Perspectives

The Development of Frequency-Response Methods in Automatic Control

ALISTAIR G. J. MACFARLANE, SENIOR MEMBER, IEEE

Editor's Note: As readers of this journal will recall, in 1976 the Control Systems Society named three distinguished control systems specialists as Consulting Editors. One of the charges to these men was to submit an invited paper on a topic of their choice for publication without the usual IDC review procedures. At the same time Professor A. G. J. MacFarlane, Professor of Control Engineering at Cambridge University, was invited by the IDC to prepare an IEEE Press reprint book of important papers on frequency-domain methods in control and systems engineering. The coincidence of these two decisions has led to the following paper. "The Development of Frequency-Response Methods in Automatic Control" is one part of the IEEE Press book, *Frequency-Response Methods in Control Systems*, edited by A. G. J. MacFarlane and sponsored by the Control Systems Society. The book will appear in mid-1979. The paper has been selected by Consulting Editor Nathaniel Nichols and should be of substantial interest to TRANSACTIONS readers. It also conveys some of the spirit and content of the book which may be purchased from IEEE Press when available.

S. KAHNE

CONTRARY to popular belief, most good engineering theory arises from work on an important practical problem; this was certainly the case with Nyquist's famous stability criterion [169]. His attack on the problem of feedback amplifier stability produced a tool of such flexibility and power that its use rapidly spread to the wider field of automatic control. This fusion of the dynamical interests of the control and communication engineer has been immensely fruitful. In order to appreciate fully the far-reaching implications of Nyquist's 1932 paper, one must first consider the developments in automatic control and telecommunications which led up to it.

EARLY DEVELOPMENTS IN AUTOMATIC CONTROL

Although automatic control devices of various sorts had been in use since the beginnings of technology [152], Watt's use of the flyball governor can be taken as the starting point for the development of automatic control as a science [153], [154]. The early Watt governors worked satisfactorily, no doubt largely due to the considerable amounts of friction present in their mechanism, and the device was therefore widely adopted. In fact, it has been estimated that by 1868 there were some 75 000 Watt governors working in England alone [75]. However, during the middle of the 19th century, as engine designs changed and manufacturing techniques improved, an increasing tendency for such systems to hunt became apparent; that is, for the engine speed to vary cyclically with time. This phenomenon had also appeared in governed clockwork drives used to regulate the speed of astronomical telescopes and had been investigated by Airy (when he was Astronomer Royal) [1]–[3], [75]. Airy had, not unnaturally, attacked this problem with the tools of his own trade: the theory of celestial mechanics. He carried out his investigations with great skill and insight, and essentially got to the root of the mathematical problems involved. Unfortunately, his work was rather intricate and difficult to follow; it therefore did not become widely known, and the subject remained shrouded in mystery to engineers grappling with the problem of fluctuating engine speeds. This problem of the hunting of governed engines became a very serious one (75 000 engines, large numbers of them hunting!) and so attracted the attention of a number of outstandingly able engineers and physicists [153], [154], [76]. It was solved by classic investigations made by Maxwell [150], who founded the theory of automatic control systems with his paper "On Governors," and by the Russian engineer Vyschnegradsky [226], [227], who published his results in terms of a design rule, relating the engineering parameters of the system to its stability. Vyschnegradsky's analysis showed that the engine design changes which had been taking place since Watt's time—a decrease in friction due to improved manufacturing techniques, a decreased moment of inertia arising from the use of smaller flywheels, and an increased mass of

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The author is with the Control and Management Systems Division, Department of Engineering, Cambridge University, Cambridge, England.

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flyball weights to cope with larger steam valves—were all destabilizing, and their cumulative effect had inevitably led to the ubiquitous phenomenon of hunting speed. Maxwell's fundamentally important contribution lay in recognizing that the behavior of an automatic feedback control system in the vicinity of an equilibrium condition could be approximated by a linear differential equation, and hence that the stability of the control system could be discussed in terms of the location of the roots of an associated algebraic equation.

Following the presentation of his work on governors, Maxwell posed the general problem of investigating the stability of a dynamical system in terms of the location of the roots of its characteristic equation. At that time Hermite's work on this problem (which had been published some years before) was not widely known [94]. A solution was put forward by Routh in his Adams Prize Essay of 1877 [74]; this work is of great interest in the context of the Nyquist stability criterion since hindsight shows that it contains the seeds of an appropriate use of complex variable mappings for the investigation of stability. In 1895 an alternative necessary and sufficient criterion for all the roots of an algebraic equation to have negative real parts was given by Hurwitz in terms of a set of determinants [99]. Andronov [5] has given an interesting description of Vyschnegradsky's work and its effect on the subsequent development of automatic control. Andronov also discusses the important investigations made by Stodola and his collaboration with Hurwitz. Jury [106] describes the background to Routh's work and its subsequent development.

Several important advances in automatic control technology were made in the latter half of the 19th century. A key modification to the flyball governor was the introduction of a simple means of setting the desired running speed of the engine being controlled by balancing the centrifugal force of the flyballs against a spring, and using the preset spring tension to set the running speed of the engine. This brought the idea of a variable set-point of operation into automatic control. Lincke [133] proposed the use of hydraulic power amplifiers to reduce the load on the flyball mechanism, whose primary function is that of speed measurement. This enabled an integral control action to be introduced into the governor system in a simple and efficient way, and hence greatly reduced the steady-state error in governed engine speed. The idea of integral control action had previously been discussed in various ways by Preuss [186], Siemens [208], [209], and by Maxwell [150] in his 1868 paper. In his work on torpedos Whitehead made considerable use of feedback mechanisms for depth control; in some of these devices a practical form of derivative feedback action was present and used to increase the overall damping of the closed-loop system [93], [191], [84], [26]. Thus by the 1870's the use of proportional, integral, and derivative feedback action in closed-loop control systems was well established. The use of feedback for the position control of massive objects was proposed by Farcot between 1868 and 1873 for naval applications; he called such devices servomechanisms [65].

When power-amplifying devices were introduced into automatic control systems, as by Lincke, the individual functions of the various parts of the overall control system became more clearly apparent. It was appreciated that various forms of the control device had certain common features; Marié specifically drew attention to the common features of certain means of controlling speed, pressure, and temperature [149]. Lincke went even further and commented on the similarities to biological regulating systems. The growing practical importance of automatic control was marked by the award of the Nobel Prize for Physics in 1912 to the Swedish inventor Dalen "for his invention of automatic regulators to be used in conjunction with gas accumulators for lighting beacons and light buoys" [91].

The Development of the Feedback Amplifier and the Genesis of Frequency-Response Techniques

The use of ordinary differential equations, together with algebraic tests to determine the location of the roots of the associated characteristic equations, remained virtually the sole analytical tools of the automatic control engineer until well into the present century. Just as these first developments arose out of struggles with the practical problem of engine governing, so the next theoretical advances came from the work on another important technical problem: long-distance audio telephony. A revolution in the technology of communication and information processing began with L. de Forest's addition of an extra electrode to Fleming's thermionic valve to create the triode amplifying valve in 1906. This invention removed the chief obstacle to the development of long-distance telephony, namely the attenuation in cable transmission. While the mechanical engineers concerned with the problems of servomechanisms were naturally using differential equations as their basic theoretical tool, electrical engineers and communications engineers had evolved their own distinctive approaches to dynamical phenomena. The development of ac electrical power systems had led to an acute need for appropriate means of handling the "arithmetic" of ac network studies, and Steinmetz [212] developed the use of complex numbers for the representation of sinusoidal voltages and currents. Schenkel [206] discussed the representation of impedance functions by simple loci (straight lines and circles) in the complex plane. Such forms of impedance diagram were also considered by Campbell [43] and Bloch [29]. Two distinctive approaches to dynamical systems now began to develop which were associated with different ways of thinking about such systems and which, in view of their historical evolution, can be conveniently called the "mechanical engineers' viewpoint" and the "communication engineers' viewpoint," respectively. A mechanical engineer using a differential equations approach modeled his system in terms of some real or abstract "mechanism" and wrote his system-describing equations down from a detailed study of the relevant physical mechanism. The communication engineer's viewpoint, however, was quite different. It was natural for him to regard his various bits of apparatus in terms of "boxes" into which certain signals were injected and out of which emerged appropriate responses. Thus it was a natural next step for a communications engineer, in considering his system's behavior, to replace the actual boxes in which distinct pieces of physical apparatus were housed by abstract boxes which represented their effect on the signals passing through them. A combination of this "operator" viewpoint with the electrical engineer's flexible use of complex-variable representations of sinusoidal waveforms made the use of Fourier analysis-based techniques for studying dynamical phenomena in communication systems virtually inevitable. Following the pioneering work of Heaviside [92] on operational methods for solving differential equations, integral transform methods and their application to practical problems were put on a secure foundation by the work of Bromwich [37], Wagner [229], Carson [46], [47], Campbell and Foster [44], Doetsch [59], and others; thus by the late 1920's and early 1930's the integral-transform approach to the analysis of dynamical phenomena in communication systems was available for the study of feedback devices, given someone with the initiative and skill to use it.

The role of positive feedback in the deliberate generation of oscillations for high-frequency modulated-carrier radio telegraphy emerged shortly after the development of the triode amplifying valve; a patent on the use of inductive feedback to produce a high-frequency alternating current using an amplifying valve was granted to Strauss in Austria in 1912; similar developments were credited to Meissner in Germany in 1913, to Franklin and Round in England, and to Armstrong and Langmuir in the U.S.A. [191]. Armstrong, in 1914, developed the use of positive feedback in his "regenerative receiver" [28]. The central role of the feedback concept in steam engine control systems had been considered by Barkhausen [9], and Barkhausen's ideas were discussed by Möller [164] in his treatment of feedback effects in electrical circuits. Further development of the idea of a feedback loop of dependence in an oscillator circuit led Barkhausen [10] to give a "formula for self-excitation":

$$KF(j\omega)=1$$

where K is an amplifier gain factor and $F(j\omega)$ is the frequency-dependent gain of an associated feedback loop in the oscillator circuit. The *Barkhausen criterion* which developed from this formula was orginally intended for the determination of the self-excitation frequency of ac generators for use in radio transmitters. Prior to the appearance of Nyquist's 1932 paper, however, the phenomenon of conditional stability was not understood and hence it was widely believed that, for a given frequency-dependent gain function $F(j\omega)$, there was only a single value of the scalar gain parameter K which separated stable and unstable regions of behavior. Thus, particularly in the German literature, Barkhausen's equation came to be used as the basis of a stability criterion for positive and negative feedback amplifiers [135], [97]. An important early contribution to the development of frequency-response methods for the analysis of linear dynamical systems was made in 1928 by Küpfmüller. In this paper [124] he gave a comprehensive discussion of the relationships between frequency transmission characteristics and transient response behavior. In another paper published in the same year, Küpfmüller [125] dealt with the problem of closed-loop stability. Here, however, he did not use a fully-developed frequency-domain approach. Küpfmüller represented the system's dynamical behavior in terms of an integral equation, and hence developed an approximate criterion for closed-loop stability in terms of time-response quantities measured and calculated from the system's transient response. Küpfmüller's technique of approximately determining closed-loop stability from such time-response measurements seems to have remained relatively unknown outside Germany. In his history of automatic control, Rörentrop [191] refers to further work done in Germany in the 1930's on frequency-response criteria for feedback system stability, and in particular he refers to the development by Strecker of a frequency-domain stability criterion of what we would now call Nyquist type. This work appears to have remained virtually unknown and was only described in the scientific literature available after the end of the Second World War [213]–[216]. In his book Strecker [215] refers to having presented a frequency-response stability criterion at a colloquium at the Central Laboratory of Siemens and Halske in 1930, and to having presented his results to a wider audience at a seminar held by the Society of German Electrical Engineers in 1938. Rörentrop [191] says that the manuscript of this lecture is still available and that in it Strecker considered the case of open-loop unstable systems.

The truly epoch-making event in the development of frequency-response methods was undoubtedly the appearance of Nyquist's classic paper [169] on feedback amplifier stability, which arose directly from work on the problems of long-distance telephony. In 1915 the Bell System completed an experimental telephone link between New York and San Francisco which showed that reliable voice communication over transcontinental distances was a practicable proposition. This link used heavy copper open-wire circuits (weighing half a ton/mi) and was inductively loaded to have a cut-off frequency of 1000 Hz. The attenuation over a 3000 mi distance was 60 dB and, with a net gain of 42 dB provided by six repeating amplifiers, this was reduced to a tolerable net attenuation figure of 18 dB overall. The use of carrier systems on open-wire circuits was soon well advanced and had resulted in a substantial economy in conductor costs with multiplex operation in a frequency range well above the audible.

A change to cable operations, however, posed a number of severe technical problems. In particular, because the conductors were small, the attenuation was large and this required the use of many repeating amplifiers. Thus a crucial technical problem had to be overcome, that of repeatedly passing signals through amplifiers, each of which contained unavoidable and significant nonlinearities, while keeping the total distortion over transcontinental distances within acceptable limits. It required an effective amplifier linearity to within better than several parts in a thousand in order to maintain intelligibility of the transmitted audio signals. Such an acute difficulty could only be overcome by a major invention, and this was provided by H. Black of the Bell Telephone Laboratory when he put forward the idea of a feedback amplifier. Black's important discovery was that high gain in a nonlinear and variable amplifying device could be traded for a reduction in nonlinear distortion, and that an accurate, stable, and highly linear overall gain could be achieved by the suitable use of a precision linear passive component in conjunction with a high-gain nonlinear amplifer. By 1932 Black and his colleagues could build feedback amplifers which performed remarkably well. They had, however, a tendency to "sing," the telephone engineer's expressive term for instability in amplifers handling audio signals. Some "sang" when the loop gain of the feedback amplifier was increased (which was not unexpected), but others "sang" when the loop gain was reduced (which was quite unexpected). The situation was not unlike that associated with the hunting governors of around 1868—an important practical device was exhibiting mysterious behavior. Moreover, it was behavior whose explanation was not easily within the compass of existing theoretical tools, since a feedback amplifier might well have of the order of 50 independent energy-storing elements within it (such as inductors, capacitors, etc.). Its description in terms of a set of differential equations, as in the classical analyses of mechanical automatic control systems, was thus hardly feasible in view of the rudimentary facilities available at that time for the computer solution of such equations. Nyquist's famous paper solved this mystery; it opened up wholly new perspectives in the theory of feedback mechanisms and hence started a new era in automatic control. Prior to 1932 the differential-equation-based approach had been the major tool of the control theorist; within the decade following Nyquist's paper these techniques were almost completely superseded by methods based on complex-variable theory which were the direct offspring of his new approach. The background to his invention and its subsequent development have been described in a fascinating article by Black [28]. It is clear from this that Black used a stability argument of frequency-response type, saying there that "...consequently, I knew that in order to avoid self-oscillation in a feedback amplifier it would be sufficient that at no frequency from zero to infinity should $\mu\beta$ be real, positive, and greater than unity." The prototype Black feedback amplifier was tested in December 1927 and development of a carrier system for transcontinental cable telephony, its first application, started in 1928. Field trials of a system using a 25-mi section of cable with 2 terminal and 68 repeater amplifiers were held at Morristown, NJ in 1930, and successfully completed in 1931. Black [27] described his amplifier in a paper which makes interesting references to the stability problem. In particular he mentions the phenomenon of conditional stability in the following words: "However, one noticeable feature about the field of $\mu\beta$ is that it implies that even though the phase shift is zero and the absolute value of $\mu\beta$ exceeds unity, self-oscillations or singing will not result. This may or may not be true. When the author first thought about this matter he suspected that owing to practical nonlinearity, singing would result whenever the gain around the closed loop equalled or exceeded the loss and simultaneously the phase shift was zero, i.e., $\mu\beta = |\mu\beta| + j0 \geqslant 1$. Results of experiments, however, seemed to indicate something more was involved and these matters were described to Mr. H. Nyquist, who developed a more general criterion for freedom from instability applicable to an amplifier having linear positive constants." Nyquist himself has also briefly described the events which led to his writing the 1932 paper [170].

Nyquist's open-loop gain frequency-response form of solution of the feedback stability problem was of immense practical value because it was formulated in terms of a quantity (gain) which was directly measurable on a piece of equipment. This direct link with experimental measurements was a completely new and vitally important development in applied dynamical work. The application of Nyquist's stability criterion did *not* depend on the availability of a system model in the form of a differential equation or characteristic polynomial. Furthermore, the form of the Nyquist locus gave an immediate and vivid indication of how an unstable, or poorly damped, system's feedback performance could be improved by modifying its open-loop gain versus frequency behavior in an appropriate way.

It seems clear that when Nyquist set out to write his 1932 paper he was aware that the fundamental phenomenon which he had to explain was that of conditional stability. The successful theoretical explanation of this counter-intuitive effect is given due prominence in the paper. It is also clear that Nyquist was fully aware of the great generality and practical usefulness of his stability criterion. He, therefore, attempted to prove its validity for a wide class of systems, including those involving a pure time-delay effect, and he took the primary system description to be a directly-measurable frequency response characteristic.

The importance of the feedback amplifier to the Bell Laboratories' development of long-distance telephony led to a careful experimental study of feedback amplifier stability by Peterson *et al.* [178]. These experiments fully supported Nyquist's theoretical predictions and thus completely vindicated his analysis. It is altogether too easy, with hindsight and our exposure to current knowledge, to underestimate the magnitude of Black's invention and Nyquist's theoretical achievement. Things looked very different in their time. The granting of a patent to Black for his amplifier took more than nine years (the final patent, No. 2,102,671, was issued on December 21, 1937). The

U.S. Patent Office cited technical papers claiming that the output could not be connected back to the input of an amplifier, while remaining stable, unless the loop gain were less than one; and the British Patent Office, in Black's words, treated the application "in the same manner as one for a perpetual-motion machine."

Nyquist's work had shown the great power of complex-variable theory for the analysis of feedback system behavior, and it was inevitable that a tool of such promise would be further developed for design purposes. It was a natural inference from the developments presented in his paper that the closeness of approach of the Nyquist locus to the critical point in the complex plane gave a measure of closed-loop damping. This was investigated by Ludwig [135] who gave a neat formula for estimating the real part of a pair of complex conjugate roots associated with the dominant mode in the case where the Nyquist locus passes near the critical point. Thus, it soon became clear that the key to making an unstable (or otherwise unsatisfactory) feedback system stable (or better damped) lay in an appropriate modification of the amplitude and phase characteristics of the open-loop gain function for the feedback loop involved. Extensive and fruitless experimental studies were made, particularly by F. B. Anderson in the Bell Telephone Laboratories, in attempts to build feedback amplifiers having loops which combined a fast cutoff in gain with a small associated phase shift. It therefore became important to analyze the way in which the amplitude and phase frequency functions of a loop gain transfer function are related. In another of the classic papers which lie at the foundation of feedback theory Bode [30] carried out such an analysis, extending previous work by Lee and Wiener [128]. This paper, written in a beautifully clear and engaging manner, showed how there is associated with any given amplitude/gain frequency function an appropriate minimum-phase frequency function. Bode was thus able to give rules for the optimum shaping of the loop-gain frequency function for a feedback amplifier. He introduced logarithmic units of amplitude gain and logarithmic scales of frequency, and hence the logarithmic gain and linear phase versus logarithmic frequency diagrams which bear his name. The critical point in the gain plane was put at its now standard and familiar location of $(-1+j0)$, and the concepts of gain and phase margin were introduced. Bode's classic work appeared in an extended form in his book *Network Analysis and Feedback Amplifier Design* [31].

Nyquist's criterion is not easy to prove rigorously for the class of systems which he far-sightedly attempted to deal with in his classic paper and the need for a rigorous approach to a simpler class of systems soon became apparent. For the case when the open-loop gain is an analytic rational function, Nyquist himself had given a simple complex variable argument in an Appendix to his 1932 paper. This approach was soon realized to provide a simple route to a satisfactory proof for the restricted class of system functions which could be specified as rational functions of a complex frequency variable. MacColl [140] gave such a proof using the Principle of the Argument, and this became the standard form of exposition appearing in influential books by Bode [31], James *et al.* [101], and many others. Such simplified presentations did scant justice to the far-reaching nature of Nyquist's classic paper, but they soon made the stability criterion a cornerstone of frequency-response methods based on complex function theory.

The treatment given in Nyquist's 1932 paper had specifically excluded systems having poles in the closed right-half plane. A pure integration effect, however, often occurs in the open-loop transmission of servomechanisms incorporating an electric or hydraulic motor, and the appropriate extension to the Nyquist criterion to handle transfer function poles at the origin of the complex frequency plane was described in various wartime reports such as MacColl's [139] and in a paper by Hall [86]. Using the complex-variable approach based on the Principle of the Argument which had by then become the standard one, Frey [73] extended the Nyquist stability criterion to deal with the case where the feedback system may be open-loop unstable, and this simple first version of the Nyquist criterion finally assumed the form which became familiar in a multitude of textbooks.

The Spread of the Frequency-Response Approach

By the beginning of the twentieth century the basic concepts of automatic control and their analytical discussion in terms of ordinary differential equations and their related characteristic algebraic equations were well established. These techniques were consolidated in review papers by Hort [98] and Von Mises [228], and in early textbooks on automatic control by Tolle [219] and Trinks [220]. The further development of automatic control devices received great impetus from important studies carried out by Minorsky [161] on the automatic steering of ships, and by Hazen [90] on shaft-positioning servomechanisms. Minorsky proposed the use of a proportional-plus-derivative-plus-integral control action for the steering control. His work was of particular significance in being practically tested in a famous series of trials on the automatic steering of the USS New Mexico in 1922–23 [162]. Both Minorsky's and Hazen's work was explained in terms of ordinary differential equations, and their success with practical devices led to the widespread use of this approach to the analysis of automatic control systems.

In the chemical process industries the introduction of feedback control tended at first to develop in isolation from the developments in mechanical and electrical engineering. One very important difference in the process industries was (and still, to a large extent, is) that the time-scale of controlled-variable behavior was sufficiently slow on many process plants to make manual feedback control action a feasible proposition. In the chemical industry the first step along the road to automatic feedback control was the introduction of indicating instruments to monitor plant operation, followed by the attachment of pen recorders to these indicators to secure a record of plant behavior. The natural development was then to go one step further and use the movement of the pen on the recorder to effect feedback action on control valves in the plant through the use of pneumatic transducers, amplifiers, and transmission lines. During the 1930's these pneumatic controllers were steadily developed, and the idea of using an integral action term, long standard in mechanical governing, transferred to this field of control. Here, however, it was called "reset action" since the behavior of the pneumatic controller with the integral control term added was analogous to that which would have been obtained if the reference input had been slowly adjusted (or reset) to the appropriate new value required to cancel out a steady-state disturbance. In the late 1930's and early 1940's, derivative action (usually called pre-act in this context) was introduced for these pneumatic controllers to give the full "3-term" controller or "PID" (Proportional, Integral, and Derivative) controller. A theoretical basis for applied process control was laid by papers by Ivanoff [100] on temperature control, and by Callander *et al.* [42] on the effect of time-lags in control systems. Ziegler and Nichols [244] made an important study which led to formulas from which proportional, reset (integral) and pre-act (derivative) controller settings could be determined from the experimentally measured values of the lag and "reaction rate" of a process which was to be controlled.

By the late 1930's there were thus two separate but well-developed methods of attacking the analysis of feedback system behavior.

1) The "time-response approach" which involved ordinary differential equations and their associated characteristic algebraic equations, and which was much used in mechanical, naval, aeronautical, and chemical engineering studies of automatic control systems; and

2) the "frequency-response approach" which involved Nyquist and Bode plots, transfer functions, etc., and which was used for studies of feedback amplifiers.

The frequency-response approach had the appealing advantage of dealing with pieces of apparatus in terms of abstract "boxes" or "blocks" which represented their effect on the signals passing through them. This proved to be a very flexible and general way of representing systems, and it was found that when such "block" diagrams were drawn for different kinds of control systems the ubiquitous loop of feedback dependence, which is the hallmark of a feedback mechanism in a representation of this sort, sprang into sudden prominence. The power and flexibility of the tools developed by Nyquist and Bode were such that their spread to other fields in which feedback principles were used was inevitable. Some early work on using the techniques of the feedback amplifier designer for the analysis of more general systems was done by Taplin at MIT in 1937 [101]. A crucial step in the transference of the telephone engineer's viewpoint to the analysis of other kinds of system was taken by Harris, also of MIT, who made the fundamentally important contribution of introducing the use of transfer functions into the analysis of general feedback systems [87]. Harris's idea enabled a mechanical servomechanism or a chemical process control system to be represented in block diagram terms, and thus analyzed using the powerful tools available to the feedback amplifier designer.

In 1938 Mikhailov gave a frequency response criterion for systems described by a known nth order constant coefficient linear differential equation and thus having an explicitly known characteristic polynomial $p(s)$ [159]. It was stated in terms of the locus of $p(j\omega)$ in a complex p-plane and so bore a superficial resemblance to the Nyquist criterion. It is, however, an essentially different thing in that it requires that the governing differential equation of the system being investigated must be known, whereas the essential virtue of the Nyquist criterion is that the Nyquist locus is something which can be directly measured for a plant whose behavior in terms of a differential equation description may well not be available. A criterion of this form was also formulated by Cremer [51] and Leonhard [131], independently of each other and of Mikhailov. In the German literature the criterion is accordingly known as the Cremer–Leonhard criterion; in the French literature it is usually called the Leonhard criterion. In the Russian technical literature the Nyquist stability criterion is often called the Mikhailov–Nyquist criterion. Work on generalizing the Nyquist criterion to deal with neutrally stable and unstable open-loop systems was done by Mikhailov [160] and Tsypkin [221].

The 1939–45 world war created an urgent need for high-performance servomechanisms and led to great advances in ways of designing and building feedback control systems. From the point of view of the development of automatic control design techniques, the chief result of the immense pooling of effort and experience involved was to spread rapidly the use of frequency-response ideas into the mechanical, aeronautical, naval and later the chemical fields, and to produce a unified and coherent theory for single-loop feedback systems. Important reports written by Brown and Hall [101] were circulated among defense scientists and engineers and soon, accelerated by the end of the war, a number of classic publications and textbooks became available which resulted in the widespread dissemination and adoption of frequency-response ideas. Herwald [95] discussed the use of block diagrams and operational calculus for the study of the transient behavior of automatic control systems, including the use of compensating networks. Ferrel [69] laid particular stress on the parallels between electromechanical control system design and electrical network design. He suggested the use of the now-familiar asymptotic Bode diagrams. Graham [83] made notable use of these diagrams and discussed dynamic errors, the effects of noise, and the use of tachometric feedback compensation. Brown and Hall [39] gave a classic treatment of the analysis and design of servomechanisms and Harris [88] gave a wide-ranging and thorough treatment of analysis and design in the frequency domain. The work done at the MIT Radiation Lab was summarized in a notable book by James *et al.* [101]; the first use of inverse Nyquist diagrams is discussed in their book and credited to Marcy [148]. Gardner and Barnes [77] gave a widely used treatment of the mathematical background to these developments.

British contributions were summarized in papers by Whiteley [231], [232]. The historical background to British work by Daniel, Tustin, Porter, Williams, Whiteley, and others has been described by Porter [183] and Westcott [230]. Several of the wartime and post-war historical developments in Britain and America have been discussed by Bennett [24]. German work during and after the war has been summarized by Rörentrop [191]. Applications of the Nyquist criterion to feedback control loops were treated in the German literature in papers by Feiss [66]–[68] and an important textbook was produced by Oldenbourg and Sartorious [171]. Leonhard [130] discussed frequency-response design techniques and extended the Mikhailov criterion approach [131]. Tsypkin [221], [222] discussed the effect of a pure delay in the feedback loop. Among the many textbooks used by designers, the two-volume work of Chestnut and Meyer [49] had a notable impact.

Since the rotating aerial of a radar system only illuminates its target intermittently, many of the fire-control systems developed during the Second World War had to be designed to deal with data available in a pulsed or sampled form. The basis for an effective treatment of sampled-data automatic control systems was laid by Hurewicz whose work is described in [101]. In particular, in his contribution to this book, Hurewicz developed an appropriate extension of the Nyquist stability criterion to sampled-data systems. The development of digital computing techniques soon led to further work on such discrete-time systems. Digital control systems operating on continuous-time plants require analysis techniques which enable both discrete-time and continuous-time systems, and their interconnection through suitable interfaces, to be looked at from a unified standpoint. Linvill [134] discussed this problem from the transform point of view, including a consideration of the Nyquist approach to closed-loop stability. Frequency-response methods of analyzing sampled-data systems were studied by Tsypkin [223]. A "z-transform" theory for systems described by difference equations emerged to match the "s-transform" theory for systems described by differential equations [188] and was treated in textbooks by Ragazzini and Franklin [188], Jury [104], [105], Freeman [72], and others. The "equivalence" between continuous-time and discrete-time system analysis methods has been discussed by Steiglitz [211].

The unique feature of the Nyquist–Bode diagram approach to closed-loop system stability and behavior is that it can make a direct use of experimentally-measurable gain characteristics. Using such data one can make inferential deductions about the behavior of the closed-loop system's characteristic frequencies. Nevertheless, there are many situations in which one does have a direct knowledge of the form of the plant transfer function and it then becomes a natural question to ask: what direct deductions can be made from this of the way in which the closed-loop characteristic frequencies vary with a gain parameter? This question was answered in 1948 by Evans who brought the complex-variable-based approach to linear feedback systems to its fully developed state by the introduction of his root-locus method [62]–[64].

The effect of random disturbances on automatic control systems was also studied during the Second World War [101]. In 1920 the autocorrelation function had been introduced by G. I. Taylor in his work on turbulent flow in fluids [217]; N. Wiener realized that this function was the link between the time and frequency-response descriptions of a stochastic process, and based his classic studies of random process analysis [233] and their relationships to communication and control theory [235] on the generalized Fourier transform of this function. Wiener became deeply interested in the relationships between control and communication problems and in the similarities between such problems in engineering and physiology. In addition to his important wartime report on time-series analysis he wrote a seminal book on cybernetics [234]. His books had the important effect of propagating feedback-control ideas in general, and frequency-response methods in particular, into the fields of stochastic system theory and physiology.

The "harmonic balance" methods developed in studies of nonlinear mechanics by Krylov and Bogoliubov [122] led to attempts to extend frequency-response methods to nonlinear feedback control problems. From these efforts emerged the describing function method which extended the use of Nyquist diagrams to the study of nonlinear feedback system stability. This was developed independently in a number of countries: by Goldfarb [80] in Russia, by Daniel and Tustin in England [225], by Oppelt [173] in Germany, by Dutilh [60] in France, and by Kochenburger [120] in the United States. Although at first resting on rather shaky theoretical foundations, this technique proved of great use in many practical studies and its introduction marked an important consolidation in the use of frequency-response methods. Investigations by Bass [18] Sandberg [204], Bergen and Franks [25], Kudrewicz [123], and Mees [156], [157] have subsequently placed the method on a sounder basis.

Aizerman [4] greatly stimulated the study of nonlinear feedback problems by putting forward his famous conjecture on the stability of systems incorporating a "sector-bounded" nonlinearity. This led to work on what is known in the Russian literature as the "problem of absolute stability." Despite the fact that Pliss [179] demonstrated by means of a counterexample that the Aizerman conjecture is not generally true, it became manifestly important to discover for what classes of system the conjecture did hold. Such a class of systems was found by Popov [182] in a classic study which led to his famous stability criterion. Popov's work led to a resurgence of interest in frequency-response treatments of nonlinear problems out of which emerged the various forms of "circle criteria" for stability [205], [243], [166]. Some important early work on the application of frequency-response methods to nonlinear systems was done by Tsypkin [224] and Naumov and Tsypkin [167].

The Development of Optimal and Multivariable Control

By the early 1950's frequency-response methods reigned virtually supreme over the applied control field; they were the routinely-used tools for the analysis and design of feedback mechanisms and automatic control systems. A good impression of the classical frequency-response approach at this period can be obtained from the collection of papers edited by Oldenburger [172]. Block diagrams, with their associated transfer functions, had become widely familiar to engineers handling many different kinds of linear dynamical models of physical systems, and were being used with great flexibility and insight. In many ways frequency-response concepts had developed into a vital medium of communication, giving a unified means of approaching and analysing a wide range of feedback phenomena from a common point of view. The Nyquist diagram and Bode diagram, with their direct relationship to physically-measurable plant responses, had become an indispensable means of assessing closed-loop stability for a wide range of practical control systems, and the describing function technique had emerged as a useful, though somewhat heuristic, means of handling many common types of nonlinearity. Evans' root-locus method had provided a further powerful tool for the design of linear feedback systems of fairly high order, and the representation of stochastic disturbances and their effects was well established in frequency-response terms. The position had changed completely from that of the late 1920's, when the time-response methods were unchallenged. However, the pendulum of fashion was about to swing back rapidly.

The emergence of the stored-program digital computer as a reliable and widely available engineering device by the late 1950's was a necessary prerequisite for the next developments in automatic control systems analysis and design. It was now reasonable to attempt much deeper and more comprehensive studies in automatic control theory, since the computing power and versatility of the big scientific machines made the lengthy and intricate calculations involved a practicable proposition. At the same time, the development of small and reliable special-purpose digital computers offered the possibility of implementing more ambitious control schemes via information-processing devices of unprecedented computing speed and flexibility. It was a natural step, therefore, to consider the simultaneous control of a number of interacting variables, and to consider different types of controller objective, such as the minimization of fuel consumption, for which the now-classical frequency-response theory was quite inappropriate.

As with the previous major developments in automatic control theory, these next advances arose out of an important technical problem, in this case the launching, maneuvering, guidance, and tracking of space vehicles. Both in the USA and the USSR, an enormous research and development effort was expended on these problems, and from this came rapid progress. The nature of these next developments in automatic control theory was profoundly influenced by two things.

1) The fact that the objects being controlled and tracked were essentially ballistic in nature meant that accurate mechanical models of the devices being controlled were normally available. Moreover, the systems involved could be fitted with measuring devices of great precision.

2) Many of the performance criteria which the final control schemes had to satisfy were of an "economic" nature. For example, a satellite position-control scheme might have to operate in such a way that a desired maneuver was executed for the minimum expenditure of fuel. A natural result of these aspects of the related control problems was to refocus attention on an approach to control via sets of ordinary differential equations. For dynamical systems having an overall performance specification given in terms of making some functional of the behavior (performance index) achieve an extremum value there was an obvious and strong analogy with the classical variational formulations of analytical mechanics given by Lagrange and Hamilton [127]. In the USSR, Pontryagin laid the foundations of what came to be called optimal control theory by an elegant generalization of the Hamiltonian approach to geometrical optics in the form of his famous maximum principle [35], [181].

An important aspect of this treatment of multivariable control problems in terms of sets of differential equations was the systematic use of sets of first-order equations. Moigno [163] had shown that any nth-order ordinary differential equation may be reduced to an equivalent set of first-order equations by means of a set of simple substitutions, and Cauchy [48] had previously studied the conditions under which such a system of equations had a unique solution. Poincaré saw the deep significance of formulating general dynamical theories in terms of sets of first-order differential equations, and introduced the now familiar idea of considering the relevant set of dynamical system variables in terms of the trajectory of a point in an n-dimensional space. He established this approach as a standard one by building the whole of his famous treatise on celestial mechanics around it [180]. One of the first major applications of the Poincaré formulation of dynamical theory was Lyapunov's celebrated study of stability [132]. Poincaré's approach to dynamics rapidly became the standard one for control engineers working on aerospace problems with these revitalized time-domain techniques, which collectively became known as the state-space approach. The concept of state now dominates the whole of applied dynamical theory. What is fundamental about dynamical systems is that their present behavior is influenced by their past history; dynamical system behavior cannot, therefore, be specified simply in terms of "instantaneous" relationships between a set of input and a set of output variables. An extra set of variables is required whose purpose is to take into account the past history of the system; these variables are the state variables of the system. The use of state-space treatments of dynamical and feedback systems immediately led to a deeper study of the scientific and mathematical problems of automatic control than had ever before been attempted, and their introduction can be said to mark the emergence of control studies as a mature scientific discipline. Even the most cursory study of the literature of the subject will show what a profound change occurred between the mid 1950's and the late 1960's.

Pontryagin's maximum principle proved invaluable in dealing with situations where there were constraints on system inputs reflecting limitations of resources, and gave a dramatic demonstration of the power and potential of this new differential-equation-based approach. Bellman's work on dynamic programming was also concerned with the problem of dynamic optimization under constraint [19]–[22]. Bellman made clear the great usefulness of the concept of state for the formulation and solution of many problems in decision and control. It was inevitable that the linear multivariable feedback control problem would now be thoroughly examined from this point of view, and Kalman gave a definitive treatment of the linear optimal control problem with a quadratic form of performance index [108], [111]. This work had one particular feature which distinguished it from most previous studies of the feedback design problem—it gave a synthesis procedure by means of which the feedback system design was obtained directly from the problem specification [115]. This, at first sight at any rate, eliminated the trial and error procedures normally associated with feedback system design. Previous attempts had been made to treat feedback system design within an analytical framework [168] but never before had the multivariable problem been so treated. Although it can be argued that such a synthesis procedure simply shifts the burden of design decision on to the choice of performance index, there is no doubt that the emergence of this elegant and powerful synthesis solution to a multivariable feedback problem marked a new high point in the development of feedback system design procedures.

The rapidly growing importance of state-space methods led to an investigation of the relationships between state-space models and transfer function representations by Gilbert [79] and Kalman [113], and algorithms were developed for obtaining minimal-order state-space dynamical models from given transfer function matrices. Such studies led to the introduction of the fundamental structural concepts of controllability and observability [109], [79]. Certain classical dynamical ideas such as those associated with the characteristic modes of vibration of a linear dynamical system were now seen to be useful in the state-space formulation and their relevance to control ideas and problems was examined. Rosenbrock put forward the idea of modal control [192] in which the action of a feedback controller was envisaged in terms of a shift of characteristic (modal) frequency. This important and physically appealing concept eventually led to a huge literature on the problem of "pole-shifting" and its use for design purposes [82], [61], [238], [96], [210], [53], [54], [184]. Wonham [238] proved that a sufficient condition for all the closed-loop characteristic frequencies of a controllable system to be arbitrarily allocatable under feedback (within mild constraints imposed by physical considerations) is that all the states of the system are accessible. This key result further underlined and reinforced the importance of the concept of state.

The use of these revitalized time-response methods had a profound effect on control work, and made crucially important contributions to solving the guidance problems of the space program. In the research literature of automatic control frequency-response methods went into a steep decline. Even worse, from the frequency-response protagonist's point of view, was to follow. Filtering theory, at one time a seemingly impregnable bastion of frequency-response ideas, was also undergoing the state-space-method revolution. Kalman and Bucy had realized that the problem of signal recovery from corrupted measurements which, following Wiener's work, had been almost invariably attacked along a frequency-response route, was also amenable to the multivariable time-response approach [110], [41]. Because of the ease with which it handled the nonstationary case, their work led to an immediate advance in filtering technique. From the point of view of the development of general feedback theory, however, it had an especial significance, since it clearly demonstrated the basic role of feedback in filtering theory. The form of multivariable filter which emerged from their studies, the Kalman–Bucy filter, essentially consisted of a dynamical model of the message-generating signal process with multivariable feedback connected around it. This work showed that a deep and exact duality existed between the problems of multivariable feedback control and multivariable feedback filtering [109].

It was now a natural next step to put together the optimal control treatment of a deterministic linear plant, whose performance is specified in terms of a quadratic cost function, with the Kalman–Bucy filtering method of extracting state estimates from observations corrupted by Gaussian noise processes. Thus emerged the standard treatment of the "LQG" (linear-quadratic-Gaussian) optimal control problem [7] which became the linch-pin of the state-space treatment of multivariable control, and which was treated in many standard textbooks [6], [129], [40], [116], [201]. The key ideas of the LQG problem and its background, and an excellent survey of the relevant literature up to 1970, are given in [245] which was devoted to this topic.

The Development of a Frequency-Response Approach to Multivariable Problems

Optimal feedback control and optimal feedback filtering theory had such a great success when applied to aerospace problems that this naturally led to attempts to apply these techniques to a wide range of earth-bound industrial processes. It soon became clear that they were less than immediately applicable in many such cases, principally because the plant models available were not sufficiently accurate, and the performance indices required to stipulate the desired controlled plant behavior were much less obvious in form than in the aerospace context. Moreover, the controller which resulted from a direct application of optimal control and optimal filtering synthesis techniques was in general a complicated one; in fact, if it incorporated a full Kalman–Bucy filter, it would have a dynamical complexity equal to that of the plant it was controlling, since the filter essentially consisted of a plant model with feedback around it. What was needed for many process control problems was a relatively simple controller which would stabilize a plant, for which only a very approximate model might be available, about an operating point and which would have some integral action in order to mitigate the effect of low-frequency disturbances. The sophisticated optimal control methods proved difficult to use by industrial engineers brought up on frequency-response ideas who essentially needed to use a mixture of physical insight and straightforward techniques, such as the use of integral and derivative action, to solve their problems. For these reasons an interest in frequency-response methods slowly began to revive. It was obvious that a huge gap in techniques existed between the classical single-loop frequency-response methods, which were still in use for many industrial applications, and the elegant and powerful multivariable time-response methods developed for aerospace applications.

An important first step towards closing the yawning gap between an optimal control approach and the classical frequency-response approach was taken by Kalman [112] who studied the frequency-domain characterization of optimality. A systematic attack on the whole problem of developing a frequency-response analysis and design theory for multivariable feedback systems was begun in a pioneering paper by Rosenbrock [193] which ushered in a decade of increasing interest in a rejuvenated frequency-response approach. Prior to this new point-of-departure, some fairly straightforward work had been done on the multivariable control problem. Boksenbom and Hood [34] put forward the idea of a noninteracting controller. Their procedure consisted simply of choosing a cascaded compensator of such type that the overall transfer function matrix of the compensated system had a diagonal form. If such a compensator could be found then the controller design could be finished off using standard single-loop design techniques. The required compensating matrix usually arising from such a procedure is necessarily a complicated one, and the most succinct objection to this approach is simply that it is not necessary to go to such drastic lengths merely to reduce interaction. A natural further step in this initial approach to multivariable control was to see what could be achieved by way of standard matrix calculations using rational matrices; papers studying the problem in this way were produced by Golomb and Usdin [81], Raymond [190], Kavanagh [117]–[119], and Freeman [70], [71]. Rosenbrock, however, opened up a completely new line of development by seeking to reduce a multivariable problem to one amenable to classical techniques in a more sophisticated way. In his inverse Nyquist array design method [194] the aim was to *reduce* interaction to an amount which would then enable single-loop techniques to be employed, rather than to eliminate it completely. The Rosenbrock approach was based upon a careful use of a specific criterion of partial interaction, the diagonal dominance concept. The success of his inverse Nyquist array method led other investigators to develop ways of reducing the multivariable design problem to an eventual succession of single-loop problems [151].

In the noninteracting, or partially noninteracting, approach to multivariable control the motivation was the eventual deployment of classical single-loop frequency-response techniques during the final stages of a design study. An alternative approach, however, is to investigate the transfer-function matrix representation as a single object in its own right and to ask: how can the key basic concepts of the classical single-loop approach be suitably extended? What are the relevant generalizations to the multivariable case of the specific concepts of pole, zero, Nyquist diagram, and root-locus diagram and, further, what essentially new frequency-domain ideas can be developed in the multivariable context? It soon emerged that there was no single line of attack suitable for finding the answer to such very deep and far-reaching questions. The various aspects of the main research lines which developed can be conveniently labeled as the algebraic, geometric, and complex-variable approaches.

The algebraic approach developed from further studies of the relationships between state-space and frequency-response representations and of the problem of generalizing the concepts of pole and zero to the multivariable case. In his study of the minimal realization problem, Kalman [113] had made use of McMillan's canonical form [155] for a rational transfer-function matrix. The so-called Smith–McMillan form was used by Rosenbrock in his treatment of multivariable zeros [195], [196], [198]. Rosenbrock gave a particularly comprehensive treatment of the multivariable zero problem as a part of his important and pioneering work on an algebraic theory of linear dynamical systems. In this work he made a systematic use of a particular polynomial matrix representation which he called the system matrix [195]. These studies by Kalman and Rosenbrock showed the great power and relevance of algebraic theories for fundamental studies of the linear multivariable control problem, and they were soon followed by a strong and sustained research effort on the algebraic approach. Surveys of work on algebraic systems theory have been given by Barnett [12] and Sain [203]. Kalman's work has shown the importance of using module theory in the algebraic approach to dynamical systems; from the mathematical point of view this leads to a particularly "clean" treatment [114], [116].

The central role of a system's state in discussing its feedback control had been established by its part in optimal control theory and by Wonham's pole-shifting theorem. Kalman and Bucy had shown how to estimate unknown states from noise-corrupted system outputs. It was thus natural to seek ways of using system model information to recover inaccessible system states from the uncorrupted outputs of deterministic dynamical systems, and Luenberger [136]–[138] introduced the use of observers for this purpose. The idea had been emerging of separating a feedback problem into the two steps of 1) working out what to do if a system's state was completely accessible, and 2) devising a means of estimating the system's inaccessible states from the information contained in its accessible outputs. In the stochastic linear optimization problem a certainty-equivalence principle had been established [102], [239], [236], [13] which had shown that the stochastic optimal control problem could indeed be solved in this way. A similar sort of "separation principle" was established for the problem of pole-shifting using an observer: the same closed-loop poles are obtained using an observer (constructed with perfect plant model information) as would have been obtained if all the system's states had been available for feedback purposes [52]. These results and ideas led naturally to a deeper study of the problems of dynamic compensation [36], [237] which further closed the gap between the classical frequency-response methods and those of what was (unfortunately) becoming known as "modern" control theory.

A linear vector space approach to control problems obviously has geometrical as well as algebraic aspects. Wonham and Morse [240] carried out a definitive and far-ranging study of the geometrical treatment of multivariable control problems, which culminated in Wonham's elegant and important book on this topic [241]. This definitive text opened up a whole new prospect for control studies. In this work the dynamical significance of certain classes of subspaces of the state space plays a key role, and investigations of such topics as decoupling is carried out in a crisp and intuitively appealing way. Independent studies of a geometrical approach were carried out by Basile and Marro [14]–[17]. It seems clear that the geometrical theory has a key role to play in bringing together state-space and frequency-response approaches to the multivariable case [147].

Yet another line of approach to the multivariable feedback problem arises from the observation that the classical Nyquist–Bode–Evans formulation of the single-loop case is based on complex-variable theory. Surely, it was thought, complex-variable ideas must have a role to play in the multivariable context, particularly when the algebraic studies had shown how to extend to the multivariable case such basic complex-variable concepts as poles and zeros. An early attempt to extend Nyquist diagram ideas to the multivariable problem was made by Bohn [32], [33]. In a series of papers MacFarlane and his collaborators demonstrated that algebraic functions could be used to deploy complex variable theory in the multivariable feedback context [141]–[146]. It was shown that the poles and zeros associated with transfer-function matrices by algebraic means, via the Smith–McMillan form for a matrix of rational transfer functions, were related to the poles and zeros of an appropriate function of a complex variable. This line of investigation in turn led to a generalization of the classical Nyquist stability criterion to the multivariable case [141]. Following earlier heuristic treatments of this generalization, complex-variable proofs were provided by Barman and Katzenelson [11] and MacFarlane and Postlethwaite [142]. The generalization of the Nyquist stability criterion to the multivariable situation was soon followed by complementary generalizations of the root locus technique [121], [142], [143], [185], [145].

Together with these counter-revolutionary developments of the classical frequency-response approaches came an increasing interest in the existence of links between state-space models and methods and the various algebraic, geometric, and complex-variable techniques and results. It was discovered that deep and important links existed between the poles, zeros, and root-locus asymptotes of the complex-variable characterizations and the basic operators of a state-space description [145]. These findings emphasized the deep significance for control studies of the algebraic and geometric approaches which were being so rapidly developed.

Since much of the motivation for work on frequency-response methods arose from the need to develop robust design methods for plants described in terms of models derived from sketchy experimental data, a number of different design approaches in the frequency domain began to emerge. Many of these techniques were conceived in terms of interactive graphical working, where the designer interacts with a computer-driven display [198]. As such, they placed great stress on the insight and intuition which could be deployed by an experienced designer; this was in great contrast to the specification-and-synthesis approach which had been the hallmark of the optimal control solution.

Many of these approaches naturally sought to capitalize on the experience and insight existing for single-loop frequency-response designs. The most straightforward way to do this is to somehow reduce a multivariable design problem to a set of separate single-loop design problems. In Rosenbrock's inverse Nyquist array method [194], [198], [165] this was done by using a compensator to first make the system diagonally-dominant. A careful use of this specific form of criterion of partial noninteraction enabled the stability and performance of the closed-loop system to be inferred from its diagonal transmittances alone, and hence enabled a multivariable design to be completed using single-loop techniques. Mayne's sequential return difference method [151] took a different line of approach to the deployment of single-loop techniques. It was built around a series of formulas for the transmittance seen between an input and output of a feedback system, having one particular feedback loop opened, when all the other feedback loop gains were made large. Providing that one could find a suitable place to start, this enabled the designer to proceed to design one loop at a time, and give it a suitably high value of loop gain before proceeding to the next one.

Other investigators were less concerned with the direct deployment of single-loop techniques and looked for ways of using the generalized Nyquist and root-locus results as the basis of design methods. MacFarlane and Kouvaritakis [144] developed a design approach based on a manipulation of the frequency-dependent eigenvalues and eigenvectors of a transfer function matrix. This line of attack was later extended to handle the general case of a plant having a differing number of inputs and outputs by incorporating a state-space-based root-locus approach as an integral part of the overall procedure [145].

The interest and importance of the multivariable control problem generated a wide range of other investigations. Owens [174]–[176] studied ways of expanding transfer function matrices as sums of dyads and developed a design approach on this basis. Wolovich developed multivariable frequency-response approaches to compensation, decoupling, and pole placement [237]. Sain investigated design methods based on transfer-function matrix factorization and polynomial matrix manipulation [202], [78], [177]. Bengtsson used geometrical ideas in the spirit of Wonham's work to devise a multivariable design approach [23]. Davison made extensive investigations of the multivariable control problem and developed an approach which, although state-space based, was in the same engineering spirit as the more frequency-biased work. His studies emphasized the importance of robustness to parameter variation [55]–[58]. Youla and Bongiorno extended the analytical feedback design technique developed by Newton [168] to the multivariable case [242].

As the broad outlines of the frequency-response theory of linear multivariable control systems began to emerge, interest rose in the appropriate extensions of nonlinear criteria such as the describing function and circle criterion, and work to this end was started by several workers [224], [157], [197], [50], [8], [189], [158].

Multidimensional Filtering

The need to enhance the quality of pictures transmitted back to earth from exploring satellites and space probes resulted in work on the "multidimensional" filtering of video signals, that is on their simultaneous processing in more than one spatial dimension. This further generalization of the problem of dynamic filtering led to a study of the stability of multidimensional feedback filters, and this in turn led to an appropriate extension of frequency-domain techniques, including that of determining closed-loop stability via Nyquist-type criteria. Jury [107] has given a very comprehensive survey of work in this area.

Epilogue

From our present vantage point we can attempt to put frequency-response methods into perspective. Nyquist started a completely new line of development in automatic control when he analyzed the problem of closed-loop stability from the signal-transmission viewpoint rather than the mechanistic viewpoint. In so doing he showed the engineers designing and developing feedback devices and automatic control systems how to use the powerful tools which can be forged from the theory of functions of a complex variable. His famous stability criterion had an immediate and lasting success because it related to quantities which could be directly measured and because it was expressed in terms of variables which could be immediately understood and interpreted in terms of appropriate actions to be taken to improve a feedback system's performance. The frequency-response concepts, and the immensely popular and useful design techniques based upon them, satisfied a criterion of great importance in engineering work—they enabled engineers to quickly and fluently communicate to each other the essential features of a feedback control situation. Complex-variable methods are of such power and potential that their continued use and development is surely not in doubt. Even at the height of the "state-space revolution" the classical Nyquist–Bode–Evans techniques were the workhorses of many designers for their single-loop work.

The real significance of the introduction of state-space methods is that it marked the beginning of a new, more general, more rigorous, deeper, and more far-reaching approach to automatic control. We are now beginning to see that automatic control is a vast subject, still in the early stages of development, and requiring a great breadth of approach in setting up adequate theoretical foundations. Its scope is such that no single approach, via the "time domain" or the "frequency domain" alone, is going to be sufficient for the development of adequate analysis and design techniques. What it is hoped will emerge clearly from the contents of this book is that Nyquist's ideas, and the frequency-response approach developed from them, are alive at the frontiers of current research, and that they will continue to play an indispensable role in whatever grand theoretical edifice emerges in time. Nyquist made truly outstanding contributions to engineering. He carried on a great tradition in the applied sciences going back to Fourier whose epochal work first appeared in 1811 [45], and in doing so transformed the arts of telegraph transmission and feedback systems development into exact sciences. May his spirit live on in the work collected here, and in the future developments of feedback and control.

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Alistair G. J. MacFarlane (SM'75) was born in Edinburgh, Scotland in 1931. He received the B.Sc. and D.Sc. degrees from the University of Glasgow in 1953 and 1968; the Ph.D. degree from the University of London in 1964; the M.Sc. degree from the University of Manchester in 1973; and the M.A. and Sc.D. degrees from the University of Cambridge in 1974 and 1979, respectively.

After graduating in electrical engineering from the University of Glasgow, he joined the Metropolitan-Vickers Electrical Company in Manchester in 1953 to complete his postgraduate training in the Radar and Servomechanisms Division. After some years working on a variety of feedback and signal-processing problems he became group leader of the Moving Target Indication and Receiver Laboratories of that company in 1956–58. He was then appointed Lecturer in Electrical Engineering in Queen Mary College, University of London, and was promoted to Reader in Electrical Engineering in 1965. In 1966 he moved to the University of Manchester Institute of Science and Technology as Reader in Control Engineering in the newly established Control Systems Centre, becoming Professor of Control Engineering in 1969. Following this he was elected to a Chair of Engineering in the University of Cambridge in 1974 and became Head of the Control and Management Systems Division of the Engineering Department at Cambridge. His research interests are in the fields of feedback theory, dynamical theory, and automatic control. For papers on these topics he was awarded the Institution Premium of the IEE in 1966, the IEE Control and Automation Division Premium in 1970 and the ICI Prize of the Institute of Measurement and Control in 1975.

Dr. MacFarlane is a Fellow of Selwyn College Cambridge, a Fellow of the IEE and of the Institute of Measurement and Control, a Vice-Chairman of the IFAC Theory Committee, and Editor of the *International Journal of Control*.