INSTRUMENTATION AND CONTROL

TUTORIAL 11 – CONTROL ACTION

This tutorial is of interest to any student studying control systems and in particular the EC module D227 – Control System Engineering.

On completion of this tutorial, you should be able to do the following.

- Explain the term control action.
- Explain proportional control action.
- Explain integral control action.
- Explain differential control action.
- Explain 3 term or P.I.D. control action.
- Discuss the Zeigler - Nichols methods of adjusting a 3T controller.
- Explain the use of Lead and Lag Phase Compensation methods.

If you are not familiar with instrumentation used in control engineering, you should complete the tutorials on Instrumentation Systems.

In order to complete the theoretical part of this tutorial, you must be familiar with basic mechanical and electrical science.

You must also be familiar with the use of transfer functions and the Laplace Transform (see maths tutorials).

1. INTRODUCTION

The diagram shows a circuit of a typical control system. The purpose of this tutorial is to study the controller. The controller processes the error and is vital in producing the desired response from the system. One of the most common controllers for analogue systems is called the 3 term or P.I.D. controller. The symbol x is used in this tutorial to indicate a signal.

x_i = input signal x_o = output signal x_e = error signal

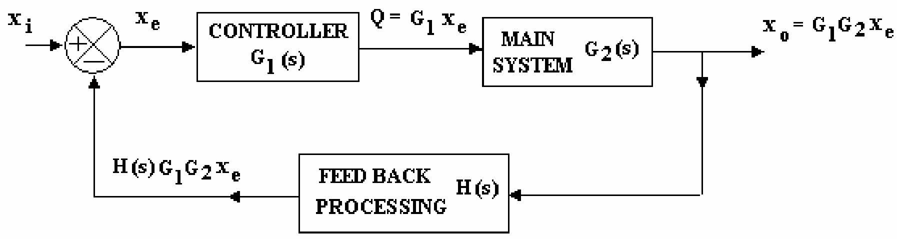

Figure 1

2. THREE TERM CONTROL

3 term control is widely used to enable a system to respond with respect to time in the best possible way. The system must respond to the error x_e such that the error is reduced to zero as quickly as possible with no oscillation. The three terms used by the controller are Proportional, Integral and Differential, abbreviated to P.I.D. control. Let's consider each in turn.

2.1 PROPORTIONAL CONTROL

The output is directly proportional to the input so $G_1 = k_p$. The constant k_p is the gain and this controls the basic response speed of the system.

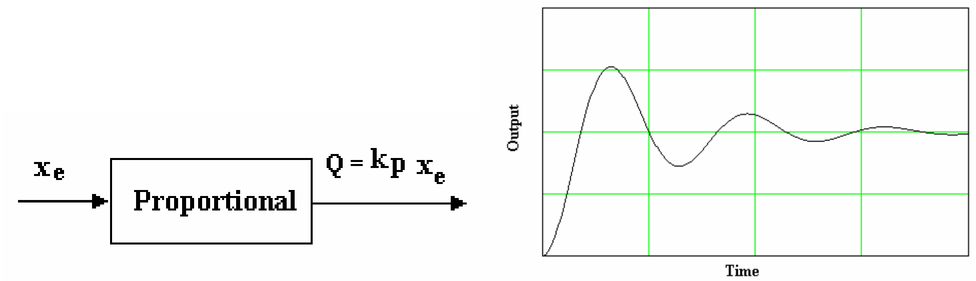

Figure 2

Increasing the gain may produce overshoot and hunting as shown above when a sudden (step) change is made to the input. This is especially true if the system is oscillatory in nature due to second order terms such as inertia or inductance and there is insufficient damping. A system will not always settle at the correct level when a step input is applied. For example the response to a system with an open loop transfer function $\frac{k}{(s+1)(s+5)}$ and unity feedback to a unit step is shown below. The system settles at a level less than unity. Increasing the gain k brings the response closer to unity but introduces a damped oscillation. This is where integral control action is needed.

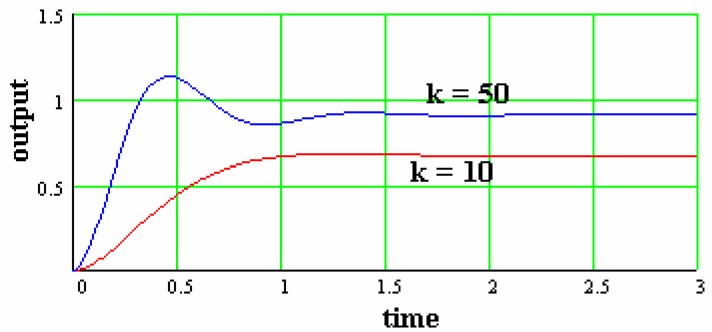

Figure 3

2.2 INTEGRAL CONTROL

This is very useful in avoiding offset error. Some systems will respond to a step input by settling at a different level to the step value. This might be due to the way the system is designed or due to a disturbance added to the output.

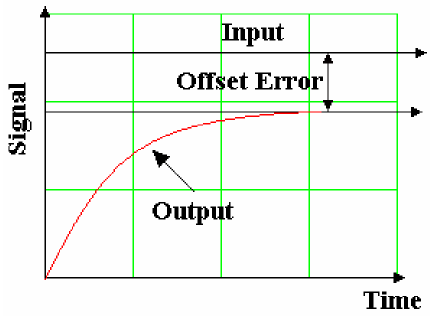

Figure 4

With integral action, the output Q will grow with time until the system responds and reduces the error to zero.

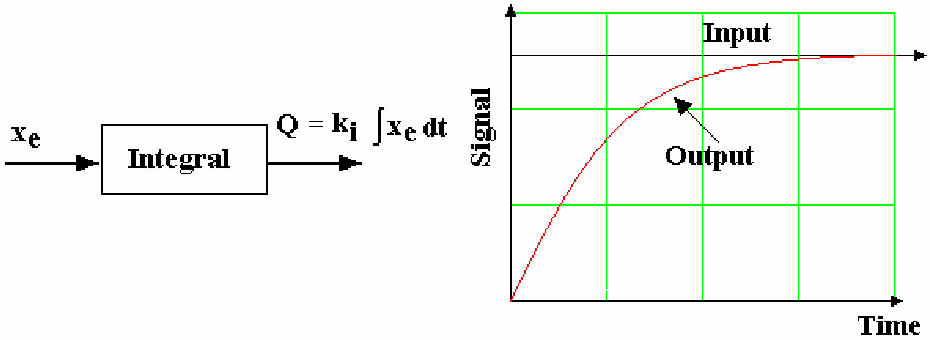

Figure 5

The equation for integral control is usually rearranged as follows.

$$Q(t) = k_i \int x_e dt = \frac{k_p}{T_i} \int x_e dt \qquad Q(s) = \frac{k_p x_e}{T_i s}$$

k_i is the integral constant of proportionality. It is usual to replace this with $k_i = \frac{k_p}{T_i}$

k_p is the proportional constant and T_i the integral time constant. The reason for arranging the equation into this form will become apparent later.

2.3 DIFFERENTIAL

The output of the controller is directly proportional to rate of change of the error.

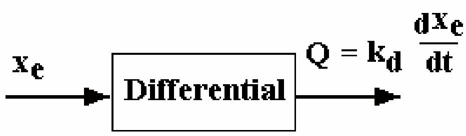

Figure 6

In the case of a step change, the rate of change is greatest at the start of the change and so the system will respond quickest in the early stages. As the error reduces, the rate of change of error also reduces and the system is slowed down in anticipation of arriving at the correct level. This form of control enables quicker response without overshoot.

The equation is usually rearranged as follows.

$$Q(t) = k_d \frac{dx_e}{dt} = (k_p T_d) \frac{dx_e}{dt} \text{ or in Laplace form } Q(s) = (k_p T_d)\, s\, x_e$$

$k_d = k_p T_d$ k_p is the proportional constant and T_d the differential time constant.

Most system controllers will have adjustments which enable the constants k_p, T_i and T_d to be set in order to optimise the system response. In modern equipment, the facility exists for the system to optimise these constants automatically. Generally they are set to produce the fastest response time possible with no overshoot.

2.4 THREE TERMS TOGETHER

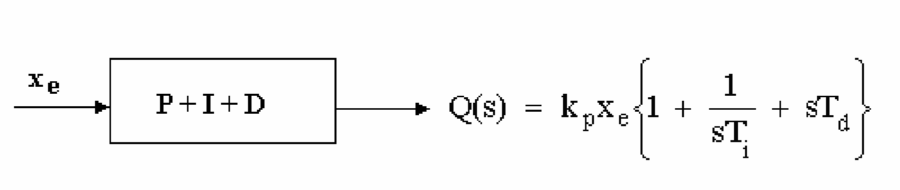

Figure 7

The output of a P.I.D. controller is

$$Q(t) = k_p x_e + \frac{k_p}{T_i} \int x_e dt + k_p T_d \frac{dx_e}{dt}$$

$$Q(s) = k_p x_e \left\{ 1 + \frac{1}{sT_i} + sT_d \right\}$$

WORKED EXAMPLE No.1

The input and output of a P.I.D. controller is related by the equation

$$Q(t) = 0.5x_e + 2\int x_e dt + 0.2 \frac{dx_e}{dt}$$

Find the value of the proportional gain, the integral time constant and the differential time constant.

SOLUTION

By comparison with the equation $Q(t) = k_p x_e + \frac{k_p}{T_i} \int x_e dt + k_p T_d \frac{dx_e}{dt}$ it is apparent that:

$k_p = 0.5$ and $k_p / T_i = 2$ hence $T_i = 0.5/2 = 0.25$ $= k_p T_d = 0.2$ hence $T_d = 0.2/0.5 = 0.4$

SELF ASSESSMENT EXERCISE No.1

The input and output of a PID controller is related by the equation

$$Q(t) = 2x_e + 0.5\int x_e dt + 4 \frac{dx_e}{dt}$$

Find the value of the proportional gain, the integral time constant and the differential time constant. (2, 4 and 2)

3. ZEIGLER – NICHOLS METHOD OF TUNING

In order to optimise the performance of a system, the controller parameters need to be set. Much has been written about this. The late Zeigler and Nichols produced a practical guide for setting up three term controllers for plant systems dating back to the 1940's. The following is still useful for that purpose.

CLOSED LOOP METHOD

In this method only proportional gain is used and this adjusted until small continuous oscillation are obtained. The system is then at the limit of instability. The gain G and periodic time T_p are noted. The three term controller is then set so that:

$k_p = 0.6\ G \qquad T_i = T_p/2 \qquad T_d = T_p/8$

This will produce a response to a step change in the form of a decaying oscillation with a damping ratio of 0.21 and the amplitude of the second cycle will be ¼ of the initial amplitude as shown. This is accepted as a reasonable setting for most process plant systems.

The resultant open loop response should be the same as in the following and the controller constants will be the same as below.

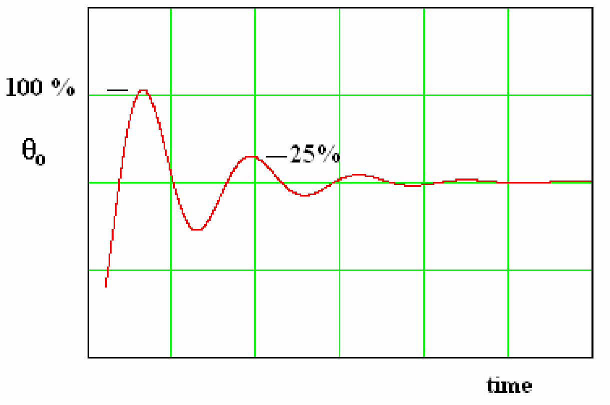

Figure 7

OPEN LOOP METHOD

With the feedback disconnected introduce a step change and measure the response. A typical plant process produces an open loop response as shown.

τ is the time delay which often occurs in plant systems due to the lag in the processes. T is the time constant, H_1 is the input step and H_2 the resultant step in the steady state. The steady state gain is H_1/H_2

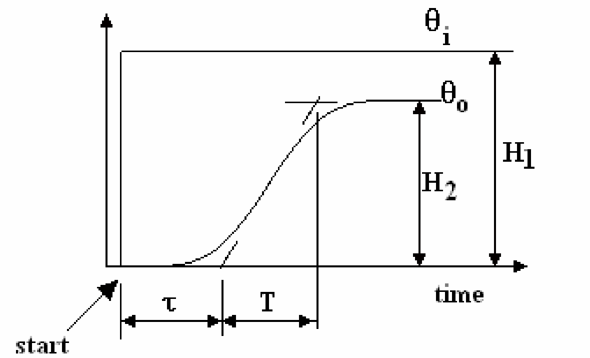

Figure 8

The settings for the controller are then adjusted as follows.

$k_p = 1.2\ T\ H_1/(H_2\ \tau) \qquad T_i = 2\ \tau \qquad T_d = 0.5\ \tau$

SELF ASSESSMENT EXERCISE No.2

1. A plant process is controlled by a PID controller. In a closed loop test using only proportional gain, the limit of stability was found to occur with a gain 4.5. Calculate the proportional, integral and differential constants required so that a ¼ decay is obtained in response to a step change.

(k_p =2.7, T_i = 40 s and T_d = 5 s)

2. The three term controller in a plant process is to be adjusted for optimal performance using the Zeigler Nichols open loop method. The proportional gain was set to give a steady state step change equal to the input change. The time delay was 24 seconds and the time constant was 50 seconds. Calculate the proportional, integral and differential constants required.

(k_p =2.5, T_i = 48 s and T_d = 12 s)

4. PHASE COMPENSATION

This is an alternative approach to using P. I. D. control and is also called Dynamic Compensation. The transfer function is denoted D(s) in the following work. The idea is to change the open loop characteristic to meet the design requirements regarding steady state error, phase margin and gain margin. A proportional gain K_p is also used. Consider a basic unit feedback system as shown. Dynamic compensation can take many forms but the most common are lead and lag.

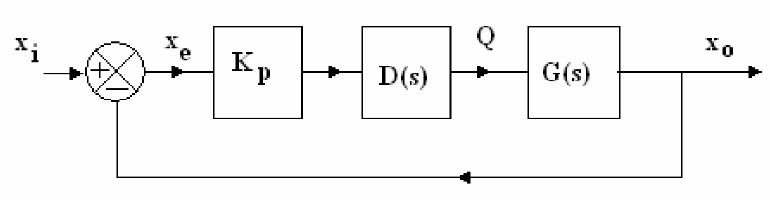

Figure 9

The dynamic compensation has transfer functions of the form $D(s) = \frac{s+z}{s+p}$ or $D(s) = \frac{\tau_1 s + 1}{\tau_2 s + 1}$.

Because of the way this was produced by analogue electronics, many text books present the transfer function differently but modern electronic controllers would have no problem using this form because it is generated by computer software.

The numerator (top line) produces lead and the denominator (bottom line) produces lag. Clearly it depends on the values of τ_1 and τ_2 as to which will dominate.

If $\tau_1 > \tau_2$ then the lead dominates.
If $\tau_2 > \tau_1$ then the lag dominates.

Although it is unlikely that either of them is used on their own, it is worth considering them separately.

LEAD COMPENSATION

This is the same as differential plus proportional. The affect of introducing this is to lower the rise time and decrease the overshoot.

$D(s) = s + z$ or $D(s) = \tau s + 1$. $D(j\omega) = j\omega\tau + 1$

The phasor on an Argand diagram is shown. The gain and phase angle are:

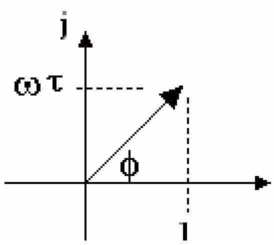

Figure 10

$$G(db) = 20\log\sqrt{\omega^2\tau^2 + 1} = 10\log\left(\omega^2\tau^2 + 1\right)$$

$$\phi = \tan^{-1}(\omega\tau)$$

LAG COMPENSATION

The affect of introducing this is to improve the steady state accuracy and so it is similar to integral action.

$D(s) = \frac{1}{s+p}$ or $D(s) = \frac{1}{\tau s + 1}$. $D(j\omega) = \frac{1}{j\omega\tau + 1}$

The phasor on an Argand diagram is shown. The gain and phase angle are:

$$G(db) = -20\log\sqrt{\omega^2\tau^2 + 1} = -10\log\left(\omega^2\tau^2 + 1\right)$$

$$\phi = -\tan^{-1}(\omega\tau)$$

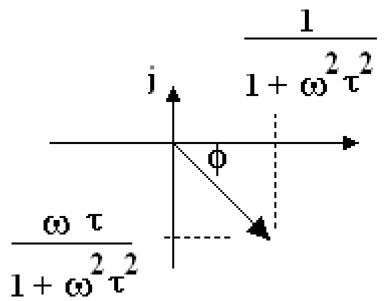

Figure 11

COMBINED

For a combination of lead and lag (which is normal) $D(s) = \frac{s+z}{s+p}$ or $D(s) = \frac{\tau_1 s + 1}{\tau_2 s + 1}$.

Basically the lag factor is useful for attenuating the signal at high frequencies and the lead factor increases the phase margin. The idea is to find values of τ_1 and τ_2 that give the required phase angle and phase gain at the required frequency. We could analyse this by looking at the position of the poles and zeros but in this tutorial we confine the work to Bode plots. The gain and phase angles are given by the following formulae.

$$G(db) = 10\log\left(\omega^2\tau_1^{\ 2} + 1\right) - 10\log\left(\omega^2\tau_2^{\ 2} + 1\right)$$

$$\phi = \tan(\omega\tau_1) - \tan^{-1}(\omega\tau_2)$$

The plot Bode plot below shows the gain and phase angle when lead dominates ($\tau_1 > \tau_2$). The gain increases above the breakpoint and the phase angle becomes more positive (stable) near the breakpoint frequency.

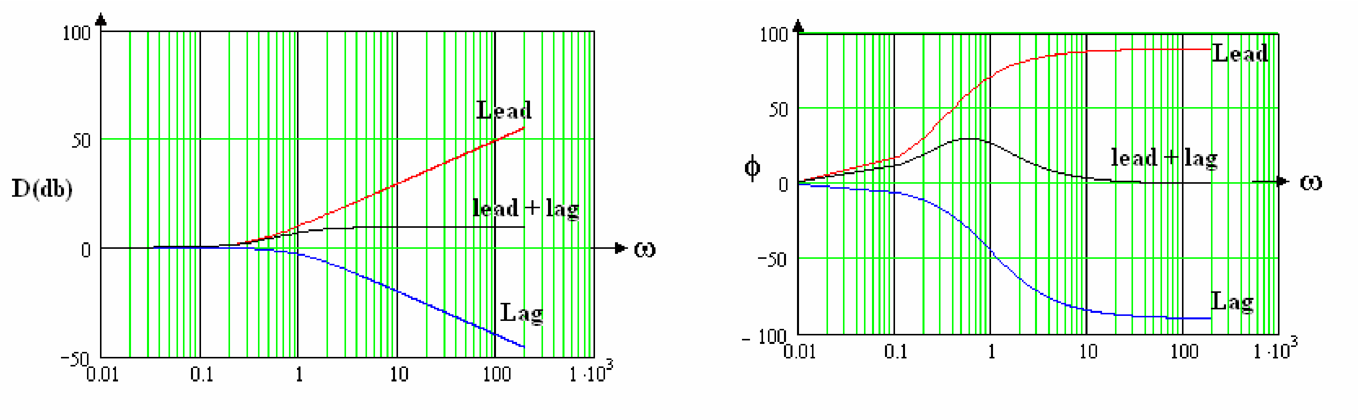

Figure 12

The plot Bode plot below shows the gain and phase angle when lag dominates ($\tau_2 > \tau_1$). The gain is attenuated above the breakpoint and the phase angle becomes more negative (less stable) near the breakpoint frequency.

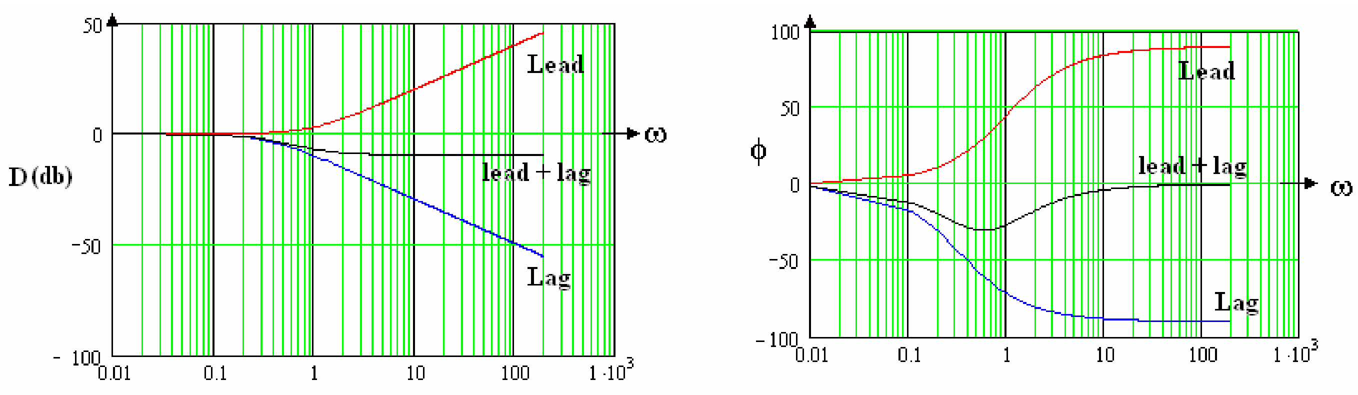

Figure 13

WORKED EXAMPLE No. 2

(Note this could also be a question about derivative control action)

The open loop transfer function for a position control system is $G(s) = \frac{1}{s(1+s)}$. This is used with unit feed back, proportional gain and a phase lead compensator. A unit ramp input of 1 m/s is applied. Determine the proportional constant K_p to produce a steady state error of 40 mm and the compensator constant τ to produce a phase margin of 30^o.

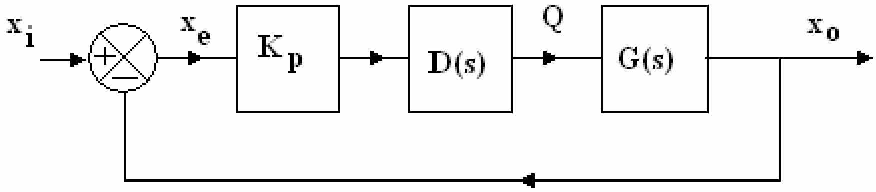

Figure 14

SOLUTION

The overall open loop transfer function is $G = \frac{K_p(1+\tau s)}{s(s+1)}$

The steady state error (from earlier tutorials) is $x_e(t) = \frac{s\theta_i(s)}{1+G(s)}$ when $s \to 0$

For a ramp or velocity input $\theta_i = ct$ and $\theta_i(s) = c/s^2$

$x_e(t) = \frac{sc}{s^2\{1+G(s)\}}$ when $s \to 0$ $\quad x_e(t) = \frac{c}{s\left\{1+\frac{K_p(1+\tau s)}{s(s+1)}\right\}}$ when $s \to 0$

$x_e(t) = \frac{c}{\left\{s+\frac{K_p(1+\tau s)}{(s+1)}\right\}}$ when $s \to 0$ $\quad x_e(t) = \frac{c}{\left\{0+\frac{K_p(1+0)}{(0+1)}\right\}} = \frac{c}{K_p}$

For a unit ramp c = 1 m/s
hence $x_e = 0.04$ m = $1(m/s)/K_p$ $\quad K_p = 1/0.02 = 25\ s^{-1}$

Now we need to find the phase margin without compensation and the frequency at which it occurs.

To do this we need a Bode plot for $G_{ol} = \frac{25}{s(s+1)}$.

The formulae to be used (from earlier tutorials) are:
$D(db) = 20\log(25) - 10\log(\omega^2T^2+1) - 20\log(\omega)$
$\phi = -\tan^{-1}(\omega T) - 90^o$ (Note T = 1)

These can be done using straight line approximations or by a full plot.

The cross over frequency is 5 rad/s and the phase angle is -169^o giving a phase margin of 11^o. This means that we need a further 19^o adding by the compensator at 5 rad/s.

The phase lag produced by the compensator is given by $\phi = \tan^{-1}(\omega\tau)$

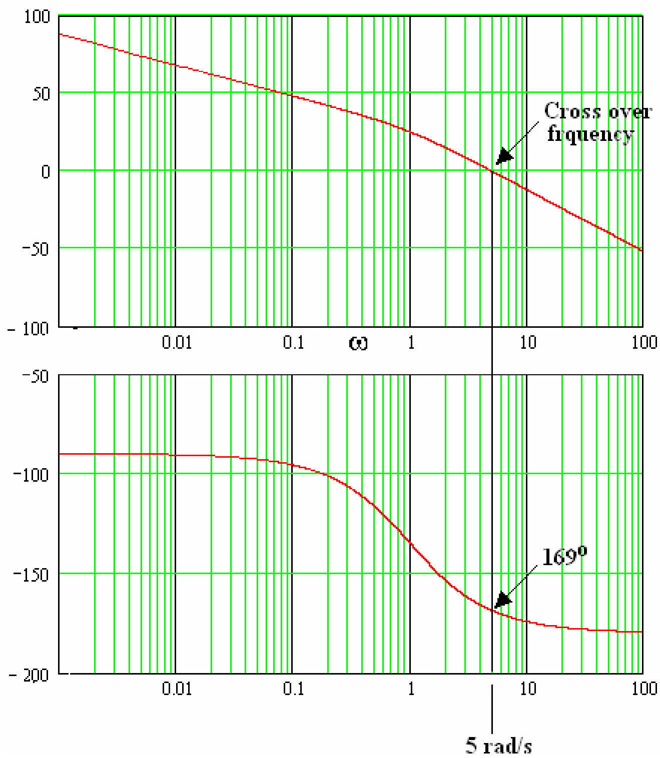

Figure 15

$\tan(19^o) = \omega\tau = 5\tau \qquad \tau = 0.344/5 = 0.069$

Now we need the Bode plot for $G = \frac{25(1+\tau s)}{s(s+1)}$ and

the formulae are:
$D(db) = 20\log(25) - 10\log(\omega^2T^2+1) - 20\log(\omega) + 10\log(\omega^2\tau^2+1)$
$\phi = -\tan^{-1}(\omega T) - 90^o + \tan^{-1}(\omega\tau)$ (Note T = 1)

The result is shown and it appears that we have met the criterion of 150^o at 5 rad/s.

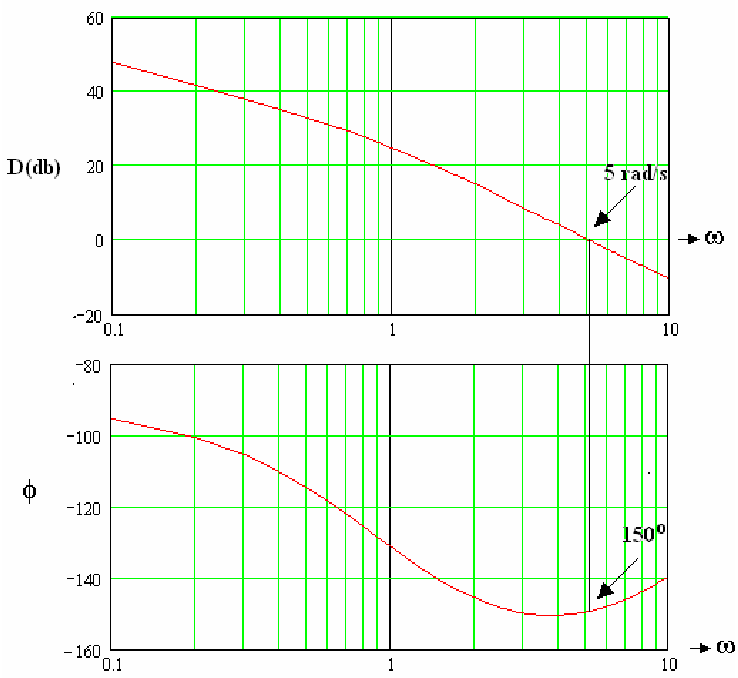

Figure 16

We need a word of warning here however. We were lucky with the figures, but adding phase lead compensation increases the cross over frequency. In this case the increase was very small. If at this stage it is found that the cross over frequency has increased significantly we might not achieve the phase margin expected and have to repeat the process with a higher frequency. This is easy with a computer package but laborious with a calculator. The best idea is to make an educated guess.

SELF ASSESSMENT EXERCISE No.3

Repeat the last example but for a phase margin requirement of 60^o. ($\tau = 0.192$)

WORKED EXAMPLE No. 4

The open loop transfer function of a system is $G(s) = \frac{K_p}{(1+4s)(1+2s)(1+0.25s)}$. Determine the best lead and lag compensation to produce a phase margin of at least 30^o and a steady state error of 1/40 in response to a unit step.

SOLUTION

We are going to add a lead and lag term of the form $D(s) = \frac{(1+\tau_1 s)}{(1+\tau_2 s)}$

The overall open loop transfer function is $G = \frac{K_p(1+\tau_1 s)}{(1+\tau_2 s)(4s+1)(2s+1)(0.25s+1)}$

The steady state error (from earlier tutorials) is $x_e(t) = \frac{s\theta_i(s)}{1+G(s)}$ when $s \to 0$

For a unit step input $\theta_i = 1/s \quad x_e(t) = \frac{1}{1+G(s)}$ when $s \to 0$

$$x_e(t) = \frac{1}{1+G(s)} = \frac{1}{1+\frac{K_p(1+\tau_1 s)}{(1+\tau_2 s)(4s+1)(2s+1)(0.25s+1)}} \text{ when } s \to 0$$

$$x_e(t) = \frac{1}{40} = \frac{1}{\frac{K_p(1)}{(1)(1)(1)(1)}} = \frac{1}{K_p}$$

Hence $K_p = 40$

The bode plot is shown for the open loop transfer function

$$G = \frac{K_p}{(4s+1)(2s+1)(0.25s+1)}$$

The cross over frequency is 2 rad/s and the phase angle at this frequency is -187° giving a phase margin of -7° which means the system is unstable.

It is relatively easy to find appropriate values of τ_1 and τ_2 with a computer package but very difficult to do it without.

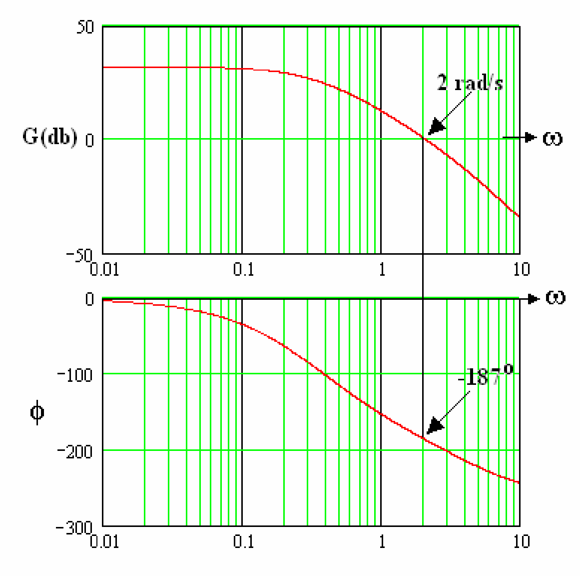

Figure 18

We require a phase margin of 30° so we need a phase angle of 150° at the cross over frequency. The frequency at which ϕ =150° is close to 0.9 rad/s and the gain is 14 db. We need to attenuate the gain by 14 db at 0.9 rad/s but this would also make the phase angle more negative. We need to make an educated guess that allowing another 5° will be more than enough so we readjust and look at ϕ = 145°. This shows we need to attenuate 15 db at 0.85 rad/s.

The change in gain is $D_c(db) = 10\log(\omega^2\tau_1^2 + 1) - 10\log(\omega^2\tau_2^2 + 1)$

The change in phase angle is $\phi_c = a\tan(\omega\tau_1) - a\tan(\omega\tau_2)$

Putting ω = 0.85 rad/s we need to do a lot of guessing and correcting to eventually find that when $\tau_1 = 21$ and $\tau_2 = 118$:

ϕ_c=-2.6° and D_c(db) = -15 db so we will achieve a phase margin of about 33 ° at the cross over frequency. We now need to plot

$$G = \frac{K_p(1+21s)}{(1+118s)(4s+1)(2s+1)(0.25s+1)}$$ and

this shows that we have achieved our objective. The blue curve shows the uncompensated result and the red curve the result after applying compensation. The phase margin is about 35°.

If the cross over point must be at some other frequency then more work would need to be done.

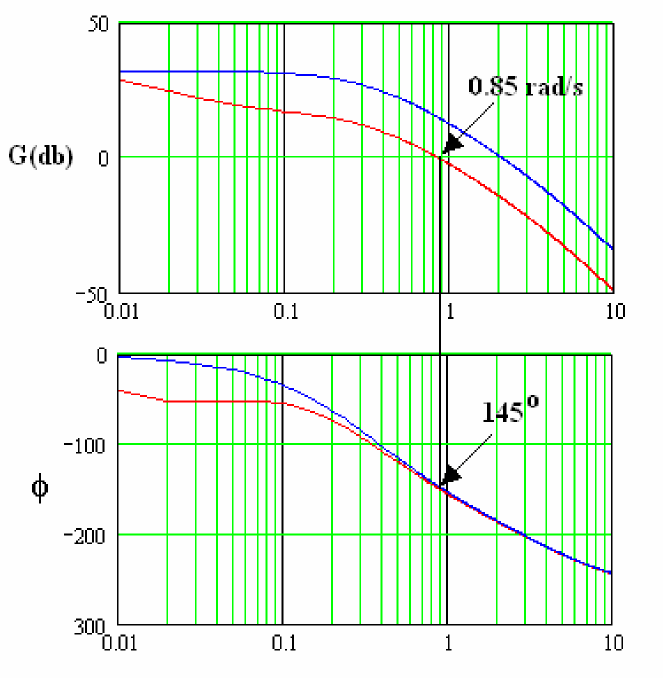

Figure 19

SELF ASSESSMENT EXERCISE No.3

The open loop transfer of a system is $G(s) = \dfrac{40}{(1+2s)(1+0.25s)}$

Determine the dynamic compensation required to produce a phase gain of 45^o with a cross over frequency between 4 and 6 rad/s.

Answer $D(s) = \dfrac{(1+8s)}{(1+15s)}$