
CCM to DCM, the transfer function changes

- In DCM, the buck voltage-mode becomes a first order system



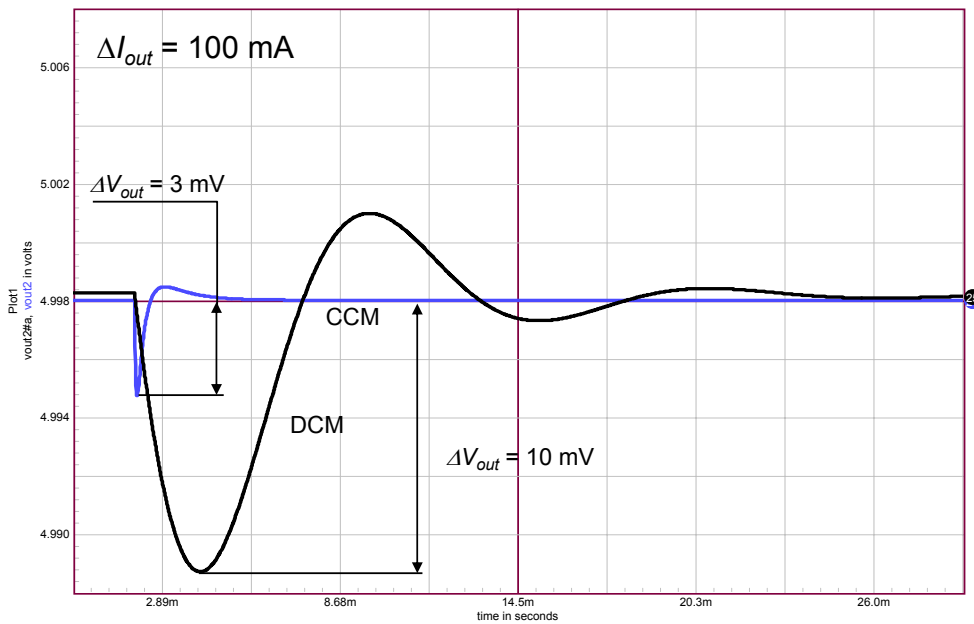
CCM to DCM, the transfer function changes

- In DCM, the compensated system becomes slower than in CCM



The DCM response is slower than in CCM

- A 100-Hz crossover in DCM versus a 4-kHz crossover in CCM



General methods for compensation

- For a buck in CCM voltage-mode:
 - Place a double zero at the LC_{out} resonance
 - If the ESR zero f_z is below f_c , put a pole $f_{p1}=f_z$
 - If the ESR zero f_z is above f_c , put a pole $f_{p1}=F_{sw}/2$
 - Put a second pole f_{p2} at $f_{p2}=F_{sw}/2$

$$\frac{V_{out}(s)}{V_{err}(s)} = \frac{V_{in}}{V_{peak}} K_c \frac{\left(1 + \frac{s}{\omega_{z1}}\right)}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

\uparrow PWM gain \leftarrow This zero is brought by the output capacitor ESR

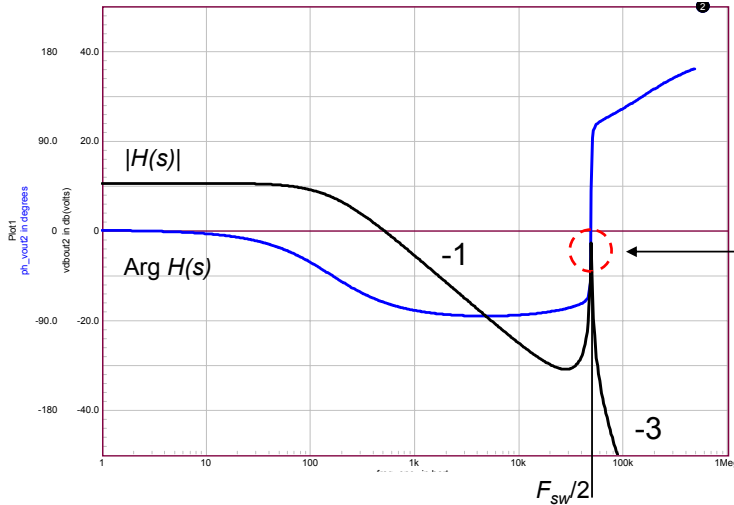
$$K_c = \frac{R_{load}}{r_{Lf} + R_{load}} \quad \omega_{z1} = \frac{1}{r_{Cf}C} \quad \omega_0 = \frac{1}{\sqrt{LC} \frac{R+r_{Cf}}{R+r_{Lf}}} \quad Q = \frac{1}{\frac{Z_o}{r_{Lf} + R_{load}} + \frac{r_{Cf} + r_{Lf} \parallel R_{load}}{Z_o}}$$

Type 3



The buck in current-mode

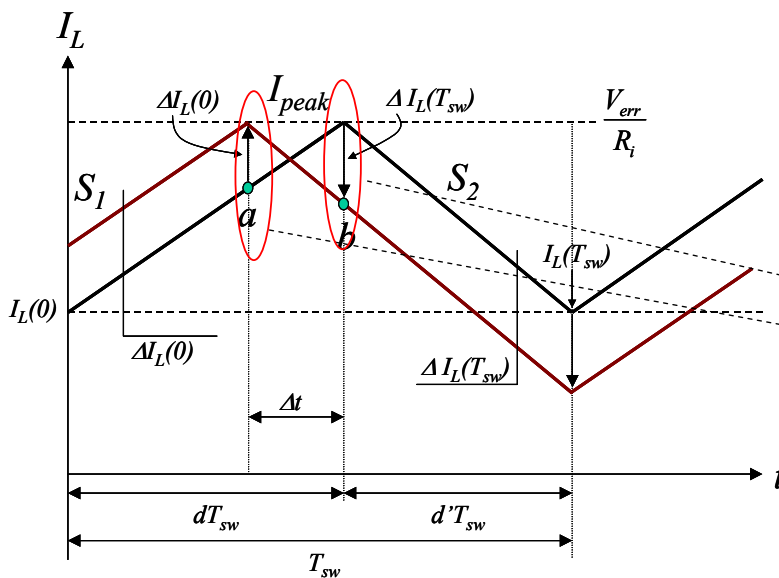
- ❑ A CCM current-mode converter acts as a third order system
- ❑ It looks like a first order system at $f_c \ll F_{sw}/2$
- No LC peaking anymore!
- But a subharmonic peaking at $F_{sw}/2$ now appears!



Subharmonic peaking now!



CCM operation, current instabilities



$$I_{peak} = a + S_1 \Delta t$$

$$b = I_{peak} - S_2 \Delta t$$

$$\begin{aligned} &\downarrow \text{Solving } \Delta t \\ \frac{I_{peak} - a}{S_1} &= \frac{I_{peak} - b}{S_2} \end{aligned}$$

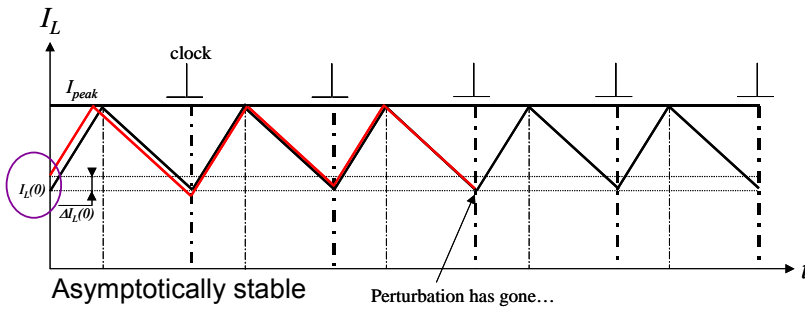
$$\frac{\Delta I_L(0)}{S_1} = \frac{\Delta I_L(T_{sw})}{S_2}$$

$$\frac{S_2}{S_1} = \frac{d}{d'}$$

$$\Delta I_L(nT_{sw}) = \Delta I_L(0) \left(-\frac{d}{d'} \right)^n$$

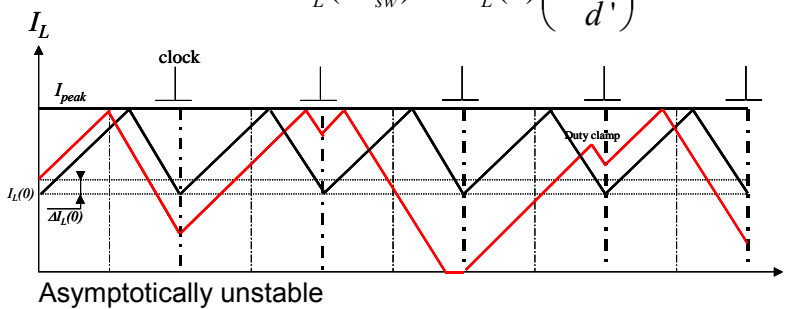


CCM operation, current instabilities



Duty-cycle < 50%

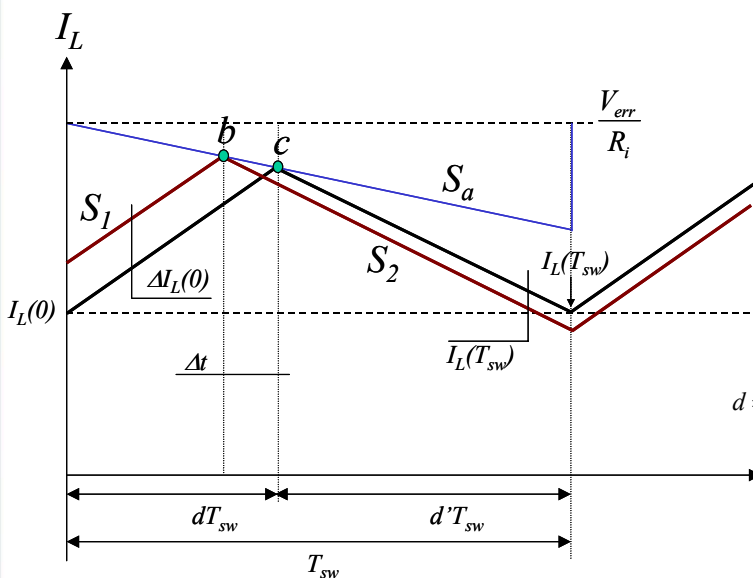
$$\Delta I_L(nT_{sw}) = \Delta I_L(0) \left(-\frac{d}{d'} \right)^n$$



Duty-cycle > 50%



CCM operation, curing current instabilities



$$\Delta I_L(nT_{sw}) = \Delta I_L(0) \left[\frac{1 - \frac{S_a}{S_2}}{\frac{d'}{d} + \frac{S_a}{S_2}} \right]^n = \Delta I_L(0) (-a)^n$$

Must stay below 1

$$\left| \frac{1 - \frac{S_a}{S_2}}{\frac{d'}{d} + \frac{S_a}{S_2}} \right| < 1$$

Up to $d = 100\%$

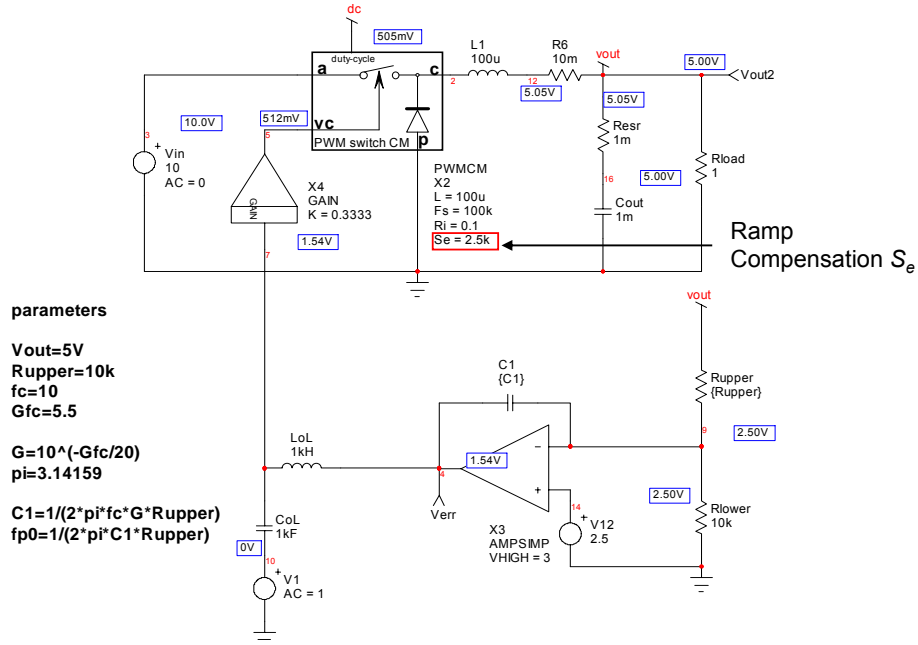
$$S_a > 50\% S_2$$

Inject ramp compensation



A buck CCM in current-mode

□ A SPICE model can predict subharmonic instabilities



parameters

Vout=5V
Rupper=10k
fc=10
Gfc=5.5

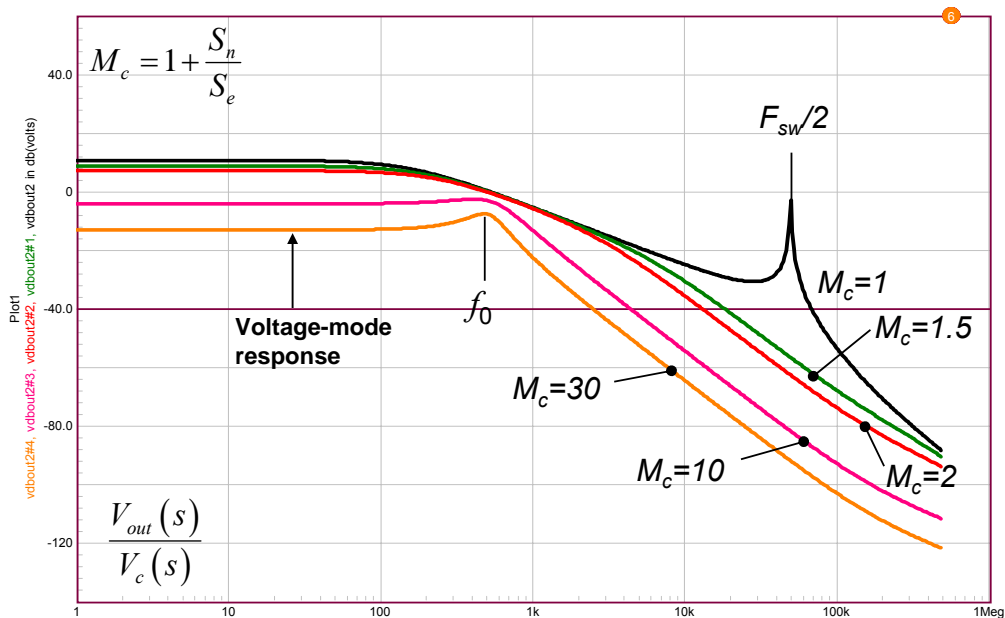
$G=10^{-(Gfc/20)}$
 $\pi=3.14159$

$C1=1/(2\pi \cdot fc \cdot G \cdot Rupper)$
 $fp0=1/(2\pi \cdot C1 \cdot Rupper)$



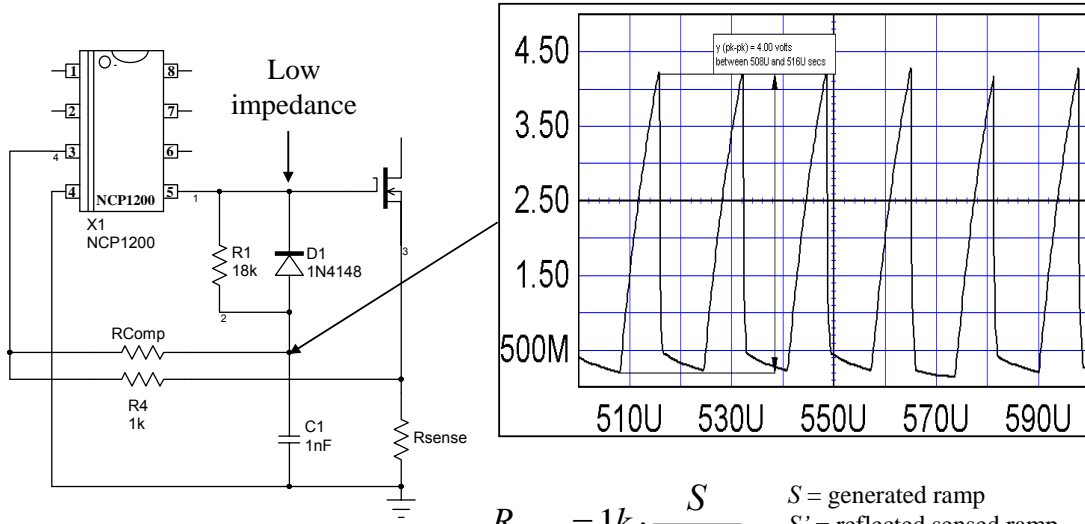
Injecting ramp damps the double pole

□ Too much ramp turns the converter into voltage-mode!





The right way to inject ramp compensation



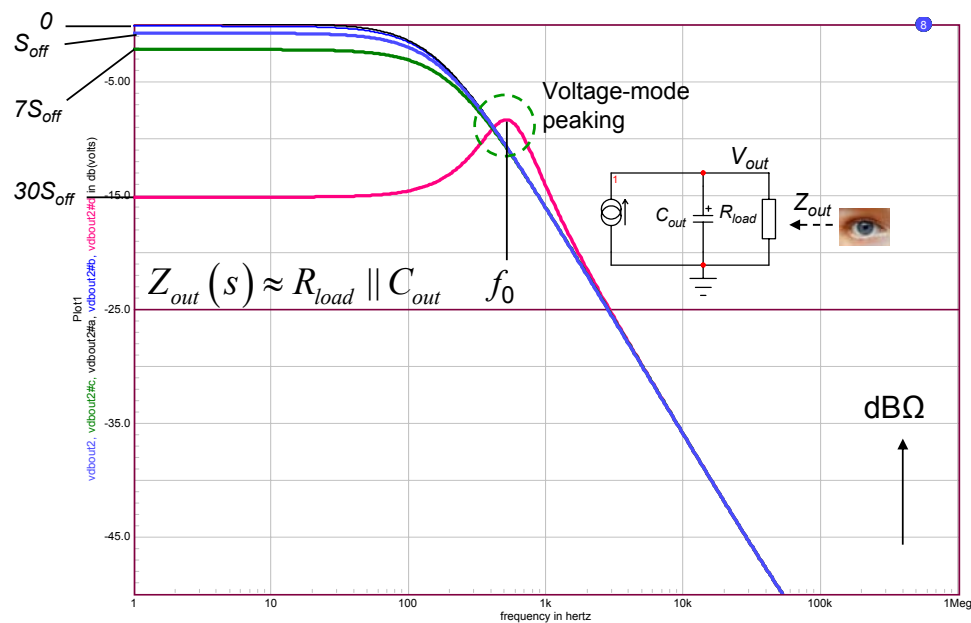
$$R_{comp} = 1k \cdot \frac{S}{S' \cdot M}$$

S = generated ramp
 S' = reflected sensed ramp
 M = amount of ramp, 0.5-0.75



The open-loop impedance of a CM converter

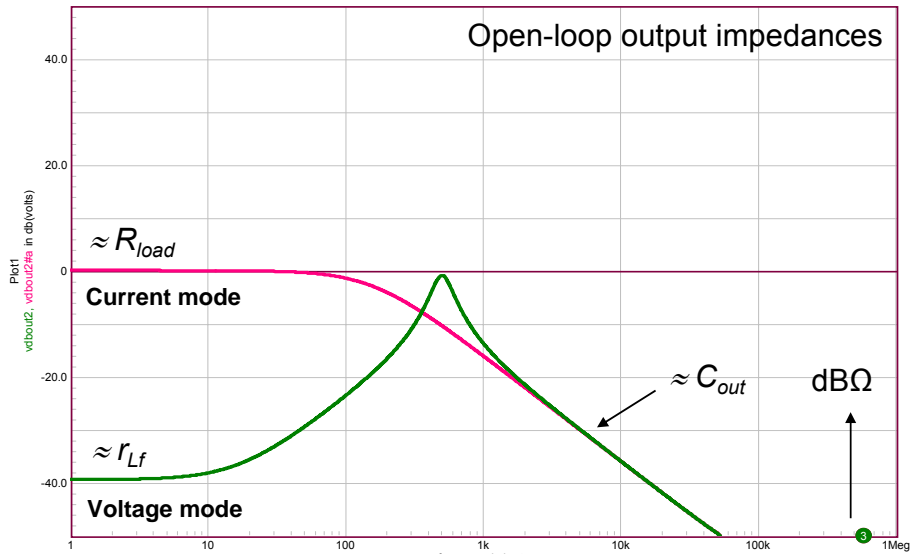
- The current-mode output impedance depends on the ramp level





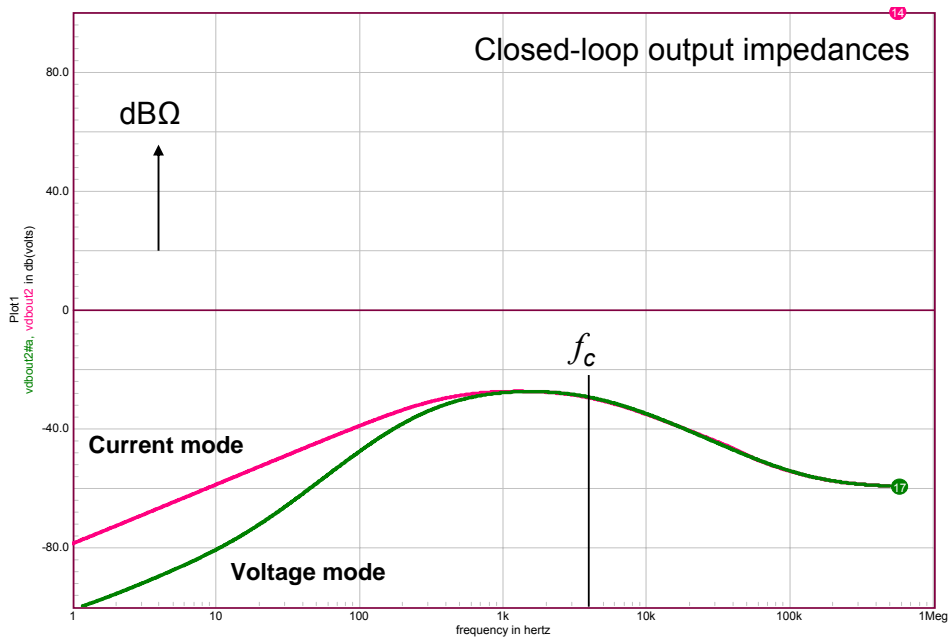
The closed-loop output impedance of CM and VM

□ Z_{out} CM is naturally larger than Z_{out} VM



The closed-loop output impedance of CM and VM

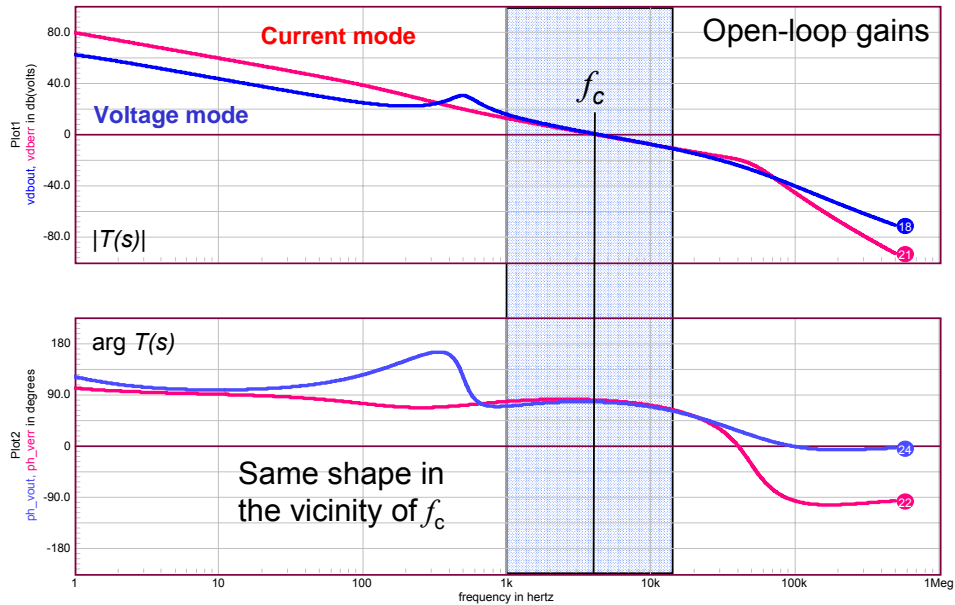
□ In closed-loop, Z_{out} in VM is still smaller...





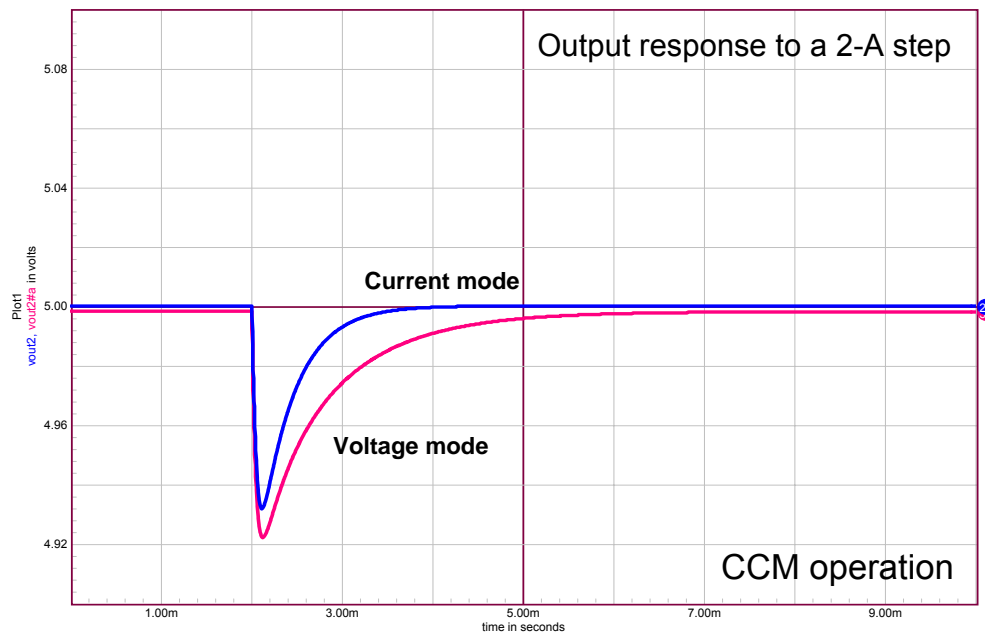
The closed-loop output impedance of CM and VM

□ The dc gain in VM is smaller because of zeros



With similar crossover frequency...

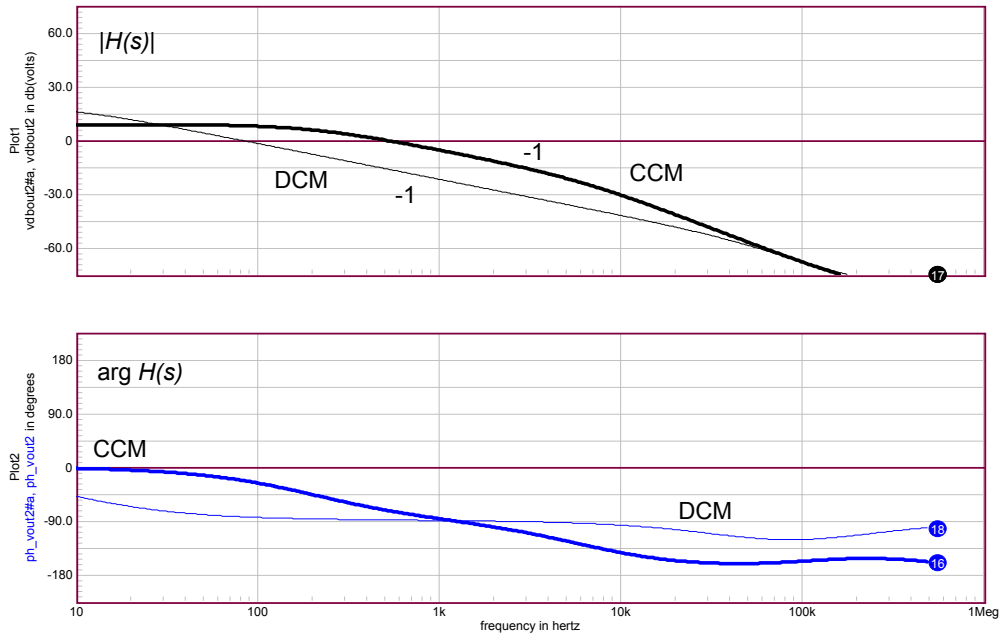
□ The voltage mode is slightly slower than current-mode





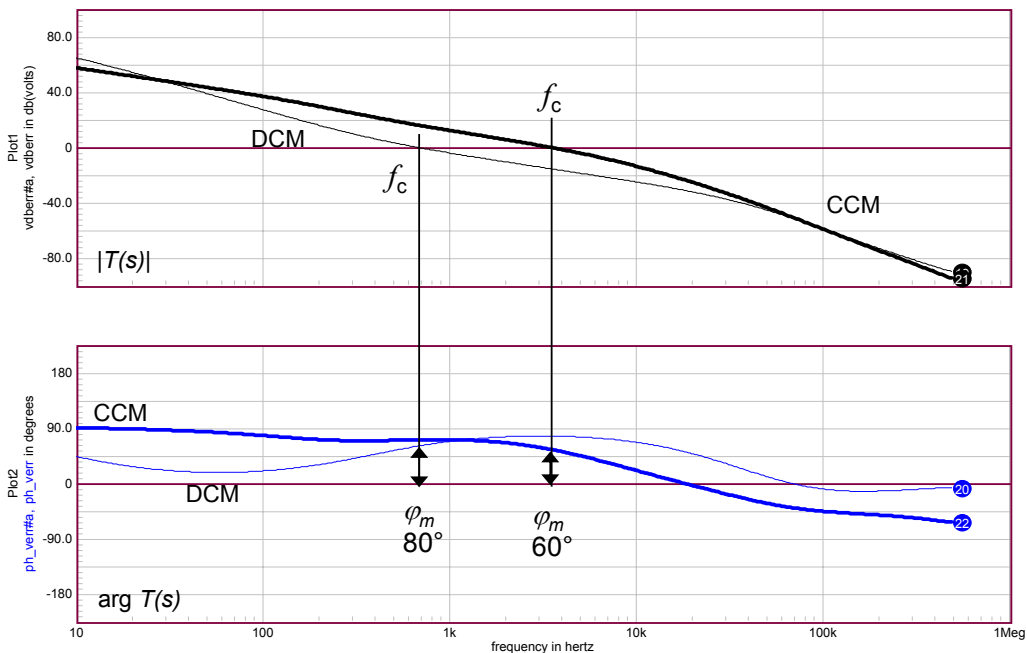
Transition from CCM to DCM in current-mode

- Current-mode remains a 1st order system in both cases



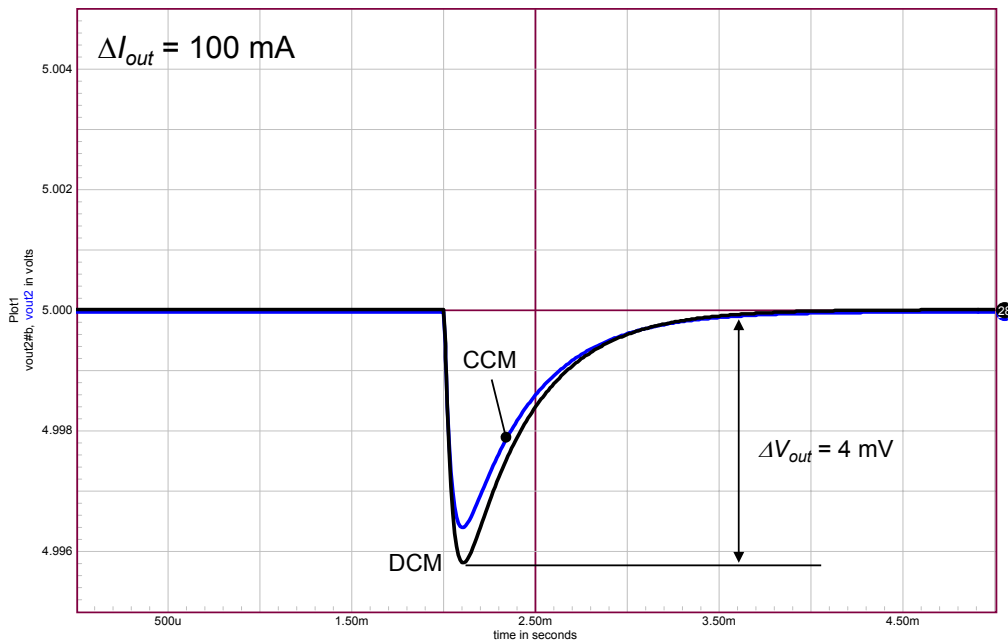
Transition from CCM to DCM in current-mode

- Once compensated, f_c still changes but φ_m is still ok



The DCM response is not too much affected

- The step response is almost unchanged...



103

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The k factor in an automated type 2

- Poles and zeros boost the phase at the crossover freq.
- How to place poles/zeros for a precise boost at f_c ?
- ✓ Use the k factor technique introduced by Dean Venable

From a 1 zero/ 1 pole compensation circuit, we have:

$$\arg(T(f_c)) = \text{boost} = \arg\left(\frac{1 + \frac{f_c}{f_{z0}}}{1 + \frac{f_c}{f_{p0}}}\right) = \arctan\left(\frac{f_c}{f_{z0}}\right) - \arctan\left(\frac{f_c}{f_{p0}}\right)$$

If we place one pole at kf_c and one zero at f_c/k , we have:

$$\text{boost} = \arctan(k) - \arctan\left(\frac{1}{k}\right)$$

$$\arctan(x) + \arctan\left(\frac{1}{x}\right) = 90^\circ \longrightarrow k = \tan\left(\frac{\text{boost}}{2} + 45\right)$$

104

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The k factor in an automated type 2

□ Suppose we have the following specs:

- $f_c = 1 \text{ kHz}$, $|H(1\text{kHz})| = -10 \text{ dB}$
- $\text{Arg } H(1\text{kHz}) = -100^\circ$
- $\varphi_m = 70^\circ$ $\text{BOOST} = \varphi_m - \arg H(f_c) - 90^\circ = 70^\circ + 100 - 90 = 80^\circ$
- For a 80° phase boost, select a type 2

□ Calculate k

$$k = \tan\left(\frac{80}{2} + 45\right) = 11.4$$

□ Place a zero at $1\text{k}/11.4 = 90 \text{ Hz}$

□ Place a pole at $1\text{k} \times 11.4 = 11.4 \text{ kHz}$

□ Adjust mid-gain to reach $10 \text{ dB}@1 \text{ kHz}$

Type 2

The k factor in an automated type 2

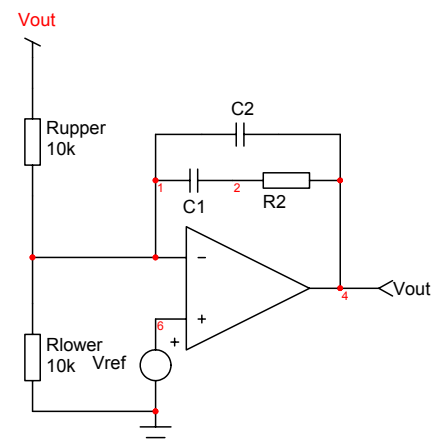
□ Calculate the elements for a type 2 amplifier

$$C_2 = \frac{1}{2\pi f_c G k R_{upper}} = \frac{1}{6.28 \times 1\text{k} \times 3.16 \times 11.4 \times 10\text{k}} = 442 \text{ pF}$$

$$C_1 = C_2 (k^2 - 1) = 442 \text{ p} \times (11.4^2 - 1) = 57 \text{ nF}$$

$$R_2 = \frac{k}{2\pi f_c C_1} = \frac{11.4}{6.28 \times 1\text{k} \times 57\text{n}} = 31.8 \text{ k}\Omega$$

$$G = 10^{\frac{Gf_c}{20}} = 10^{10/20} = 3.16$$



Type 2



The k factor in an automated type 2

Macros can be used to automate the calculation

parameters

Rupper=10k

fc=1k

pm=70

Gfc=-10

pfc=-100

$G=10^{(-Gfc/20)}$

boost=pm-(pfc)-90

pi=3.14159

$K=\tan((boost/2+45)*\pi/180)$

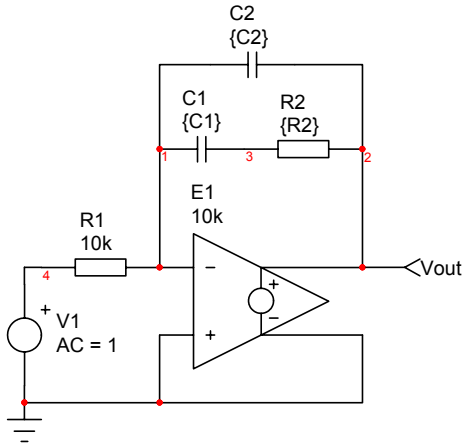
$C2=1/(2*\pi*fc*G*k*Rupper)$

$C1=C2*(K^2-1)$

$R2=k/(2*\pi*fc*C1)$

$fp=1/(2*\pi*R2*C2)$

$fz=1/(2*\pi*R2*C1)$



```
*****
* DEFAULT PARAMETERS
*****

***** mainckt

RUPPER = 1.000e+004
FC = 1.000e+003
PM = 7.000e+001
GFC = -1.000e+002
PFC = -1.000e+002
G = 3.162e+000
BOOST = 8.000e+001
PI = 3.142e+000
K = 1.143e+001
C2 = 4.403e-010
C1 = 5.709e-008
R2 = 3.187e+004
FP = 1.134e+004
FZ = 8.749e+001
```

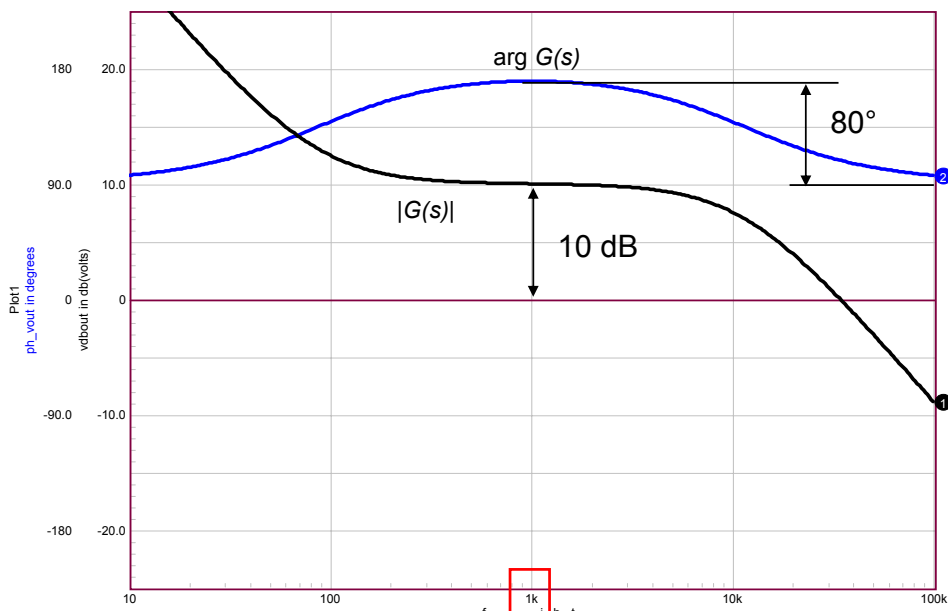
results

Type 2



The k factor in an automated type 2

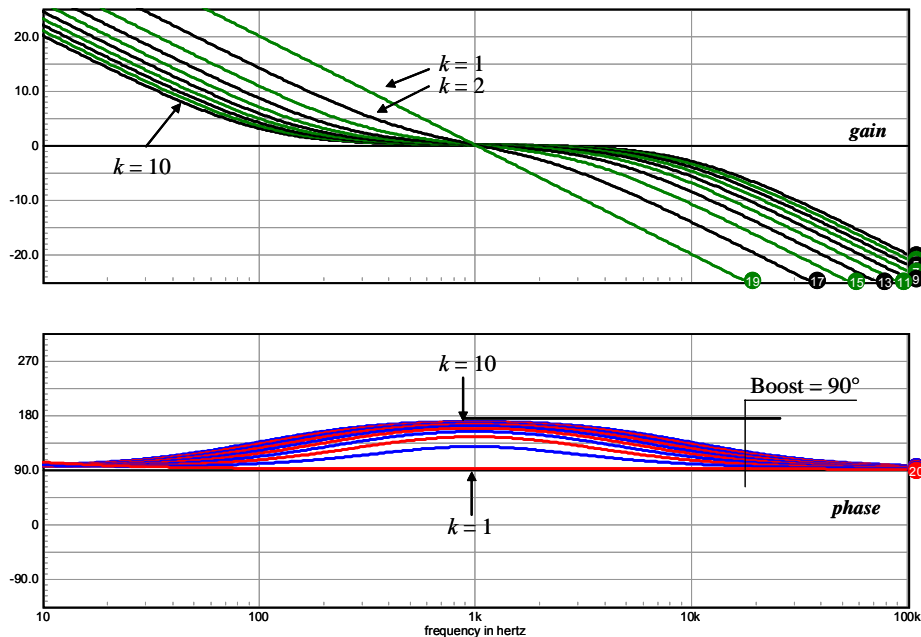
The numbers exactly match the predictions



Type 2

The k factor in an automated type 2

- Adjusting k modulates the resulting phase boost



Type 2

109

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The k factor in an automated type 3

- 2 poles and 2 zeros give a higher boost at crossover
- These 2 poles and 2 zeros are coincident
- How to place them for a precise boost at f_c ?

From a 1 zero pair / 1 pole pair compensation circuit, we have:

$$\arg(T(f_c)) = \text{boost} = \arg\left(\frac{\left(1 + \frac{f_c}{f_{z0}}\right)^2}{\left(1 + \frac{f_c}{f_{p0}}\right)^2}\right) = 2 \arctan\left(\frac{f_c}{f_{z0}}\right) - 2 \arctan\left(\frac{f_c}{f_{p0}}\right)$$

If we place two poles at $\sqrt{k}f_c$ and two zeros at f_c/\sqrt{k} , we have:

$$\text{boost} = 2 \left[\arctan(\sqrt{k}) - \arctan\left(\frac{1}{\sqrt{k}}\right) \right]$$

$$\text{boost} = 2 \left[\arctan(\sqrt{k}) + \arctan\sqrt{(k)} - 90 \right] = 4 \arctan\sqrt{(k)} - 180 \longrightarrow k = \left[\tan\left(\frac{\text{boost}}{4} + 45\right) \right]^2$$

110

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The k factor in an automated type 3

□ Suppose we have the following specs:

- $f_c = 1 \text{ kHz}$, $|H(1\text{kHz})| = -15 \text{ dB}$
- $\text{Arg } H(1\text{kHz}) = -140^\circ$
- $\phi_m = 70^\circ$ $\text{BOOST} = \phi_m - \arg H(f_c) - 90^\circ = 70^\circ + 140 - 90 = 120^\circ$
- For a 120° phase boost, select a type 3

□ Calculate k

$$k = \left[\tan\left(\frac{120}{4} + 45\right) \right]^2 = 13.9$$

□ Place a zero at $1\text{k}/3.7 = 268 \text{ Hz}$

□ Place a pole at $1\text{k} \times 3.7 = 3.7 \text{ kHz}$

□ Adjust mid-gain to reach $15 \text{ dB}@1 \text{ kHz}$

Type 3

The k factor in an automated type 3

□ Calculate the elements for a type 3 amplifier

$$C_2 = \frac{1}{2\pi f_c G R_1} = \frac{1}{6.28 \times 1\text{k} \times 5.6 \times 10\text{k}} = 2.85 \text{ nF}$$

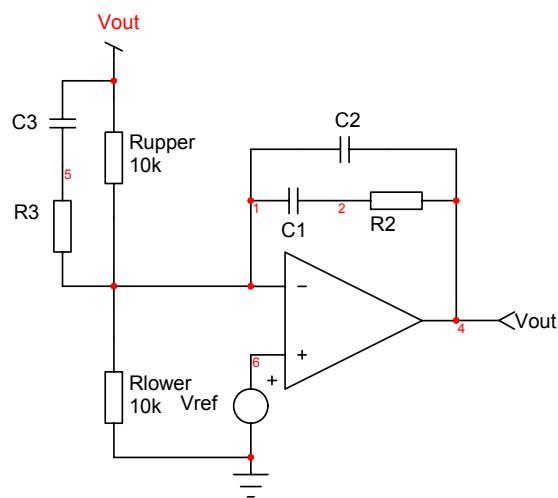
$$C_1 = C_2 (k - 1) = 2.85\text{n} \times (13.9 - 1) = 36.7 \text{ nF}$$

$$R_2 = \frac{\sqrt{k}}{2\pi f_c C_1} = \frac{\sqrt{13.9}}{6.28 \times 1\text{k} \times 36.7\text{n}} = 16.2 \text{ k}\Omega$$

$$R_3 = \frac{R_1}{k - 1} = \frac{10\text{k}}{13.9 - 1} = 775 \Omega$$

$$C_3 = \frac{1}{2\pi f_c \sqrt{k} R_3} = \frac{1}{6.28 \times 1\text{k} \times \sqrt{13.9} \times 775} = 55 \text{ nF}$$

$$G = 10^{\frac{G_f}{20}} = 10^{15/20} = 5.6$$



Type 3



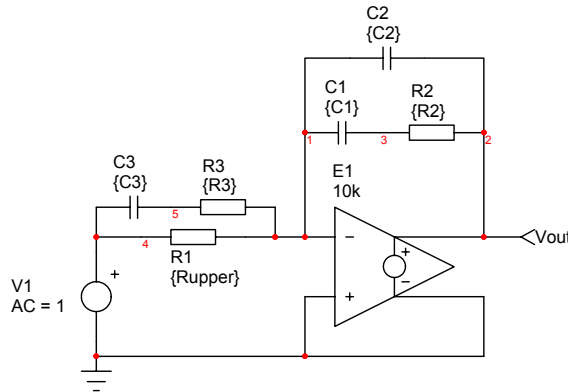
The k factor in an automated type 3

☐ Macros can be used to automate the calculation

```
parameters
Rupper=10k
fc=1k
pm=70
Gfc=-15
ps=-140
```

```
G=10^(-Gfc/20)
boost=pm-(ps)-90
pi=3.14159
K=(tan((boost/4+45)*pi/180))^2
C2=1/(2*pi*fc*G*Rupper)
C1=C2*(K-1)
R2=sqrt(k)/(2*pi*fc*C1)
R3=Rupper/(k-1)
C3=1/(2*pi*fc*sqrt(k)*R3)
```

```
fp1=1/(2*pi*R2*C2)
fp2=1/(2*pi*R3*C3)
fz1=1/(2*pi*R2*C1)
fz2=1/(2*pi*Rupper*C3)
```



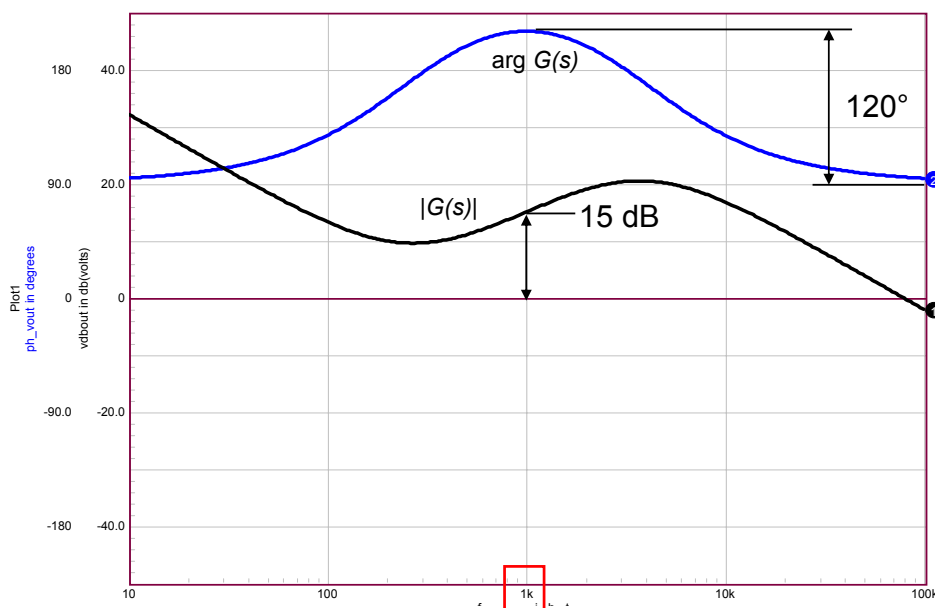
```
*****
* DEFAULT PARAMETERS
*****
**** mainckt
RUPPER = 1.000e+004
FC = 1.000e+003
PM = 7.000e+001
GFC = -1.500e+001
PS = -1.400e+002
G = 5.623e+000
BOOST = 1.200e+002
PI = 3.142e+000
K = 1.393e+001
C2 = 2.830e-009
C1 = 3.659e-008
R2 = 1.623e+004
R3 = 7.735e+002
C3 = 5.513e-008
FP1 = 3.464e+003
FP2 = 3.732e+003
FZ1 = 2.680e+002
FZ2 = 2.887e+002
```

results



The k factor in an automated type 3

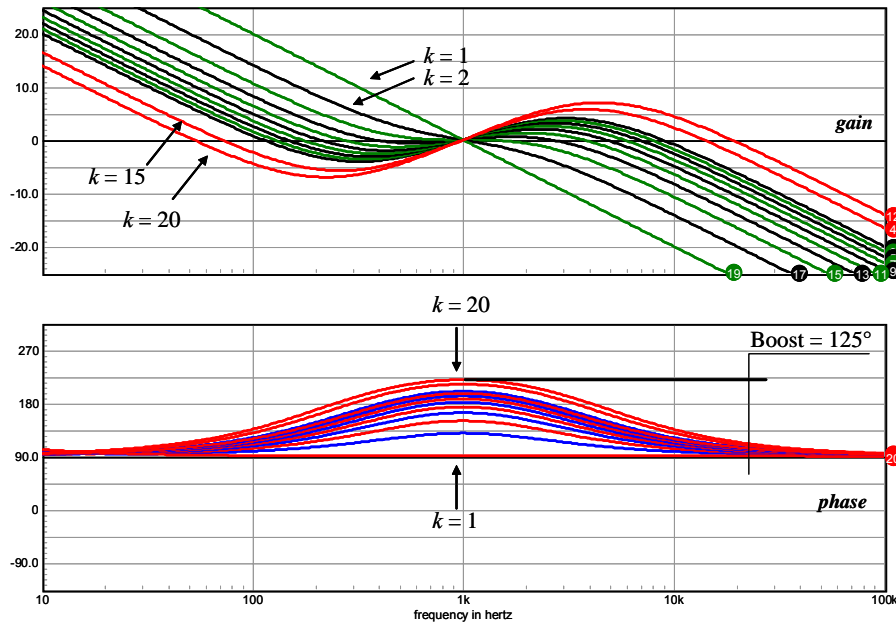
☐ The numbers exactly match the predictions



Type 3

The k factor in an automated type 3

- Adjusting k modulates the resulting phase boost



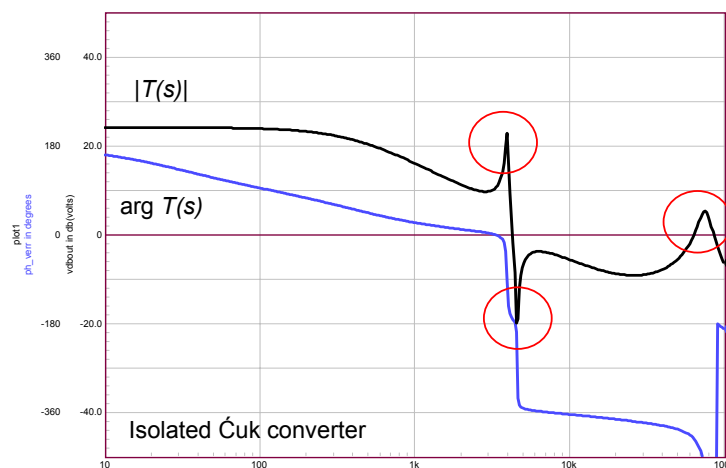
Type 3

115

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Is the k factor the panacea?

- The tool is a quick and easy means to stabilize the converter
- However, it solely focuses on the crossover frequency
 - What is happening before and beyond f_c ?
- The k factor technique is blind to these effects.
- In resonant systems, conditional stability can occur.



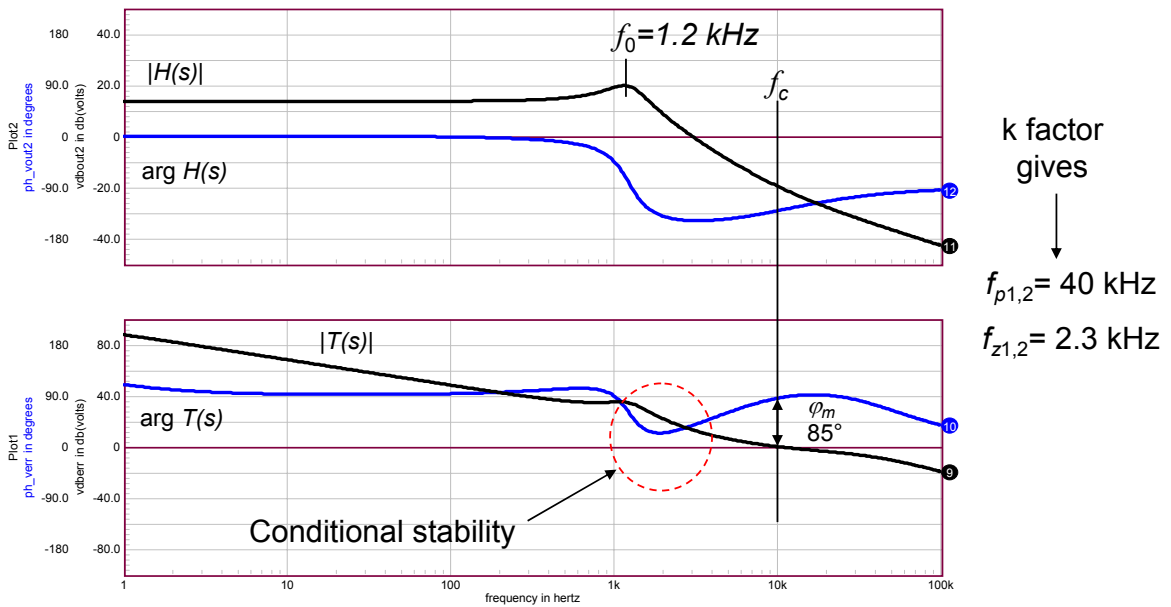
116

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The k factor can lead to conditional stability

- In CCM voltage-mode, conditional stability occurs



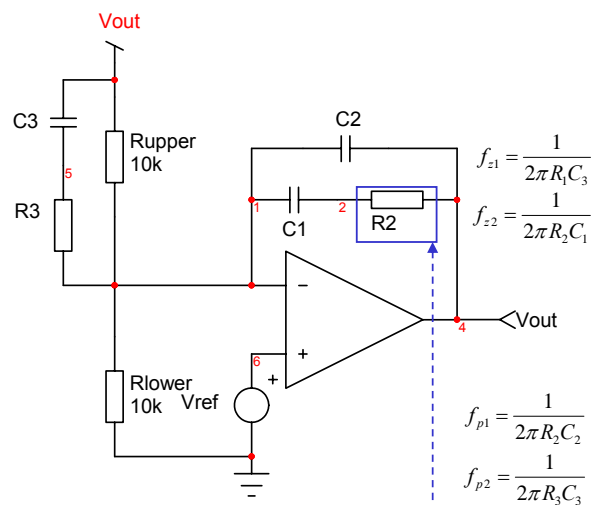
Manual pole/zero placement is a solution

- Avoid coincident poles and zeros brought by k factor
- Place them wherever you wish to shape $G(s)$

$$G(s) \approx \frac{R_2}{R_3} \frac{\left(\frac{1}{sR_1C_3} + 1\right) \left(\frac{1}{sR_2C_1} + 1\right)}{\left(1 + sR_2C_2\right) \left(\frac{1}{sR_3C_3} + 1\right)}$$

$$G(f) \approx \frac{R_2}{R_3} \frac{\left(1 + \frac{s_{z1}}{s}\right) \left(1 + \frac{s_{z2}}{s}\right)}{\left(1 + \frac{s}{s_{p1}}\right) \left(1 + \frac{s}{s_{p2}}\right)}$$

$$R_2 = \sqrt{\frac{(f_{p1}^2 + f_c^2)(f_{p2}^2 + f_c^2)}{(f_{z1}^2 + f_c^2)(f_{z2}^2 + f_c^2)}} \frac{Gf_cR_3}{f_{p1}}$$



Type 3



Manual pole/zero placement is a solution

parameters

Rupper=10k
Rlower=Rupper

fc=10k
Gfc=-20

G=10^(-Gfc/20)
pi=3.14159

fz1=1k
fz2=1k
fp1=26k
fp2=26k

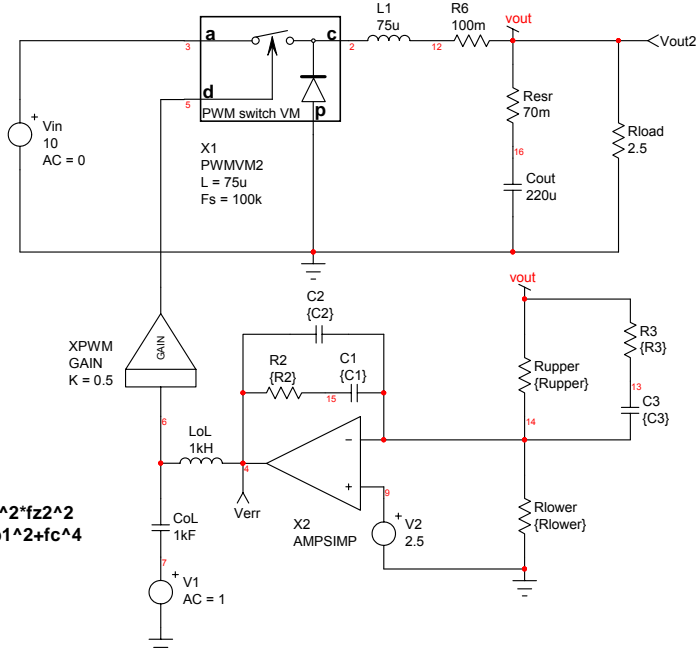
C3=1/(2*pi*fz1*Rupper)
R3=1/(2*pi*fp2*C3)

C1=1/(2*pi*fz2*R2)
C2=1/(2*pi*(fp1)*R2)

a=fc^4+fc^2*fz1^2+fc^2*fz2^2+fz1^2*fz2^2
c=fp2^2*fp1^2+fc^2*fp2^2+fc^2*fp1^2+fc^4

R2=sqrt(c/a)*G*fc*R3/fp1

fz1x=1/(2*pi*C1*R2)
fz2x=1/(2*pi*C3*(Rupper+R3))
fp1x=1/(2*pi*(C1*C2/(C1+C2))*R2)
fp2x=1/(2*pi*C3*R3)

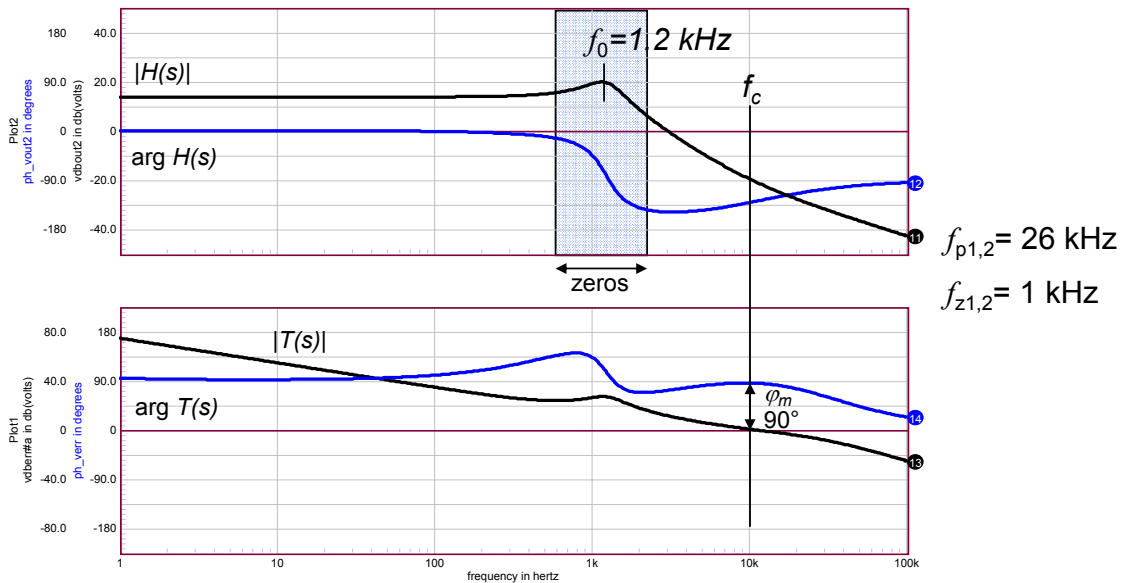


Automated calculations help iterations!



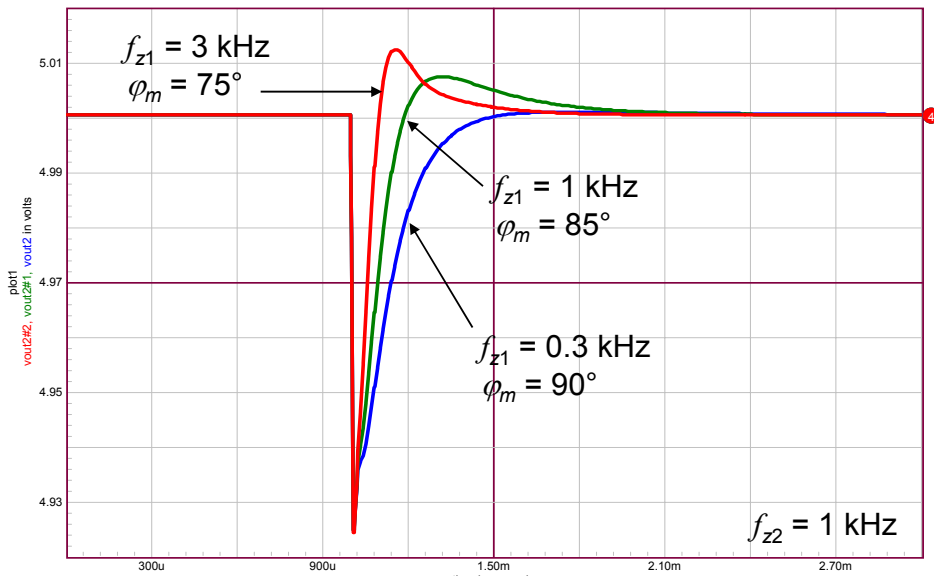
Manual pole/zero placement is a solution

- Conditional stability is gone
- Fine tuning is now possible



The zeros position affects the response

- Transient response changes as one zero position moves



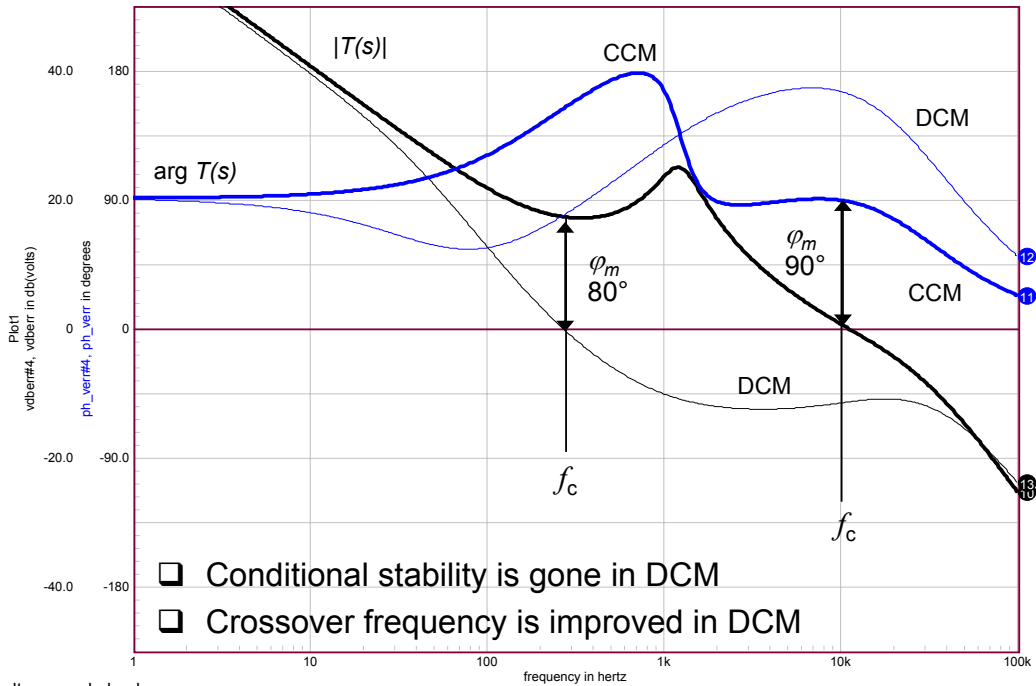
The zeros position affects the response

- Splitting the zeros can fix DCM stability issue in buck VM

	Frequency	Overshoot	Settling time	Phase margin
f_{z1}	↗	↗	faster	↘
	↘	↘	slower	↗
f_{z2}	↗	↗	faster	↘



By decreasing f_{z1} , DCM stability is improved

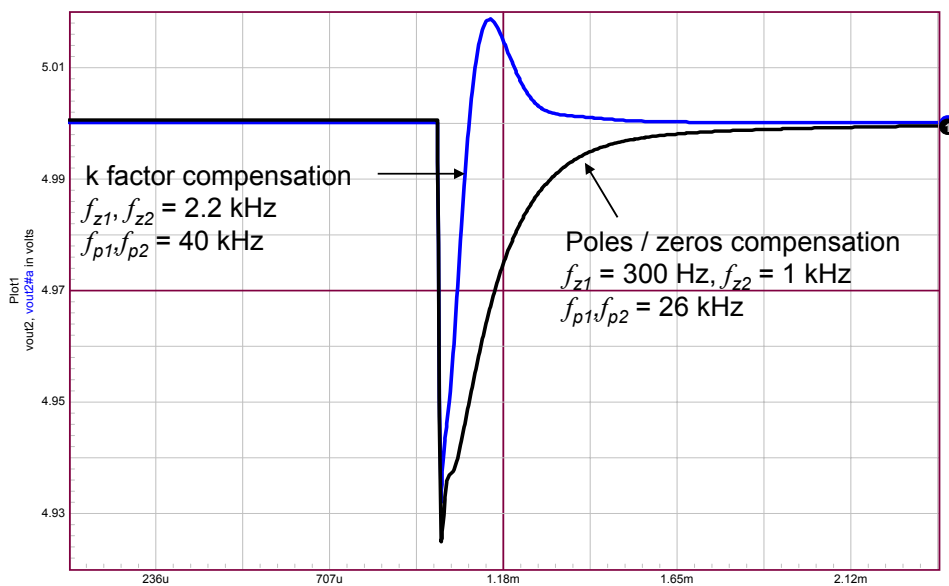


CCM/DCM voltage mode buck



Improving DCM stability slows down the buck

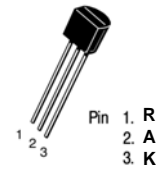
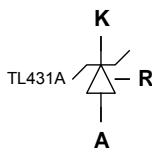
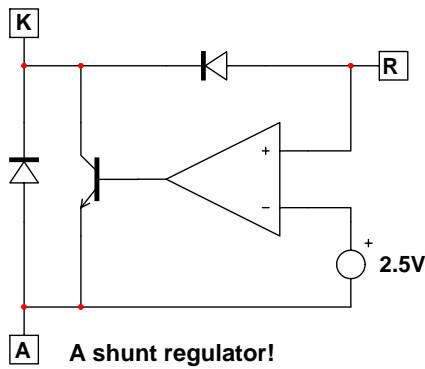
f_{z1} moving low, it hampers the response time...





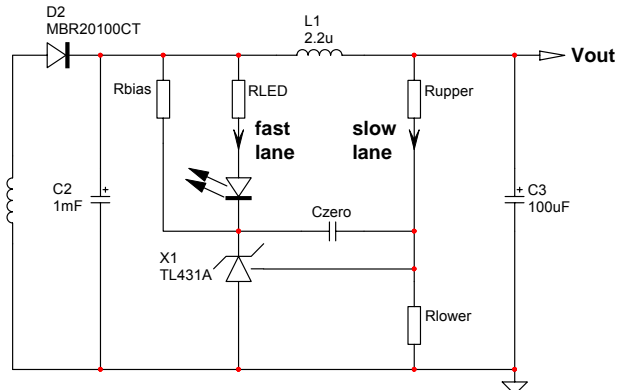
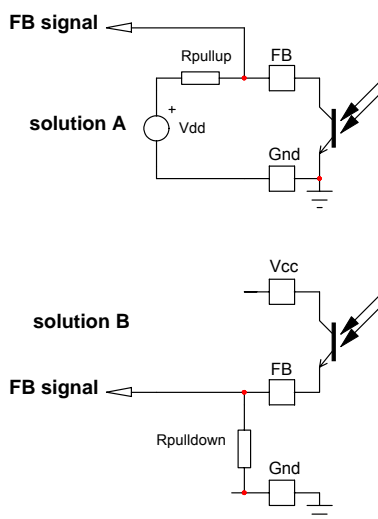
Type 2 and 3 with a TL431

- ❑ Literature examples use op amps to close the loop.
- ❑ Reality differs as the TL431 is widely implemented.
- ❑ How to adapt type 2 and 3 to a TL431 circuit?
- ❑ Question: who hides behind the TL431 anyway?



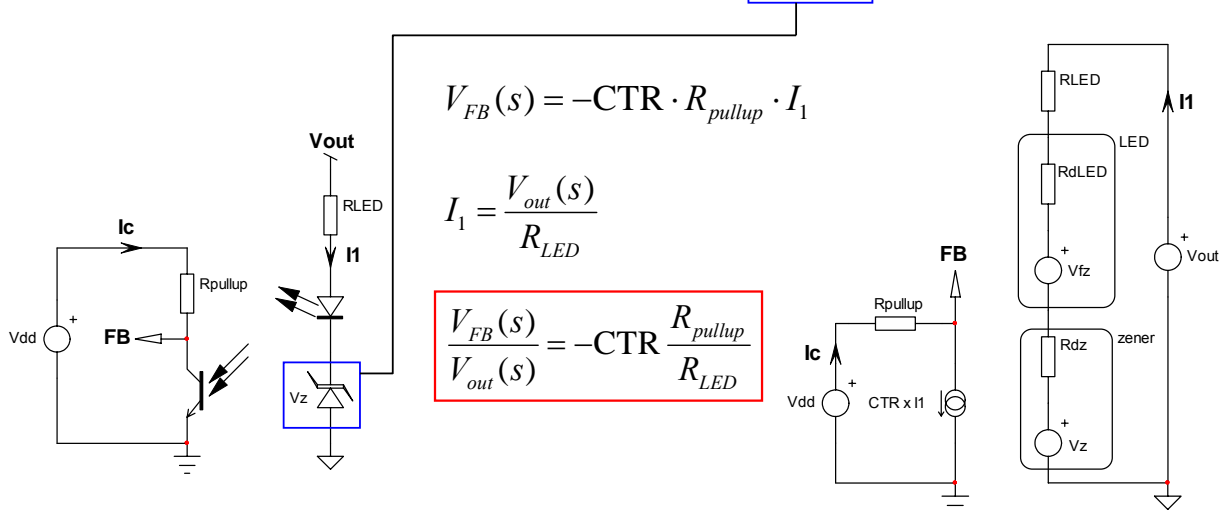
Feedback with a TL431

- ❑ A TL431 implements a two-loop configuration



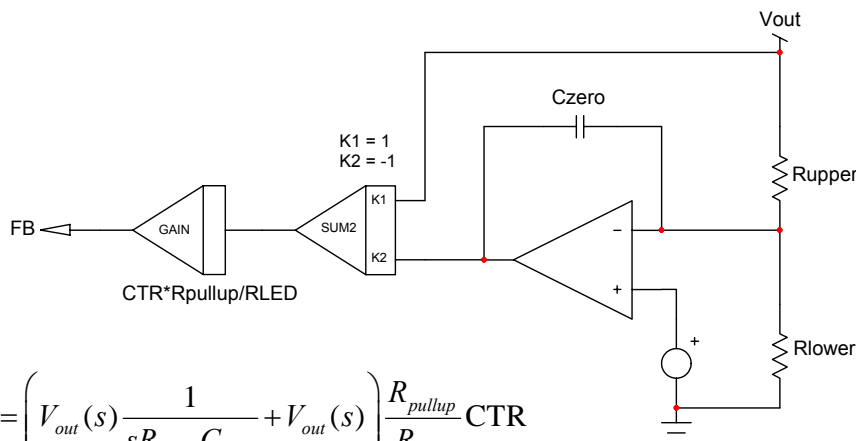
Feedback with a TL431

- When C_{zero} is a short circuit, the slow lane is off
- The TL431 turns into a static Zener diode



The TL431 and its equivalent schematic

- The fast lane presence adds a zero in the equation



$$V_{FB}(s) = \left(V_{out}(s) \frac{1}{sR_{upper}C_{zero}} + V_{out}(s) \right) \frac{R_{pullup}}{R_{LED}} CTR$$

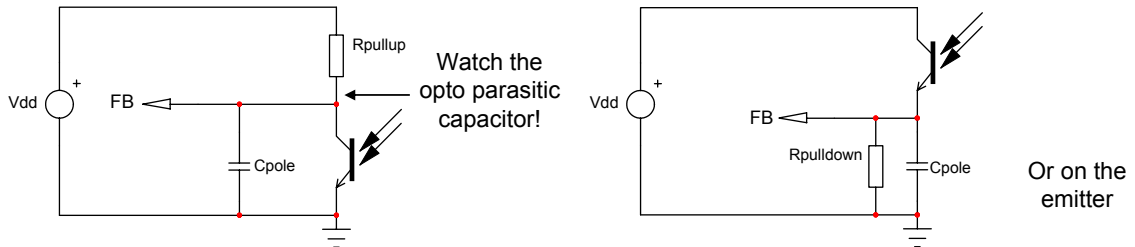
$$\frac{V_{FB}(s)}{V_{out}(s)} = - \left(\frac{1}{sR_{upper}C_{zero}} + 1 \right) \frac{R_{pullup}}{R_{LED}} CTR = - \left(\frac{sR_{upper}C_{zero} + 1}{sR_{upper}C_{zero}} \right) \frac{R_{pullup}}{R_{LED}} CTR$$

No high frequency pole?

Type 2

Adding a pole for a type 2 circuit

- The pole is a simple capacitor on the collector



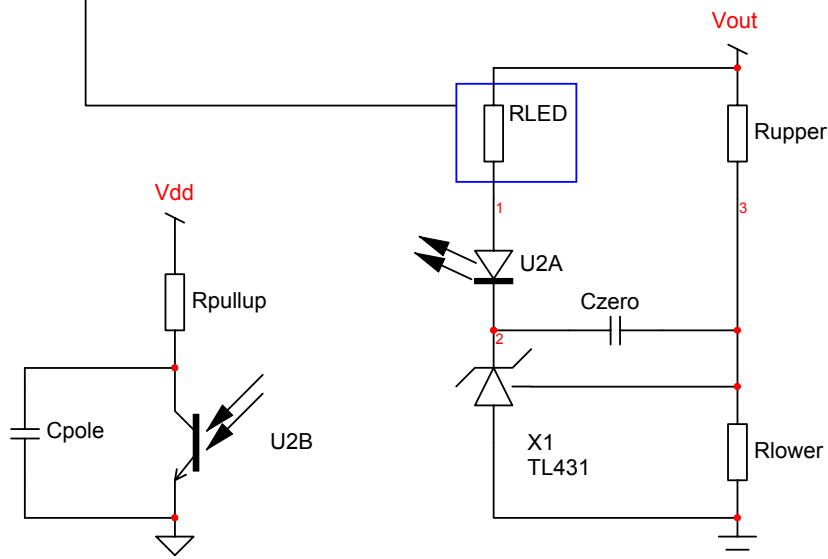
$$G(s) = \frac{V_{FB}(s)}{V_{out}(s)} = - \left(\frac{sR_{upper}C_{zero} + 1}{sR_{upper}C_{zero}} \right) \left(\frac{1}{1 + sR_{pullup}C_{pole}} \right) \frac{R_{pullup}}{R_{LED}} CTR$$

$$f_{po} = \frac{1}{2\pi R_{upper}C_{zero}} \quad f_z = \frac{1}{2\pi R_{upper}C_{zero}} \quad G = \frac{R_{pullup}}{R_{LED}} CTR \quad f_p = \frac{1}{2\pi R_{pullup}C_{pole}}$$

Pole at the origin Low frequency zero Mid-band gain High frequency pole **Type 2**

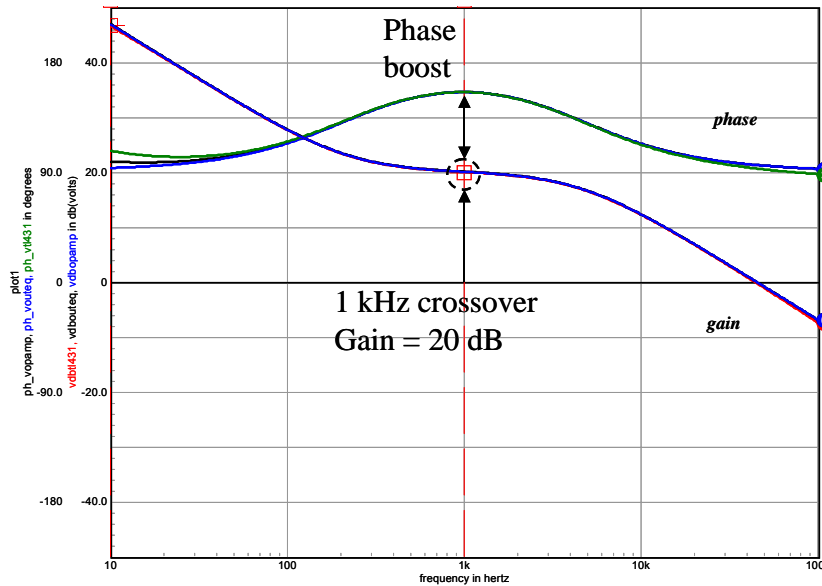
The type 2 final implementation

- The LED resistor fixes the mid-band gain



Comparing a type 2 op amp with a TL431

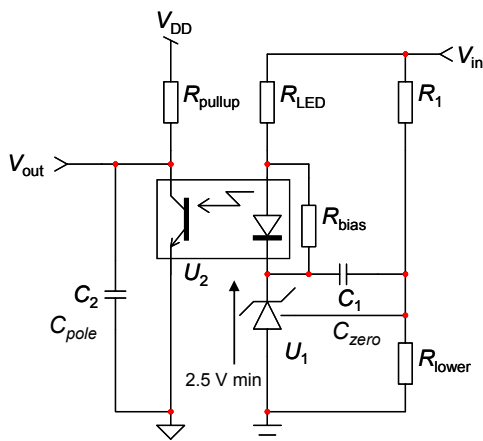
□ The ac plot shows no significant differences



Type 2

The TL431 type 2 is not a panacea!

- Because R_{LED} must also ensures sufficient bias current:
 - There is an upper resistor limit
 - The minimum mid-band gain you can obtain is clamped!
 - Worst case is low V_{out} , e.g. 5 V



$$R_{LED,max} \leq \frac{V_{out} - V_f - V_{TL431,min}}{V_{dd} - V_{CE,sat}} R_{pullup} CTR_{min}$$

$$R_{LED,max} \leq \frac{5 - 1 - 2.5}{5 - 0.3} \times 0.3 \times 20k \leq 1.9 k\Omega$$

$$G_0 \geq \frac{R_{pullup}}{R_{LED,max}} CTR_{min}$$

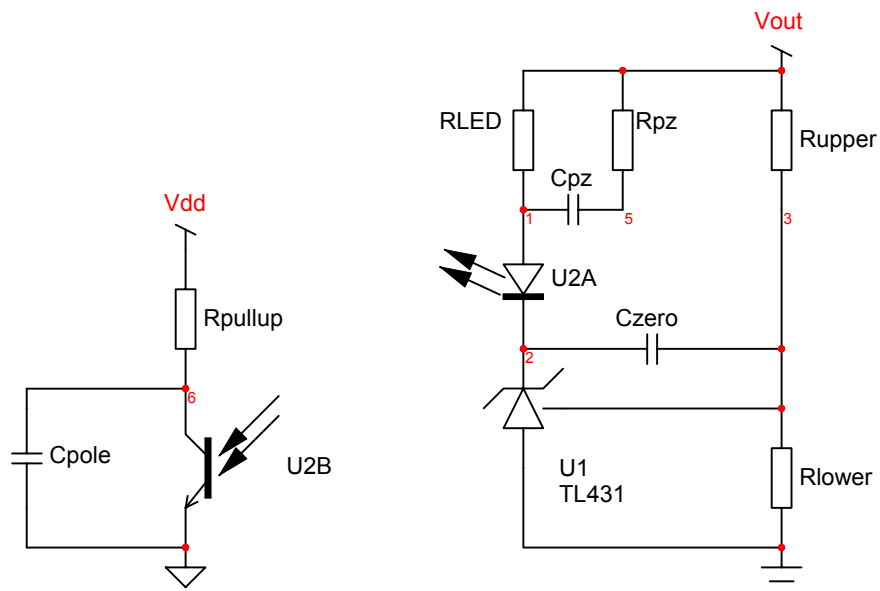
$$G_0 \geq \frac{V_{dd} - V_{CE,sat}}{V_{out} - V_f - V_{TL431,min}} \geq 3.13 \text{ or } \approx 10 \text{ dB}$$

Cannot go below this!



A type 3 is also possible

□ The type 3 is less flexible given the R_{LED} role



Type 3



If $\arg H(s)$ @ f_c is less than 45° ...

□ We can use a true type 1 without gain limits

➤ No phase boost!

$$R_{LED,max} \leq \frac{5-1-2.5}{5-0.3} \times 0.3 \times 20k \leq 1.9 \text{ k}\Omega \xrightarrow{20\% \text{ margin}} R_{LED,max} = 0.8 \times 1.9 \text{ k}\Omega = 1.5 \text{ k}\Omega$$

$$G(s) = - \left(\frac{sR_{upper}C_{zero} + 1}{s \frac{R_{upper}R_{LED}}{R_{pullup}CTR} C_{zero}} \right) \left(\frac{1}{1 + sR_{pullup}C_{pole}} \right)$$

coincident

Origin pole $\rightarrow \omega_{po} = \frac{1}{s \frac{R_{upper}R_{LED}}{R_{pullup}CTR} C_{zero}}$

$$f_{po} = f_c G \rightarrow C_{pole} = \frac{CTR}{2\pi f_{po} R_{LED}} \quad C_{zero} = \frac{R_{pullup}}{R_{upper}} C_{pole}$$



The type 3 with a TL431

□ R_{LED} affects the gain and the zero position

$$G(s) = \frac{V_{FB}(s)}{V_{out}(s)} = - \left(\frac{sR_{upper}C_{zero1} + 1}{sR_{upper}C_{zero1}} \right) \left(\frac{1}{1 + sR_{pullup}C_{pole2}} \right) \left[\frac{sC_{pz}(R_{LED} + R_{pz}) + 1}{(sR_{pz}C_{pz} + 1)} \right] \frac{R_{pullup}}{R_{LED}} CTR$$

$$f_{po} = \frac{1}{2\pi R_{upper} C_{zero1}}$$

$$f_{z1} = \frac{1}{2\pi R_{upper} C_{zero1}}$$

$$f_{p1} = \frac{1}{2\pi R_{pz} C_{pz}}$$

$$f_{z2} = \frac{1}{2\pi (R_{LED} + R_{pz}) C_{pz}}$$

$$G = \frac{R_{pullup}}{R_{LED}} CTR$$

$$f_{p2} = \frac{1}{2\pi R_{pullup} C_{pole2}}$$

Pole at the origin

Two zeros

Mid-band gain

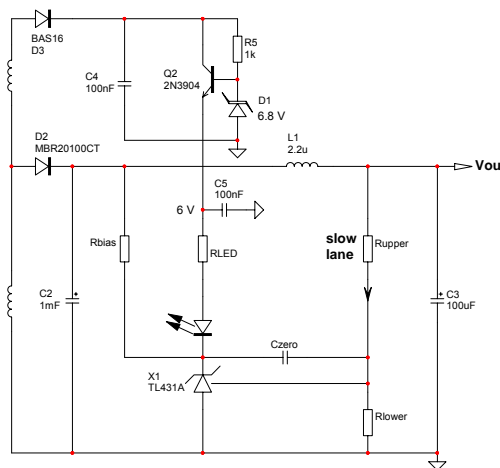
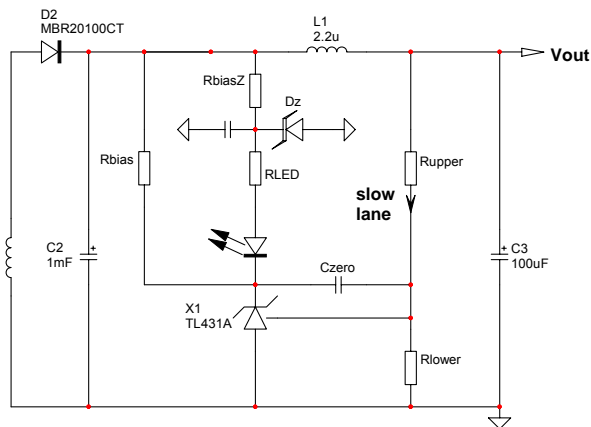
Two poles

Type 3



The problem is the fast lane...

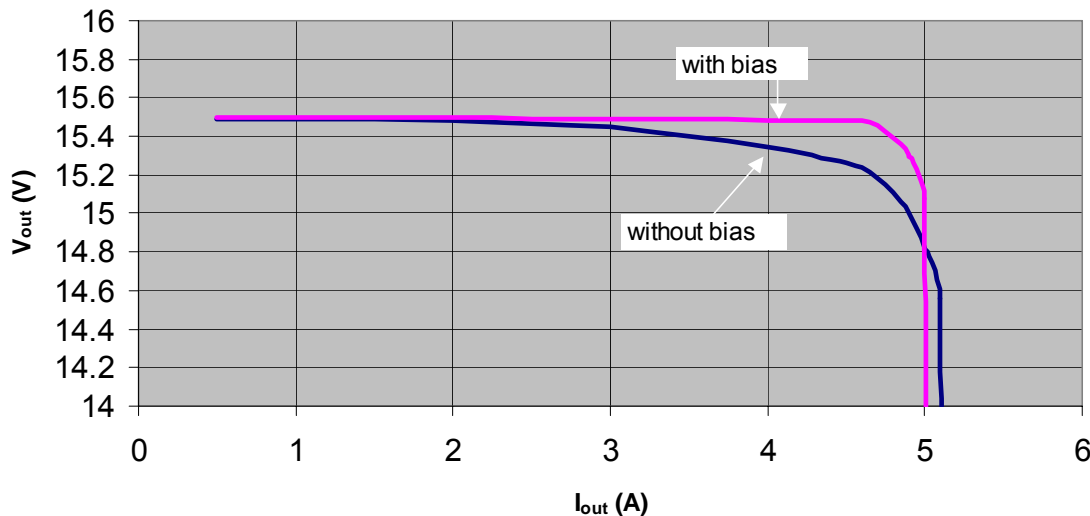
- The difference with the TL431 comes from the fast lane
- Can we get rid of it?



➤ Rather costly solutions...

Watch for the TL431 bias!

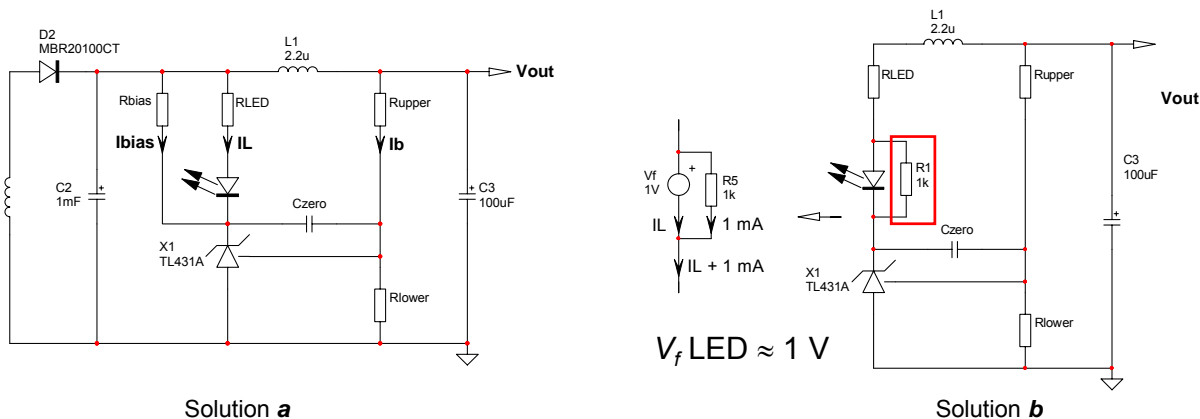
- ❑ A TL431 requires 1 mA minimum to operate within specs
- ❑ A TLV431, 100 μ A only...



- ❑ Changes in Z_{out} implies a change in $T(0)$

How to provide more bias?

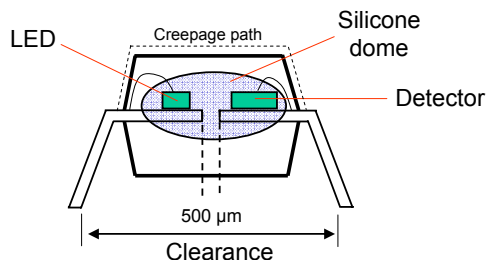
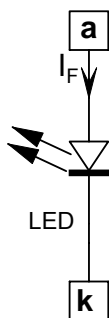
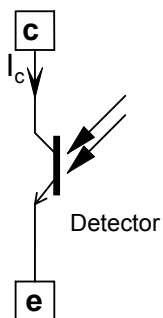
- ❑ Connect a resistor to V_{out}
- ❑ Use the LED to form a constant current source



- ❑ R_1 on the right picture steals current away from the LED

The optocoupler is the traitor here!

- ❑ You need galvanic isolation between the prim. and the sec.
- ❑ An optocoupler transmits light only, no electrical link



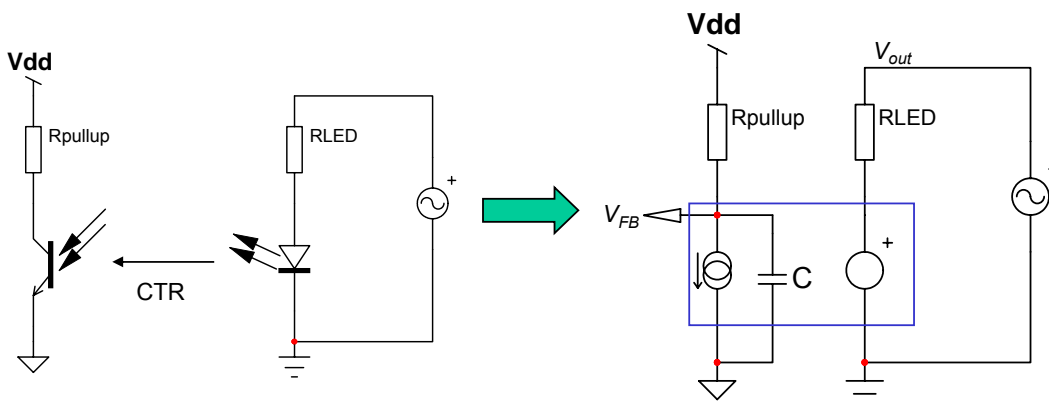
$$CTR = \frac{I_c}{I_F} \times 100$$

Current Transfer Ratio

Luigi Galvani, 1737-1798
Italian physician and physicist

The internal pole should be known

- ❑ The photons are collected by a collector-base area.
- ❑ This area offers a large parasitic capacitance.



$$\frac{V_{FB}(s)}{V_{out}(s)} = -\frac{R_{pullup} CTR}{R_{LED}} \frac{1}{1 + sR_{pullup} C}$$

If f_p is above 5 times f_c , its effect is negligible
 If f_p is close to f_c , phase margin degradation

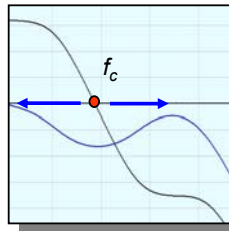
Assess the CTR variations

- CTR changes with the operating current!
- Try to select collector bias currents around 2-5 mA

Current Transfer Ratio (I_C/I_F at $V_{CE}=5.0$ V) and Collector-emitter Leakage Current

Parameter	-1	-2	-3	-4	-12	-23	-34	-13	-24	-14	Unit
I_C/I_F ($I_F=10$ mA)	40-80	63-125	100-200	160-320	40-125	63-200	100-320	40-200	63-320	40-320	%
I_C/I_F ($I_F=1.0$ mA)	30(>13)	45(>22)	70(>34)	90(>56)	30(>13)	45(>22)	70(>34)	30(>13)	45(>22)	30(>13)	
Collector-Emitter Leakage Current, I_{CEO} , $V_{CE}=10$ V	2.0(≤50)	2.0(≤50)	5.0(≤100)	5.0(≤100)	2.0(≤50)	5.0(≤100)	5.0(≤100)	5.0(≤100)	5.0(≤100)	5.0(≤100)	nA

CTR between 0.63 and 1.25
Normalized to 1 (0 dB)
0.63 gives -4 dB
1.25 gives +2 dB



Watch out for crossover frequency changes and phase margin at CTR extremes!

SFH-615

Selecting the right optocoupler

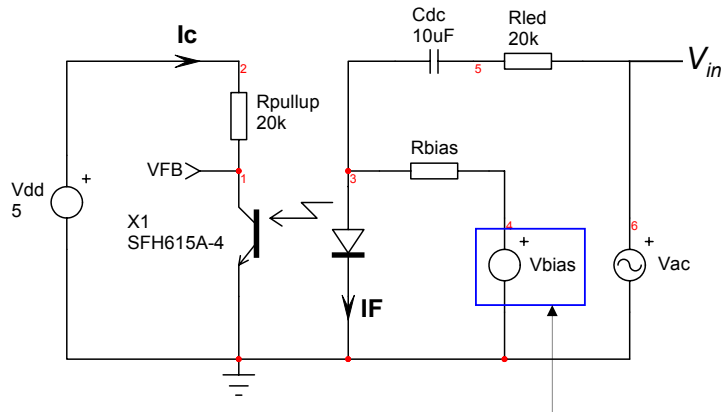
- High temperature shortens the optocoupler lifetime
- Low LED currents:
 - expand lifetime
 - CTR dispersion increases
- Low CTR optocouplers have low internal capacitance
- Higher drive currents improve speed but:
 - Shorten lifetime
 - Degrade standby power



Understand the optocoupler impact on your design!

Find the pole position on your optocoupler

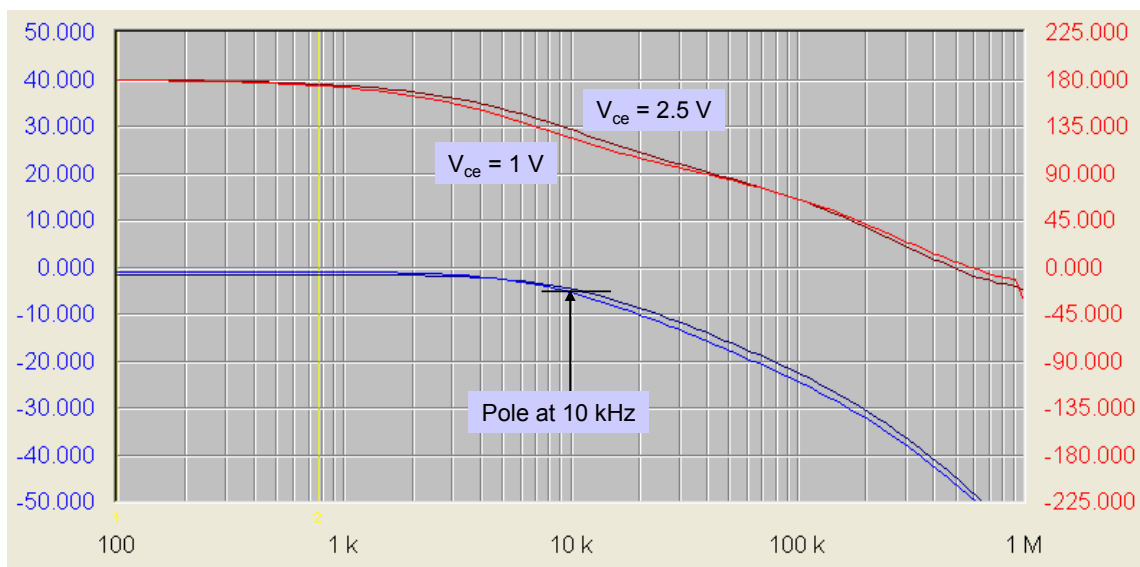
Once your optocoupler is selected, characterize it:



- Adjust V_{bias} to $V_{FB} = 2.5\text{ V}$ (or $V_{dd}/2$) then ac sweep
- Observe V_{FB} with a scope or a network analyzer

Select the bias point as in your circuit

Identify the pole position. Here it is located at 10 kHz

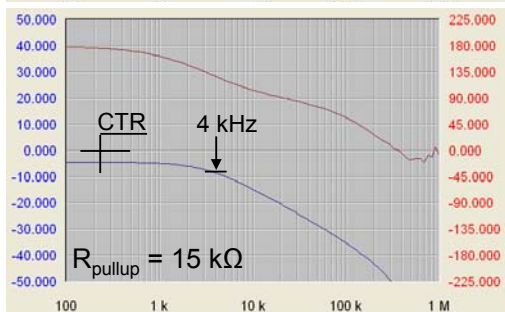
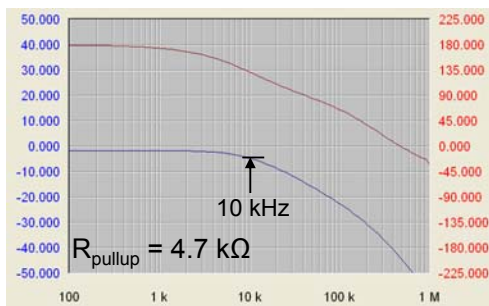
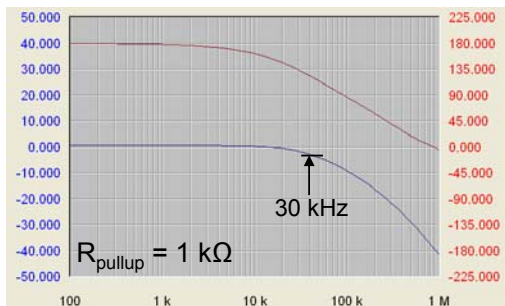


$R_{pullup} = 4.7\text{ k}\Omega$

SFH615A-2

Changing the pullup affects the pole position

□ A low pullup resistor offers better bandwidth!



□ Changing the bias point affects the CTR

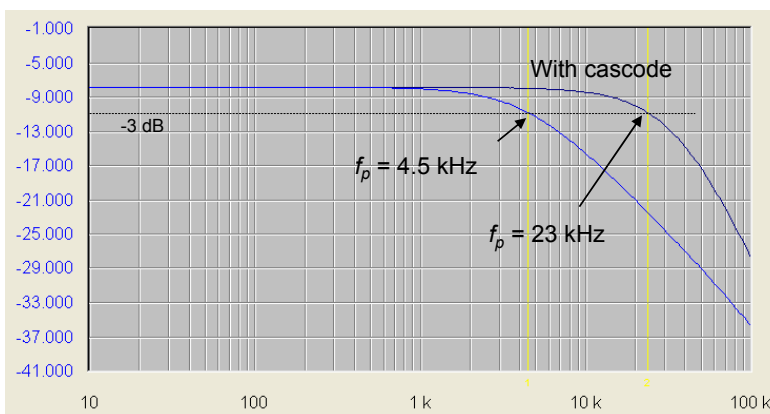
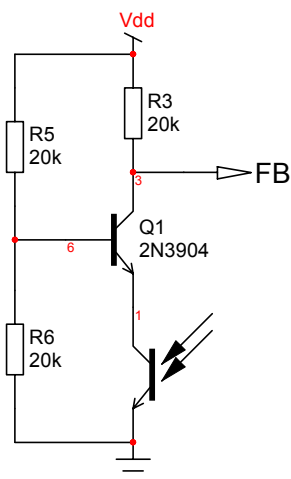
$$\frac{V_{FB}(s)}{V_{out}(s)} = -\frac{R_{pullup}}{R_{LED}} \text{CTR}$$

□ If $R_{pullup} = R_{LED}$, then $|G_0| = 0\text{ dB} \dots ?$

SFH615A-2

To push the pole farther, use a cascode

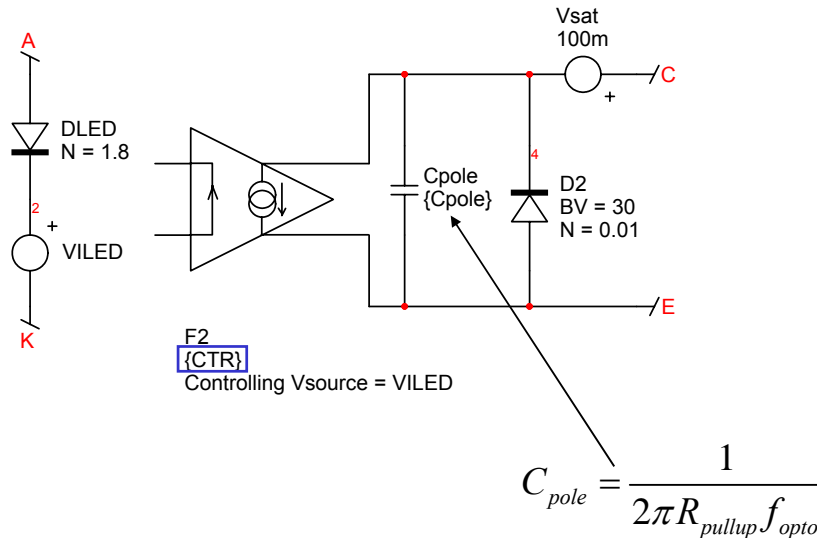
- The cascode fixes the optocoupler collector potential
- It neutralizes the Miller capacitance of the optocoupler



SFH615A-2

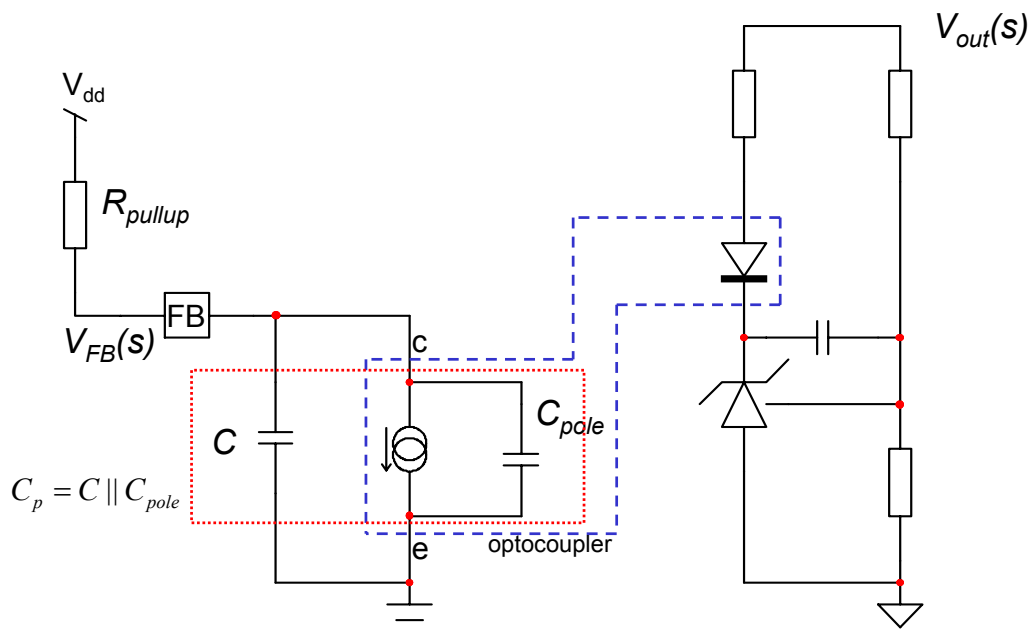
A simple optocoupler model

- ❑ Read the pole position and the CTR value
- ❑ Adjust the internal capacitor value to fix the pole



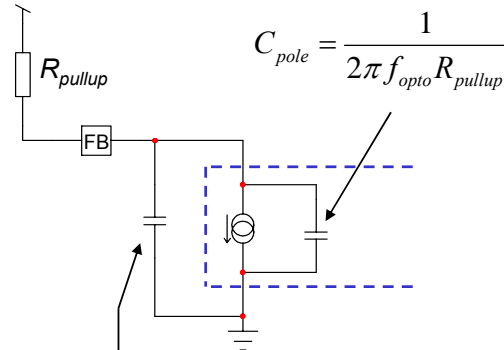
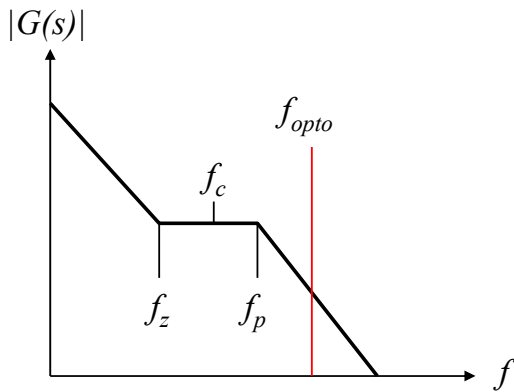
With the TL431, the situation is different

- ❑ In the TL431 application, the pole is inherently present



With the TL431, the situation is different

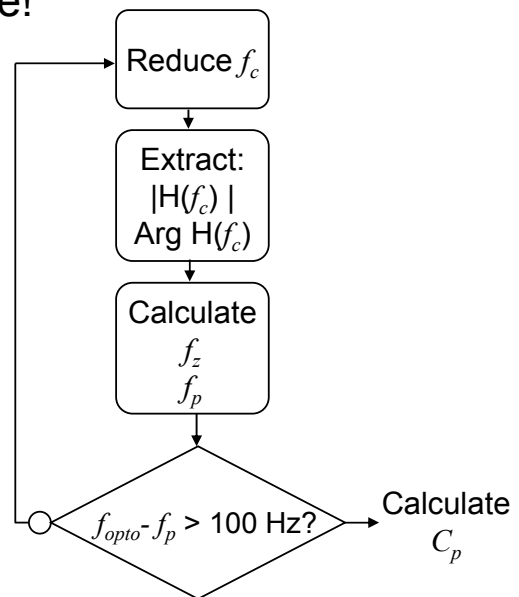
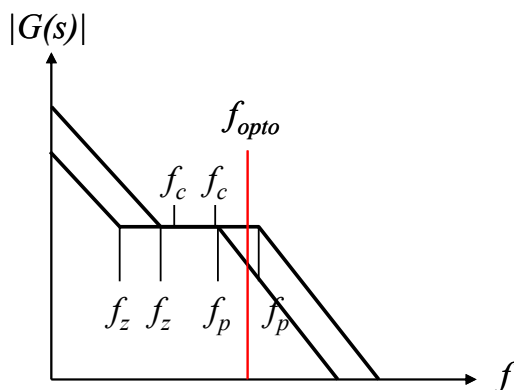
- ❑ First case: the optocoupler pole is beyond $G(s)$ pole
- Calculate C_p to combine with C_{pole}



From $G(s)$ calculations $\rightarrow C \rightarrow C_p = C - C_{pole}$

With the TL431, the situation is different

- ❑ Second case: the optocoupler pole is below $G(s)$ pole
- The optocoupler sets $G(s)$ pole!

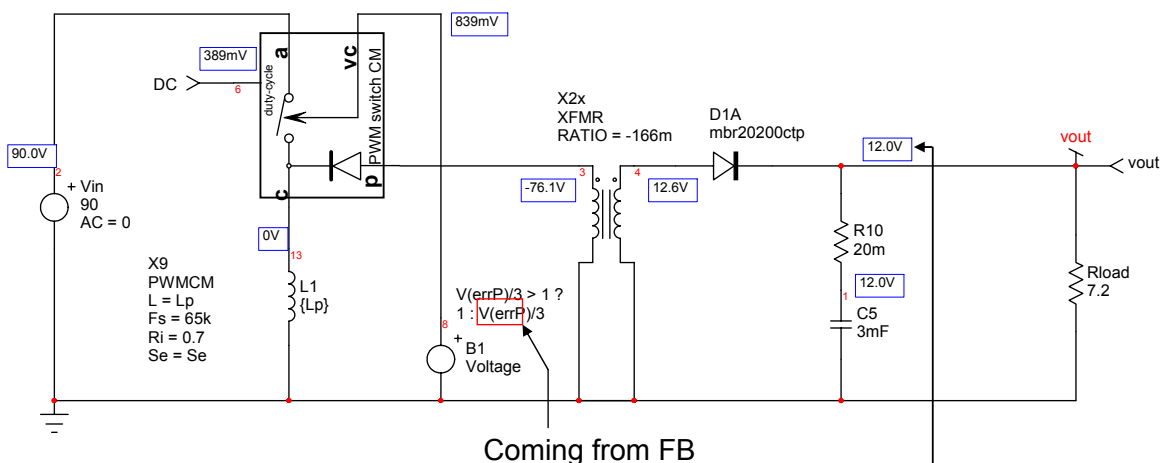


Design example: a DCM flyback converter

- ❑ We want to stabilize a 20-W DCM adapter
 - ❑ $V_{in} = 85$ to 265 V rms, $V_{out} = 12$ V/1.7 A
 - ❑ $F_{sw} = 65$ kHz, $R_{pullup} = 20$ k Ω
 - ❑ Optocoupler is SFH-615A, pole is at 6 kHz
 - ❑ Cross over target is 1 kHz
 - ❑ Selected controller: NCP1216
1. Obtain a power stage open-loop Bode plot, $H(s)$
 2. Look for gain and phase values at cross over
 3. Compensate gain and build phase at cross over, $G(s)$
 4. Run a loop gain analysis to check for margins, $T(s)$
 5. Test transient responses in various conditions

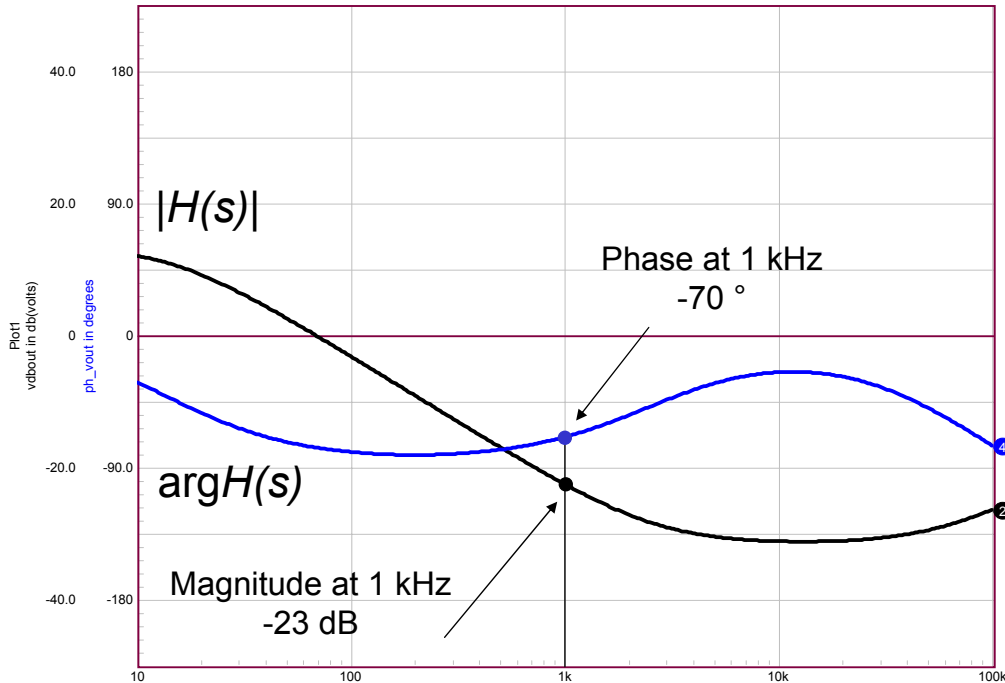
Stabilizing a DCM flyback converter

- ❑ Capture a SPICE schematic with an averaged model



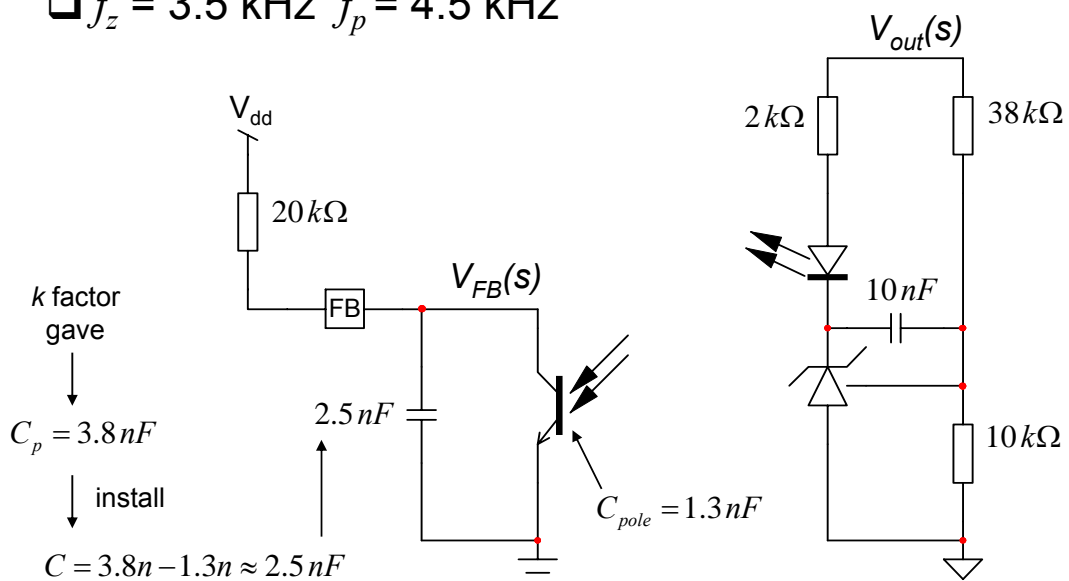
- ❑ Look for the bias points values: $V_{out} = 12$ V, ok

Stabilizing a DCM flyback converter



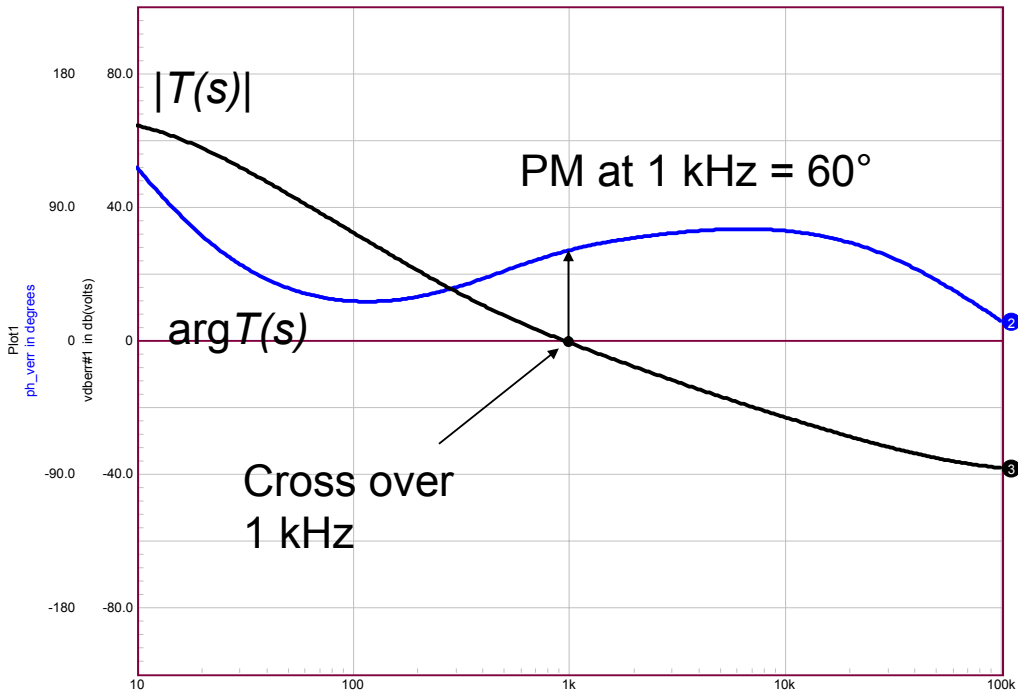
Stabilizing a DCM flyback converter

- Apply k factor or other method, get f_z and f_p
- $f_z = 3.5 \text{ kHz}$ $f_p = 4.5 \text{ kHz}$





Stabilizing a DCM flyback converter



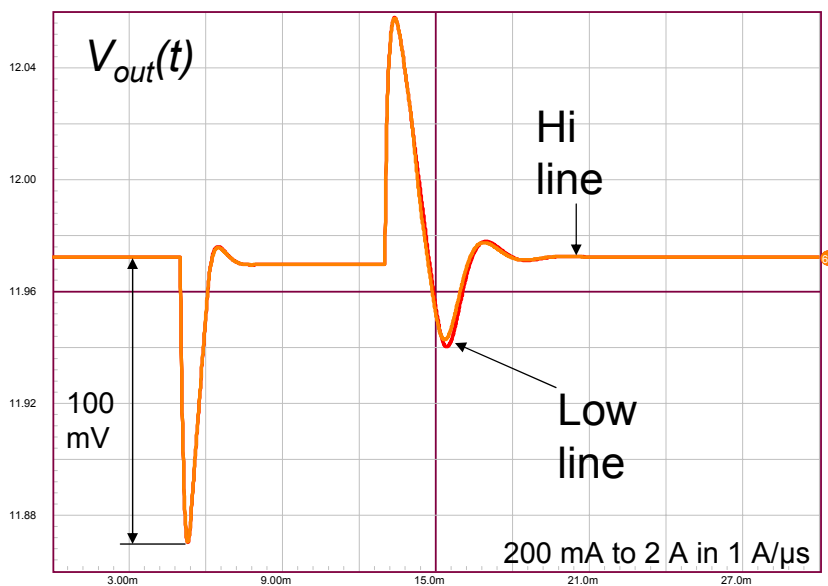
155

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Stabilizing a DCM flyback converter

- ☐ Sweep ESR values and check margins again

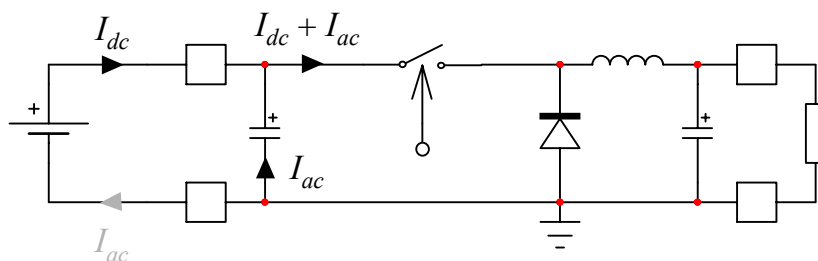


156

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The need for an input filter

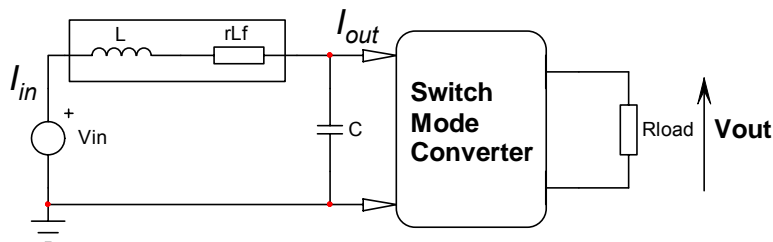
- ❑ The dc-dc converter is supplied from a dc source
- ❑ A capacitor locally decouples the line. In theory:
 - The capacitor supplies the ac current
 - The source only sees a dc current



- ❑ In reality, the capacitor is not perfect (limited cap., ESR and ESL)
- ❑ Some ac current manages to enter the source
- Source pollution, adjacent converters disturbance, radiated noise

The need for an input filter

- ❑ A front-end filter has to be installed
- ❑ The ac current will only flow in C as L opposes its circulation
- ❑ The remaining ac current in the source must pass the specs!



The need for an input filter

- Our dc-dc ensures P_{out} is constant regardless of V_{in}
 - If V_{in} increases, I_{in} reduces to keep $V_{in} \cdot I_{in}$ constant
 - If V_{in} decreases, I_{in} increases to keep $V_{in} \cdot I_{in}$ constant
- } $\eta = 100\%$

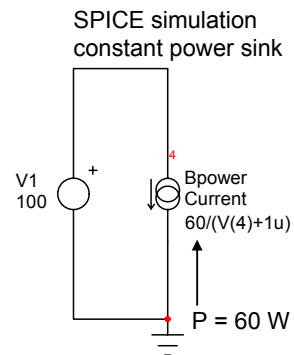
$$R_{in} = \frac{V_{in}}{I_{in}} \quad V_{in} = \frac{P_{in}}{I_{in}} \quad P_{in} = R_{load} I_{out}^2$$

$$\frac{dV_{in}}{dI_{in}} = \frac{d}{dI_{in}} \frac{R_{load} I_{out}^2}{I_{in}} = -R_{load} \left(\frac{I_{out}}{I_{in}} \right)^2$$

Neg. sign!

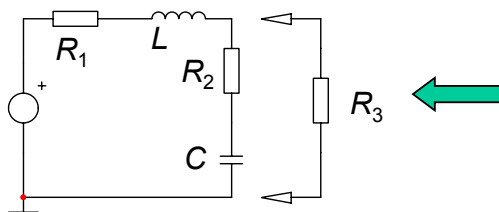
```
.TF V(4) V1 ; transfer function analysis
**** SMALL SIGNAL DC TRANSFER FUNCTION
output_impedance_at_V(4) 0.000000e+000
v1#Input_impedance -1.66667e+002
Transfer_function 1.000000e+000
```

Neg. sign!



The need for an input filter

- Ohmic paths in the RLC filter are losses that damp the network



- If these losses are compensated, the damping factor is zero

$$T(s) = \frac{1}{s^2 + \frac{s}{\omega_0 Q} + 1} \quad Q = \frac{1}{2\zeta}$$

If ohmic losses are gone, the damping factor is zero.

- In a lossy RLC filter, the damping factor is:

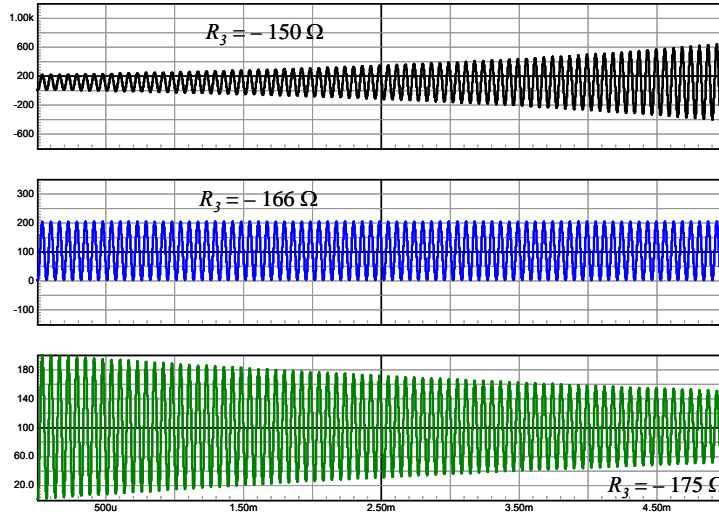
$$\zeta = \frac{L + C(R_2 R_3 + R_1 R_3 + R_2 R_1)}{2(R_1 + R_3)} \omega_0 \xrightarrow{=0} R_3 = -\frac{R_1 R_2 C + L}{C(R_1 + R_2)}$$



A tunnel-based oscillator?

- ❑ Loading an *RLC* filter with a negative impedance:
- You build a negative resistance oscillator!

$$R_3 = -\frac{R_1 R_2 C + L}{C(R_1 + R_2)} = -\frac{100m \times 500m \times 1u + 100u}{1u \times (600m)} = -166 \Omega$$



Diverging oscillations
 $\zeta < 0$

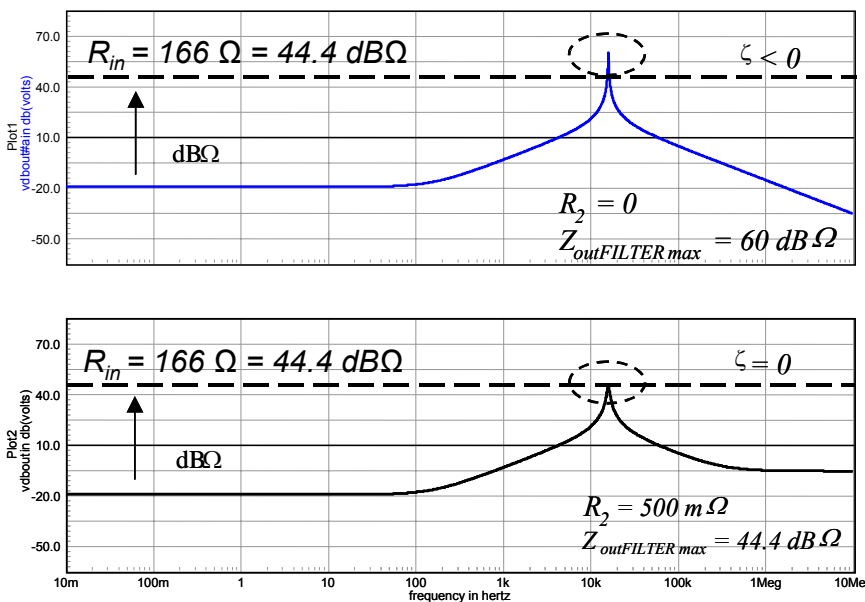
Steady-state oscillations
 $\zeta = 0$

Damped oscillations
 $\zeta > 0$

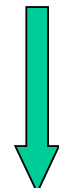


Watch the output/input impedances

- ❑ A possibility to look for oscillations is to ac sweep Z_{out} of *RLC*



$$\|Z_{out}\|_{max} = \frac{Z_0^2}{R_1} \sqrt{1 + \left(\frac{R_1}{Z_0}\right)^2}$$

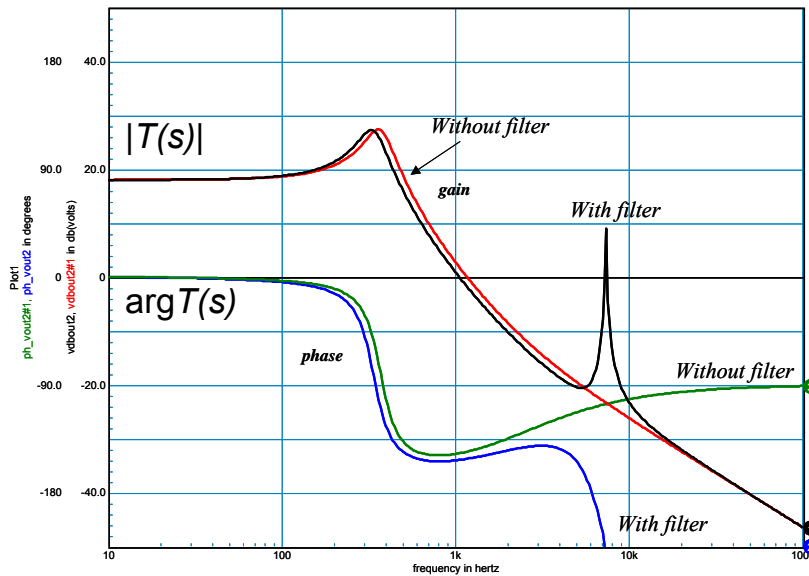


To be safe

$$\|Z_{out}\| \ll \|Z_{in}\|$$

The filter effect also appears on the Bode plot

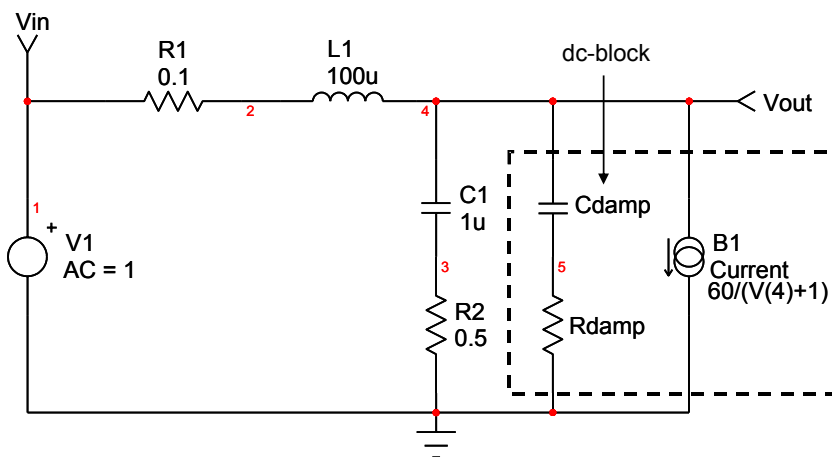
- ❑ The filter peaking brings instabilities...



Damp that RLC filter!

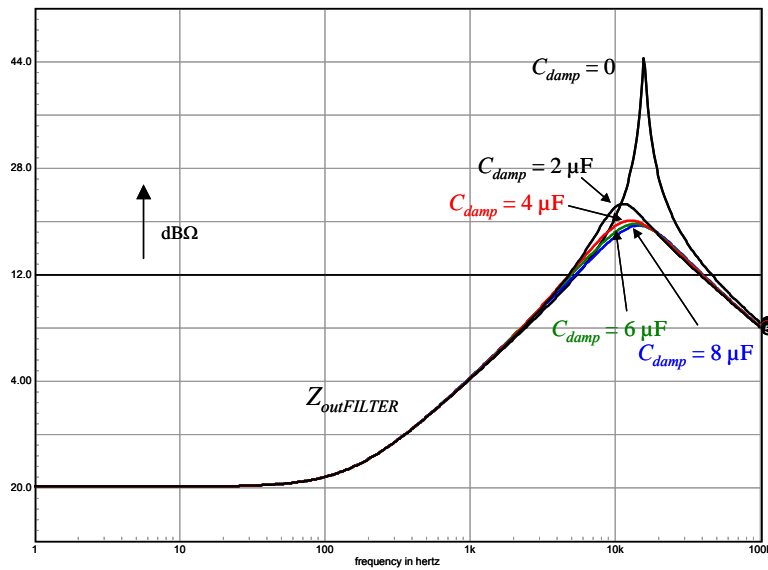
How to damp the RLC network

- ❑ Damping can be obtained through different arrangements:
 - a resistor in parallel with C_1
 - a resistor in parallel with L_1
 - Numerous other possible combinations!



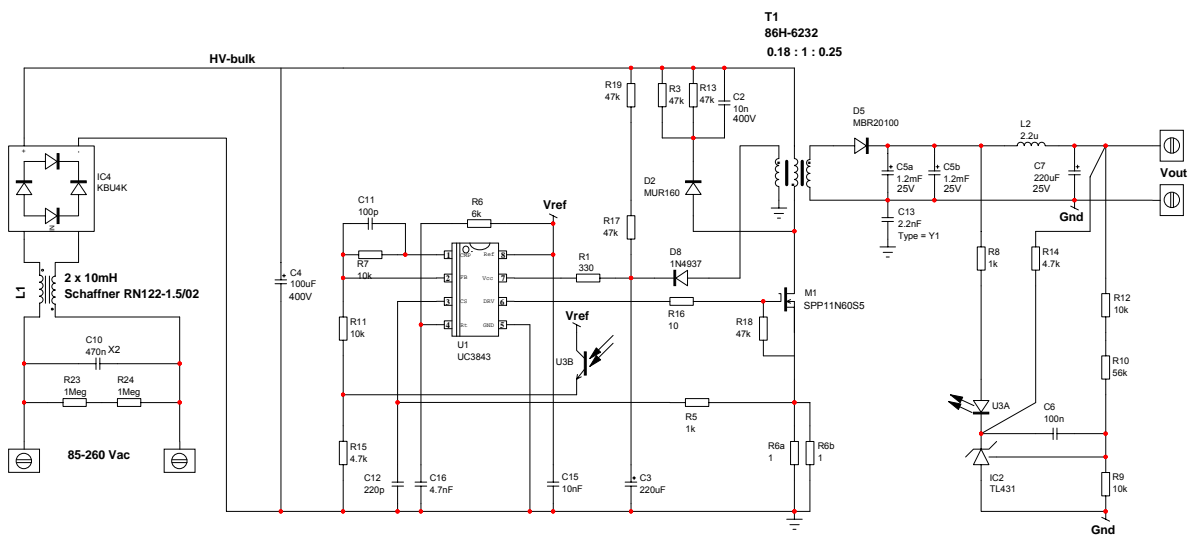
Buck, a design example

- ❑ The damping resistor is calculated to 9.5 Ω in this example
- ❑ The dc-block capacitor is swept from 2 to 10 times C_1



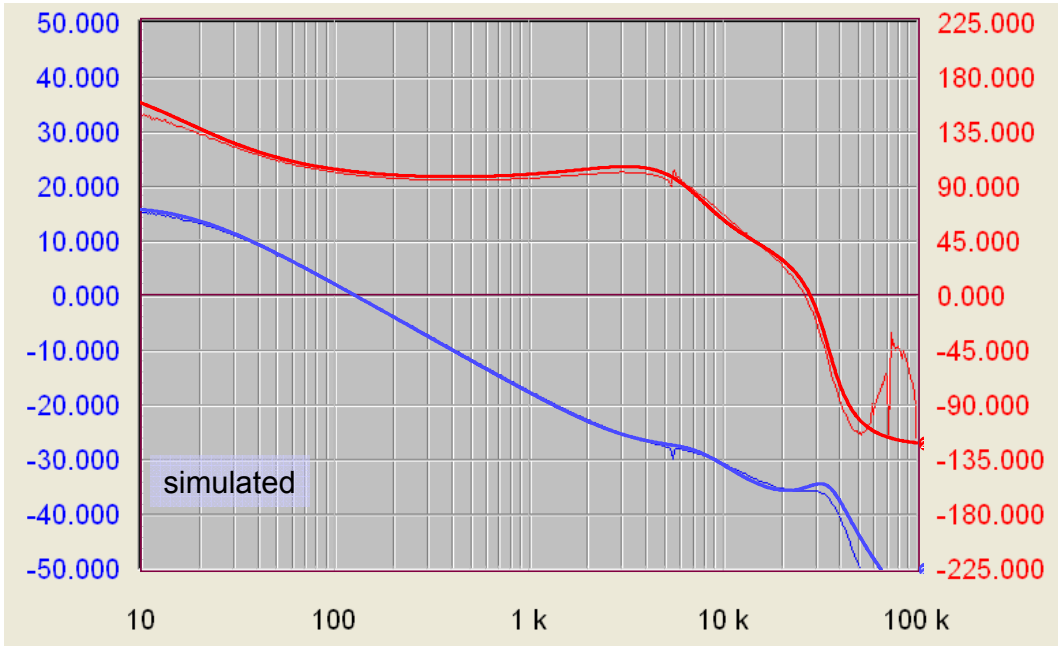
Stabilizing a UC3843 converter

- ❑ A 19 V/3 A converter is built around an UC3843





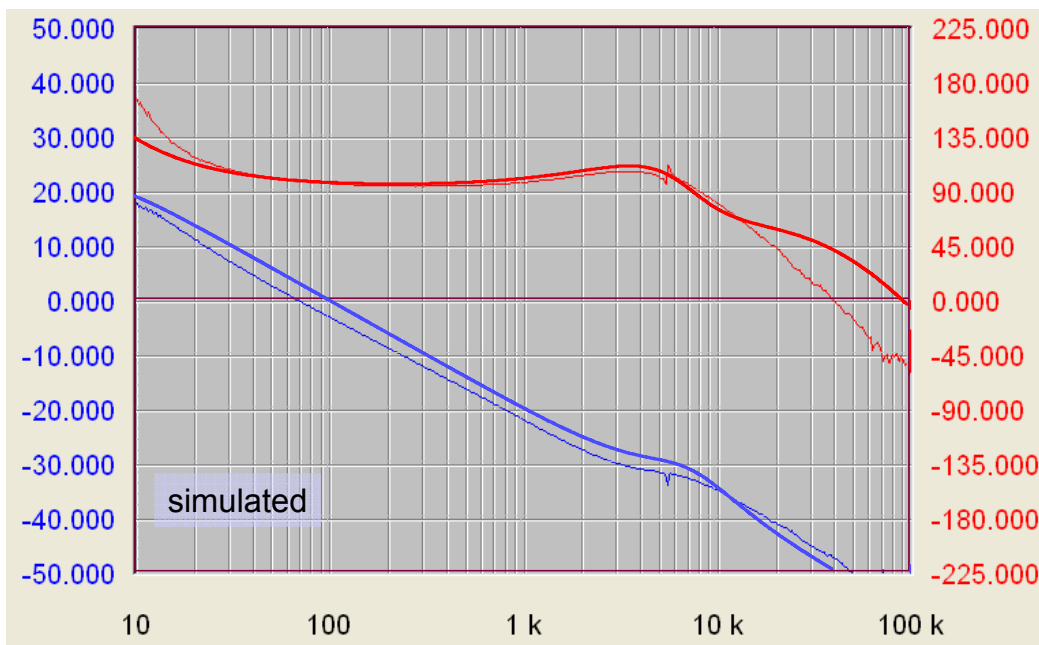
Change the operating conditions easily



CCM operation, $R_{load} = 6.3 \Omega$



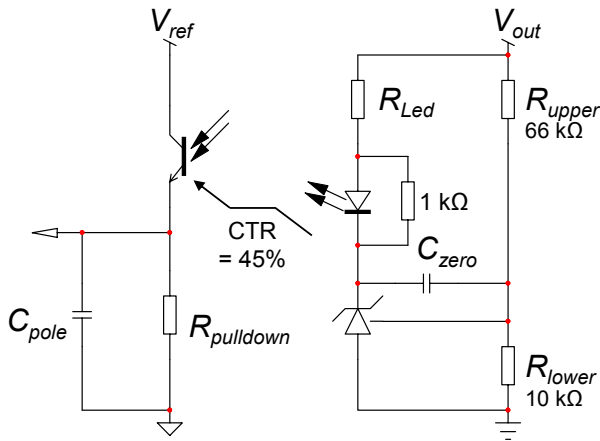
Reduce the load to enter in DCM



DCM operation, $R_{load} = 20 \Omega$

From the open-loop Bode plot, compensate

- The TL431 is tailored to pass a 1-kHz bandwidth



Calculate mid-band gain: +18 dB

$$R_{LED} = \frac{R_{pullup} \cdot CTR}{10^{\frac{18}{20}}} = \frac{4.7k \times 0.45}{7.94} = 266 \Omega$$

We place a zero at 300 Hz:

$$C_{zero} = \frac{1}{2\pi f_{zero} R_{upper}} = \frac{1}{6.28 \times 300 \times 66k} = 8 \text{ nF}$$

We place a pole at 3.3 kHz:

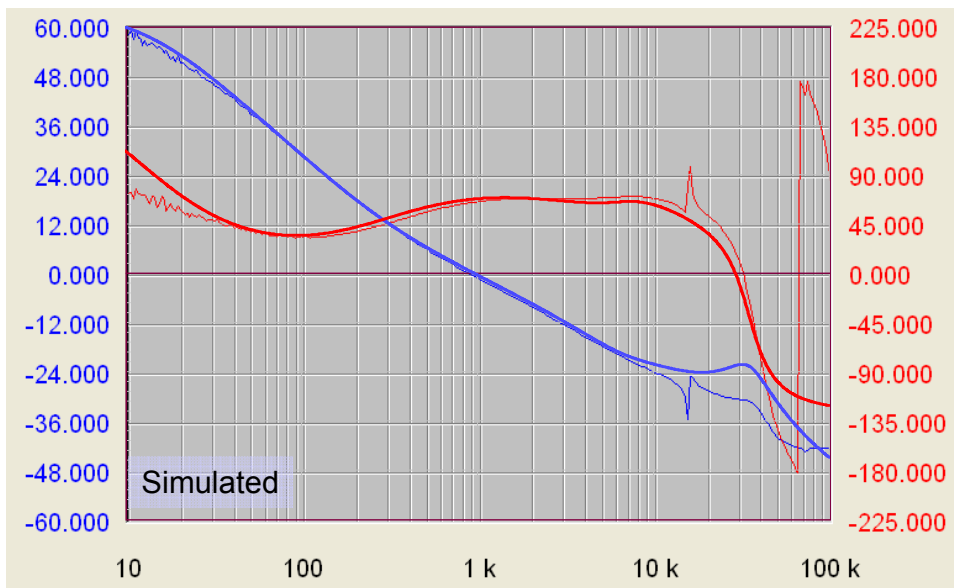
$$C_{pole} = \frac{1}{2\pi f_{pole} R_{pulldown}} = \frac{1}{6.28 \times 3.3k \times 4.7k} = 10 \text{ nF}$$

↓ k factor method

“Switch-Mode Power Supplies: SPICE Simulations and Practical Designs”, McGraw-Hill

Verify in the lab. the open-loop gain

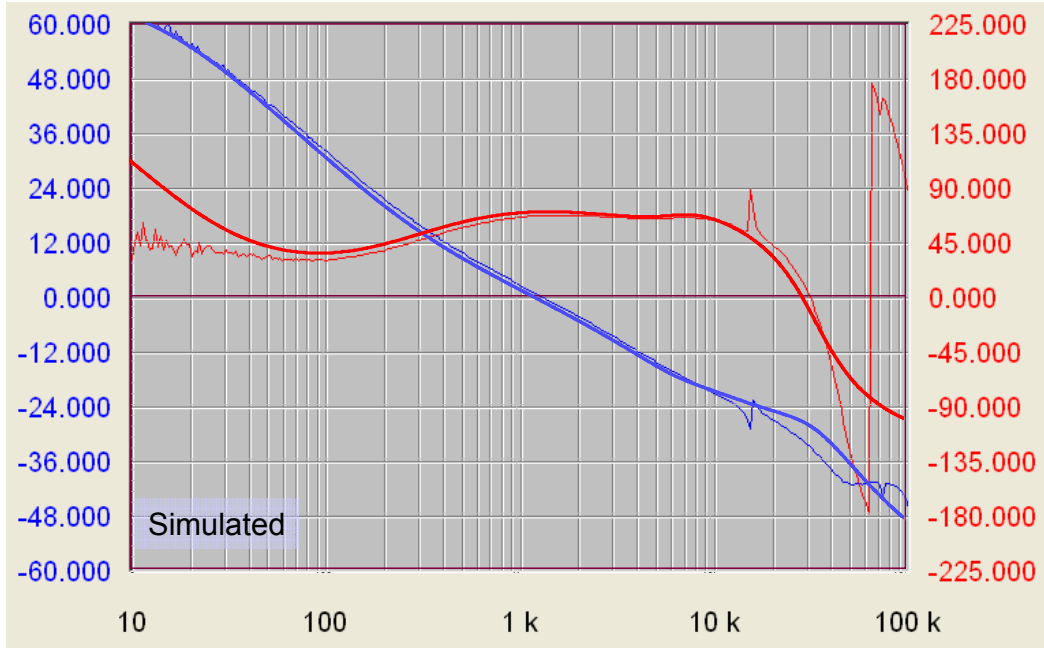
- Sweep extreme voltages and loads as well!



CCM operation, $R_{load} = 6.3 \Omega$, $V_{in} = 150 \text{ Vdc}$



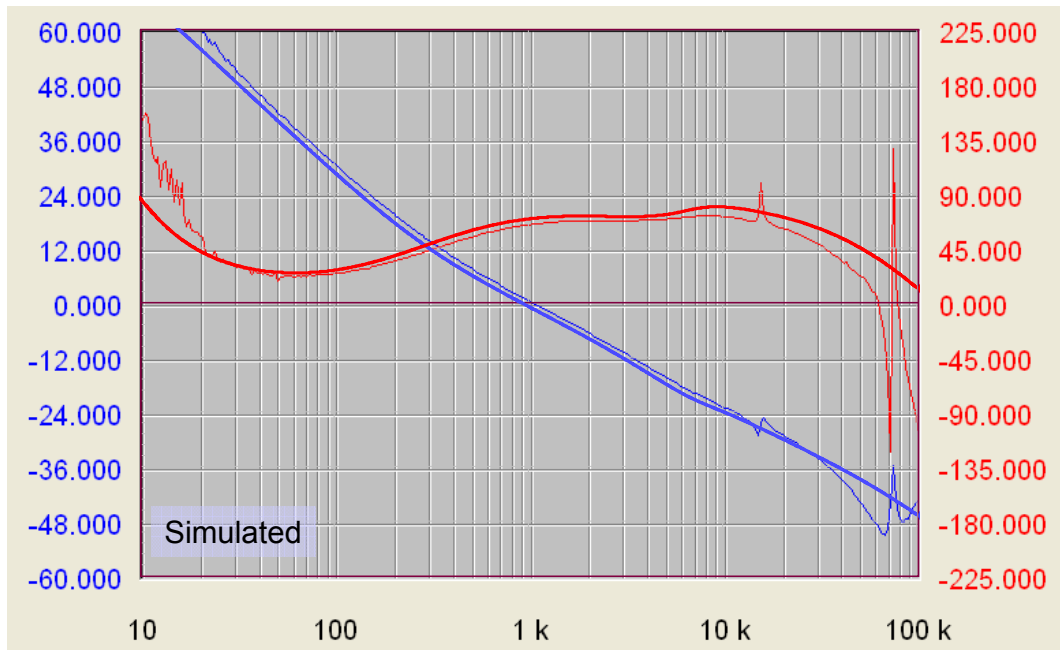
Verify in the lab. the open-loop gain



CCM operation, $R_{load} = 6.3 \Omega$, $V_{in} = 330 \text{ Vdc}$



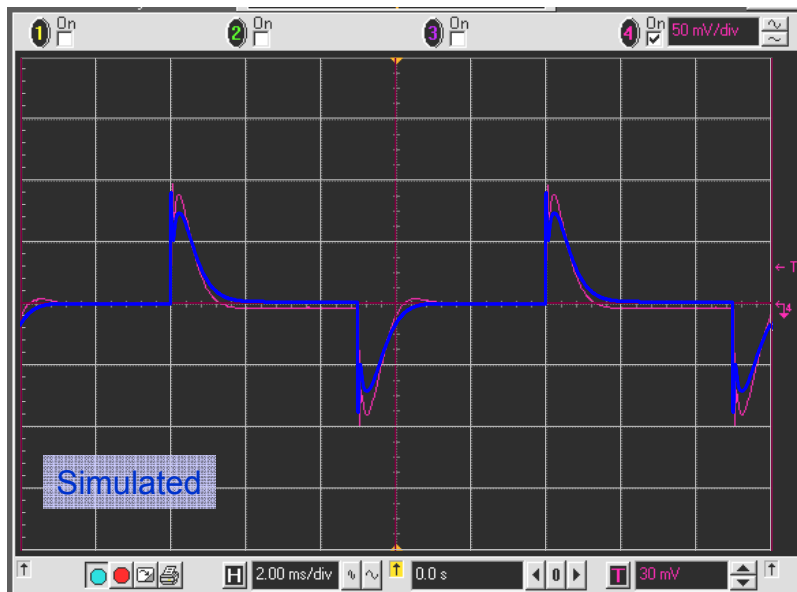
Verify in the lab. the open-loop gain



DCM operation, $R_{load} = 20 \Omega$, $V_{in} = 330 \text{ Vdc}$

As a final test, step load the output

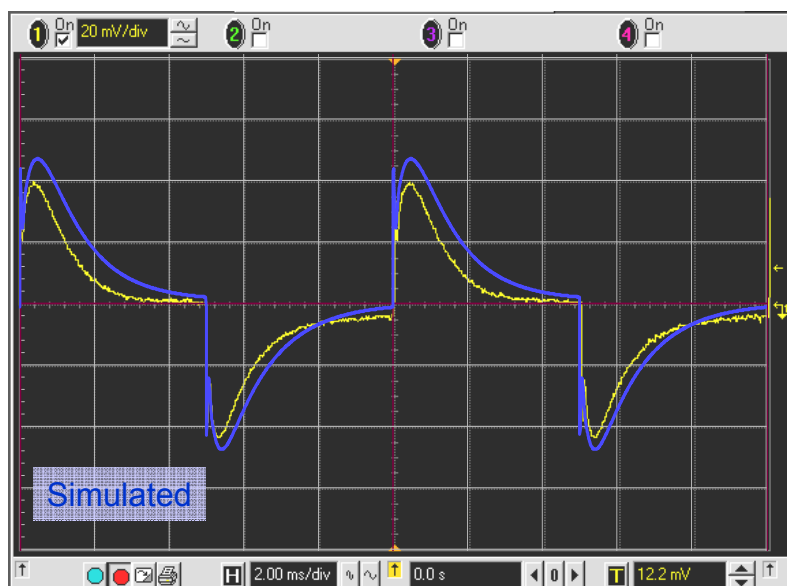
- ☐ Good agreement between curves!



$V_{in} = 150\text{ V}$
CCM
2 to 3 A
1 A/μs

As a final test, step load the output

- ☐ DCM operation at high line is also stable



$V_{in} = 330\text{ V}$
DCM
0.5 to 1 A
1 A/μs

Conclusion

- We now understand the origins of phase margin needs
- The crossover frequency value is analytically derived
- Current-mode technique simplifies the compensation
- Operating mode transition is not a problem for CM
- Type 2 and type 3 are also available with OTAs and TL431
- The optocoupler's pole can degrade the phase margin
- Do NOT forget the influence of the EMI filter
- SPICE eases the design with multi-output converters
- A real-case example confirmed the validity of the approach!



Merci !
Thank you!
Xiè-xie!