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DISCRETE-TIME SLIDING MODES OBSERVERS FOR INDUCTION MOTORS

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Abstract

An extension of the discrete-time sliding modes observer, developed by [1] for discrete-time systems subject to uncertain disturbances and varying or unknown parameters is presented in this paper. This extension allows the observation of systems with "non-matching" non-linearities. Such observer is employed to estimate the flux and torque of the induction motor. Two formulations are presented: the first for rotor resistance disturbance rejection, the other for stator resistance disturbance rejection. These formulations allow the observer design even in the case of total lack of knowledge on some motor parameters. There is no need of a priori information on variation bounds, nominal values or statistics on the unknown parameter.

Keywords: discrete observers, discrete-time sliding modes, induction motor.

1. Introduction

The most recent methods for induction motor drives design are based on the availability of some variables that are not easily measurable, as the rotor flux. This difficulty has been usually solved through a real time motor dynamics simulation scheme [2] or, alternatively, through a conventional Luenberger states observer scheme [3]. These solutions, however, may lead to significant mismatches in the flux estimation, due to the unavoidable variation or uncertainty in some model parameters, which vary with machine saturation level and temperature. In view of this, recent works have proposed the usage of parameter adaptation schemes, such as the Kalman extended filter [4]. This procedure, however, increases the observer computational complexity, so rendering difficult its real time employment.

In this work the induction motor is treated as a discrete-time non-linear dynamic system subject to parameter uncertainties. The discrete-time sliding modes observer proposed in [1] is here extended, in order to account for the "mismatched" non-linearities in the motor. Motor flux and torque observers with the property of disturbances rejection are developed. The disturbances rejection is accomplished in arbitrary subspaces of dimension less then or equal to the number of available measurements. Two structures are presented in detail: the first one suited to the rejection of rotor resistance disturbances, and the second one designed to reject stator resistance disturbances. Simulation results are presented for the first structure. The resulting observer is, in any case, computationally simple, with the structure of a linear compensator with variable parameters depending on voltage, current and velocity measurements.

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2. Sliding Modes Observer for Discrete-Time Non-Linear Systems

An observer for non-linear discrete-time systems subject to unknown disturbances has been developed in [1]. Such observer, however, has its applications limited to the cases in which the nonlinearity is "matched", meaning that it is lumped in some subspaces of the state space. An extension of that observer is developed in this section, in order to account for the case of "non-matching" nonlinearities. The observer here developed still needs, however, the "matching condition" in the disturbances to be canceled. Start from the non-linear discrete-time system:

$$x(k+1) = F\left(x(k), u(k), k\right) + \\ + \Delta F\left(x(k), u(k), k\right) + Dv(k) \qquad (1)$$
$$y(k) = Cx(k)$$

In this system, $x \in \mathbb{R}^n$ represents the state vector, $u \in \mathbb{R}^p$ is the control inputs vector, $v \in \mathbb{R}^q$ is an unknown disturbances vector, and $y \in \mathbb{R}^m$ is the measurement vector, and $k \in \mathbb{N}$. The system dynamics is composed of a "nominal" part F and of a "disturbed" part ΔF. The function $F(\cdot,\cdot,\cdot)$ and the matrix C are supposed to be known, and the function $\Delta F(\cdot,\cdot,\cdot)$ and the matrix D are supposed to be unknown, although obeying to the following perturbation "matching" condition:

$$\mathcal{R}(H) = \mathcal{R}(D) \cup \mathcal{R}(\Delta F) \tag{2}$$

$\mathcal{R}(\cdot)$ representing the range space of the matrix argument or the image space of the function argument, depending on the context. Matrix H in (2) is supposed to be known, and must satisfy, jointly with C:

$$\begin{aligned} \rho(CH) &= \rho(H) = r \\ \rho(C) &= m \geq r \end{aligned} \tag{3}$$

where $\rho(\cdot)$ stands for the rank of the argument matrix. The nominal plant (system (1), excluded the perturbations) must also be stable and observable through the measurement matrix C in the whole operation range. With condition (2), one may write:

$$\begin{aligned} &\forall (x,k,u,v)\exists \omega : \\ &H\omega(k) = \Delta F\left(x(k), u(k), k\right) + Dv(k) \end{aligned} \tag{4}$$

This allows rewriting equation (1) as:

$$x(k+1) = F\left(x(k), u(k), k\right) + H\omega(k) \tag{5}$$

The discrete time sliding modes observer is built by adding to (5) a disturbances cancellation term:

$$\hat{x}(k+1) = F\left(\hat{x}(k), u(k), k\right) + Hw(k) + Hz(k) \tag{6}$$

Consider the following output vector partition:

$$y(k) = Cx(k) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x(k) \tag{7}$$

The term $z(k)$ is deduced from the constraint:

$$C_1\hat{x}(k+1) - y_1(k+1) = L_1\left[C_1\hat{x}(k) - y_1(k)\right] \tag{8}$$

which, once substituted into (6), leads to:

$$\begin{aligned} \hat{x}(k+1) &= \left[\mathbf{I} - H(C_1H)^{-L}C_1\right] F\left(\hat{x}, u(k), k\right) + \\ &+ H(C_1H)^{-L}\left[y(k+1) + L_1\left(C_1\hat{x}(k) - y_1(k)\right)\right] \end{aligned} \tag{9}$$

where $(\cdot)^{-L}$ stands for any left inverse of the argument matrix.

Constraint (8) is equivalent to a system order reduction, the state vector being constrained to belong to a (proper) surface in the state space. The subvector y_1 of the output vector contains the measurements employed in the disturbances decoupling. The parameter matrix L_1 may be used to assign a dynamics to the reaching motion of the state error vector $e(k) = \hat{x}(k) - x(k)$ in relation to the surface $C_1e(k) = 0$. This surface is called "sliding surface", and the observer is said to be in "sliding modes" in this surface, by analogy with control systems in sliding modes [5, 6]. Finally, in order to establish some freedom in the error dynamics assignment, a proportional term is added to equation (9), similarly to the classical Luenberger observer [7]:

$$\begin{aligned} \hat{x}(k+1) &= \left[\mathbf{I} - H(C_1H)^{-L}C_1\right] F\left(\hat{x}(k), u(k), k\right) \\ &+ H(C_1H)^{-L}y(k+1) + L\left[C\hat{x}(k) - y(k)\right] \\ &+ H(C_1H)^{-L}L_1\left[C_1\hat{x}(k) - y_1(k)\right] \end{aligned} \tag{10}$$

Define:

$$\Phi_2 = H(C_1H)^{-L} \quad \Phi_1 = \mathbf{I} - \Phi_2C_1 \tag{11}$$

so leading to the expression for the observer:

$$\begin{aligned} \hat{x}(k+1) &= \Phi_1F(\hat{x}, u, k) + \Phi_2y(k+1) + \\ &+ \Phi_2L_1\left[C_1\hat{x} - y_1\right] + L\left[C\hat{x} - y\right] \end{aligned} \tag{12}$$

The index corresponding to the vectors at instant k were omitted in equation (12). This simplified notation will be employed throughout whenever it does not generate ambiguity. This equation corresponds to an observer which completely rejects the disturbances as described by (4), with an assignable error dynamics. The error convergence analysis for the general case will not be presented here. In the specific case here treated, the dynamic function F may be written as:

$$F(x, u, k) = A(x)x \tag{13}$$

Consider this, and take $\hat{A} = A(\hat{x})$. The following observer equation is proposed:

$$\begin{aligned} \hat{x}(k+1) &= \left[\Phi_1\hat{A} + \Phi_2L_1C_1 + LC\right]\hat{x} + \Phi_1Bu + \\ &+ \Phi_2y(k+1) - \Phi_2L_1y_1 - Ly \end{aligned} \tag{14}$$

Take the following expression for $\hat{A}$:

$$\hat{A}(k) = A(k) + \Delta(k) \tag{15}$$

The equation for states estimation error becomes:

$$\begin{aligned} e(k) &= \hat{x}(k) - x(k) \\ \Psi(k) &= \left(\Phi_1\hat{A} + \Phi_2L_1(k)C_1 + L(k)C\right) \\ e(k+1) &= \Psi(k)e(k) - \Delta(k)x(k) \end{aligned} \tag{16}$$

Still in the specific case of the induction motor, the term Δ may be written as:

$$\Delta(k) = f\left(Ze(k)\right)' \tag{17}$$

with Z a constant matrix and f a constant vector. The error expression becomes:

$$e(k+1) = \Psi(k)e(k) + f.e'(k).Z'.x(k) \qquad (18)$$

Manipulating this equation comes:

$$e(k+1) = [\Psi(k) + f.x'(k).Z]\, e(k) \qquad (19)$$

From equation (16) one may infer that a proper choice of matrix L at each step may arbitrarily place the eigenvalues of matrix $\Psi = \Phi_1\hat{A} + LC$, under the condition that the system matrices pair $\langle \Phi_1\hat{A}, C\rangle$ fulfills the usual linear systems observability criteria. This is the case for induction motors, in general[1]. Note that matrices $\langle \Phi_1\hat{A}, C\rangle$ are known at each step.

The matrix corresponding to the term $(f.x'(k).Z)$, however, has the parameters f and Z known and the variable $x(k)$ only approximately known. This might define a "bounded uncertainty" domain for matrix $\Psi(k)$, which would suggest the usage of a robust design method. Practical considerations reveal, however, that this unknown term may be, most of the cases, regarded to be "dominated" by the known parcel. Then, a simplifying assumption will be here adopted, considering the error dynamic matrix to be equal to $\Psi(k)$, in the remainder of this work.

The designer might select the matrices H, L, L_1 and $(CH)^{-L}$ such that a compromise is attained between the error dynamics and possibly existing non-matching disturbances attenuation (see [1]).

3. Induction Motor Discrete-Time Model

The arbitrary reference discrete-time model proposed in [4] will be employed in this work. Taking the stator and rotor flux linkages as the state variables, the following equations are obtained:

$$\begin{bmatrix} \lambda_s(k+1) \\ \lambda_r(k+1) \end{bmatrix} = \begin{bmatrix} Q_1 \\ 0 \end{bmatrix} V_s(k) + \begin{bmatrix} P_1 - aR_sQ_1 & cR_sQ_1 \\ cR_rQ_2 & P_2 - bR_rQ_2 \end{bmatrix} \begin{bmatrix} \lambda_s(k) \\ \lambda_r(k) \end{bmatrix} \qquad (20)$$

where $\lambda_s(k) = [\ \lambda_{qs}(k) \quad \lambda_{ds}(k)\ ]'$, $\lambda_r(k) = [\ \lambda_{qr}(k) \quad \lambda_{dr}(k)\ ]'$ are respectively the stator and rotor flux linkages, and $V_s(k) = [\ v_{qs}(k) \quad v_{ds}(k)\ ]'$ is the stator voltage vector. The constants in the equation are:

R_s: stator resistance;
R_r: rotor resistance;
L_{ss}: stator inductance;
L_{rr}: rotor inductance;
M: stator-rotor mutual inductance.

$$a = \frac{1}{\sigma L_{ss}} \qquad b = \frac{1}{\sigma L_{rr}}$$

$$c = \frac{M}{L_{ss}L_{rr} - M} \qquad \sigma = 1 - \frac{M}{L_{ss}L_{rr}}$$

and the matrix terms are:

$$P_1 = \begin{bmatrix} \cos\omega h & -\sin\omega h \\ \sin\omega h & \cos\omega h \end{bmatrix}$$

$$P_2 = \begin{bmatrix} \cos(\omega-\omega_r)h & -\sin(\omega-\omega_r)h \\ \sin(\omega-\omega_r)h & \cos(\omega-\omega_r)h \end{bmatrix}$$

$$Q_1 = \frac{1}{w}\begin{bmatrix} \cos\omega h & -(1-\sin\omega h) \\ 1-\sin\omega h & \cos\omega h \end{bmatrix}$$

$$Q_2 = \frac{1}{\omega-\omega_r}\times$$

$$\times\begin{bmatrix} \cos(\omega-\omega_r)h & -[1-\sin(\omega-\omega_r)h] \\ 1-\sin(\omega-\omega_r)h & \cos(\omega-\omega_r)h \end{bmatrix}$$

in which h is the sampling period, ω is the reference frame angular velocity and ω_r is the machine rotor velocity. The mechanic equations are:

$$\omega_r(k+1) = e^{h\beta}\omega_r(k) - \frac{1}{\mu}(e^{h\beta}-1)\,(T_{em}(k) - T_L(k)) \qquad (21)$$

$$T_{em}(k) = \frac{3}{2}\frac{p}{2}c\,[\lambda_{qs}(k)\lambda_{dr}(k) - \lambda_{ds}(k)\lambda_{qr}(k)]$$

The variables T_{em} and T_L are respectively the electromagnetic torque and the load torque, and the constant β is given by $\beta = -p\mu/J_m$, with p the machine number of poles, J_m the system inertia and μ the friction constant. Define the following variables:

$$K_1 = \sin\omega h \qquad K_5 = \frac{\sin\omega h}{\omega}$$

$$K_2 = \cos\omega h \qquad K_6 = \frac{1-\cos\omega h}{\omega}$$

$$K_3 = \sin(\omega-\omega_r)h \qquad K_7 = \frac{\sin(\omega-\omega_r)h}{\omega-\omega_r}$$

$$K_4 = \cos(\omega-\omega_r)h \qquad K_8 = \frac{1-\cos(\omega-\omega_r)h}{\omega-\omega_r} \qquad (22)$$

and the constants:

$$g_1 = e^{h\beta} \qquad g_2 = \frac{-3pc}{4\mu}(g_1 - 1) \qquad g_3 = \frac{g_1 - 1}{\mu}$$

[1] To be more precise, there is a loss in observability for operating points near zero velocity. Outside this region, the system is fully observable.

Define the state vector as:

$$x(k) = [\ \lambda_{qs}(k) \quad \lambda_{ds}(k) \quad \lambda_{qr}(k) \quad \lambda_{dr}(k) \quad \omega_r(k)\ ]'$$

and the control and load input vectors as:

$$u(k) = [\ v_{qs}(k) \quad v_{ds}(k)\ ]'$$
$$d(k) = T_L(k)$$

The system dynamics is given by:

$$x(k+1) = A(k)x(k) + B(k)u(k) + D(k)d(k) \qquad (23)$$

with system matrices given by:

$$A = \begin{bmatrix} K_2 - aR_sK_5 & -K_1 + aR_sK_5 & \cdots \\ K_1 - aR_sK_5 & K_2 - aR_sK_5 & \cdots \\ cR_rK_7 & -cR_rK_8 & \cdots \\ cR_rK_8 & cR_rK_7 & \cdots \\ 0 & 0 & \cdots \end{bmatrix}$$

$$\begin{matrix} \cdots & cR_sK_5 & -cR_sK_6 & 0 \\ \cdots & cR_sK_6 & cR_sK_5 & 0 \\ \cdots & K_4 - bR_rK_7 & -K_3 + bR_rK_8 & 0 \\ \cdots & K_3 - bR_rK_8 & K_4 - bR_rK_7 & 0 \\ \cdots & -g_2\lambda_{ds} & g_2\lambda_{qs} & g_1 \end{matrix}\Bigg]$$

$$B = \begin{bmatrix} K_5 & K_6 & 0 & 0 & 0 \\ -K_6 & K_5 & 0 & 0 & 0 \end{bmatrix}'$$

$$D = [\ 0 \quad 0 \quad 0 \quad 0 \quad g_3\ ]'$$

4. State Observer Design for Unknown Rotor Resistance

Take the model given in (23) and (21). The rejection of rotor resistance disturbances requires at least two measurements, since such parameter influences a subspace of dimension two, containing the state variables $x_3(\lambda_{qr})$ and $x_4(\lambda_{dr})$. Matrix H in observer equation must be chosen as:

$$H = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}' \qquad (24)$$

It is easy to take measurements of the stator currents, which are related to the states in the following way ($y_1 = C_1x$):

$$\begin{bmatrix} i_{qs}(k) \\ i_{ds}(k) \end{bmatrix} = \begin{bmatrix} a & 0 & -c & 0 & 0 \\ 0 & a & 0 & -c & 0 \end{bmatrix} x(k) \qquad (25)$$

Choose:

$$(C_1H)^{-L} = \begin{bmatrix} -1/c & 0 \\ 0 & -1/c \end{bmatrix} \qquad (26)$$

and calculate $\Phi_2 = H(C_1H)^{-L}$ e $\Phi_1 = \mathbf{I} - \Phi_2 C_1$. Define $\alpha = a/c$. Let a_{ij} denote the element of row i and column j of the induction motor state transition matrix A, given in (23). This leads to:

$$\Phi_1 A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \alpha a_{11} & \alpha a_{12} & \alpha a_{13} & \alpha a_{14} & \alpha a_{15} \\ \alpha a_{21} & \alpha a_{22} & \alpha a_{23} & \alpha a_{24} & \alpha a_{25} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \qquad (27)$$

$$\Phi_1 B = \begin{bmatrix} K_5 & K_6 & aK_5 & aK_6 & 0 \\ -K_6 & K_5 & aK_6 & aK_5 & 0 \end{bmatrix}' \qquad (28)$$

$$\Phi_2 y_1 = \frac{-1}{c} [\ 0 \quad 0 \quad i_{qs} \quad i_{ds} \quad 0\ ]' \qquad (29)$$

The error dynamics in the observer depends on matrix $\Phi_f = [\Phi_1 A + \Phi_2 L_1 + LC]$ eigenvalues, which may be arbitrarily assigned through the choice of L_1 and L. Note that such eigenvalues must be understood as the eigenvalues of a "linear" time-varying matrix. These eigenvalues may be assigned through time-varying feedback matrices L and L_1 which are calculated at each sampling time in order to fix a linear time-invariant error dynamics.

Supposing there is no mismatched disturbance, or no reason to filter it, matrix L_1 may be chosen to be zero (a discussion on this procedure is performed in [1]). This choice implies that two eigenvalues of matrix $\Phi_1 A$ are located at the origin.

Let $\Phi_f = \Phi_1 A + LC$, and let L_{ij} be the i-th row and j-th column element of L. As:

$$C = \left[\begin{array}{c} C_1 \\ \hline C_2 \end{array}\right] = \left[\begin{array}{ccccc} a & 0 & -c & 0 & 0 \\ 0 & a & 0 & -c & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \qquad (30)$$

then:

$$\Phi_f = \begin{bmatrix} a_{11} + aL_{11} & a_{12} + aL_{12} & \cdots \\ a_{21} + aL_{21} & a_{22} + aL_{22} & \cdots \\ \alpha a_{11} + aL_{31} & \alpha a_{12} + aL_{32} & \cdots \\ \alpha a_{21} + aL_{41} & \alpha a_{22} + aL_{42} & \cdots \\ aL_{51} & aL_{52} & \cdots \end{bmatrix}$$

$$\begin{matrix} \cdots & a_{13} - cL_{11} & a_{14} - cL_{12} & L_{13} \\ \cdots & a_{23} - cL_{21} & a_{24} - cL_{22} & L_{23} \\ \cdots & \alpha a_{13} - cL_{31} & \alpha a_{14} - cL_{32} & L_{33} \\ \cdots & \alpha a_{23} - cL_{41} & \alpha a_{24} - cL_{42} & L_{43} \\ \cdots & a_{53} - cL_{51} & a_{54} - cL_{52} & a_{55} + L_{53} \end{matrix}\Bigg] \qquad (31)$$

Choose an L matrix such that matrix Φ_f has the following structure:

$$\Phi_f = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & 0 & L_{13} \\ \Phi_{21} & \Phi_{22} & 0 & 0 & L_{13} \\ \alpha\Phi_{11} & \alpha\Phi_{12} & 0 & 0 & \alpha L_{13} \\ \alpha\Phi_{21} & \alpha\Phi_{22} & 0 & 0 & \alpha L_{23} \\ 0 & 0 & a_{53} & a_{54} & a_{55} + L_{53} \end{bmatrix} \qquad (32)$$

This particular choice is taken in order to allow obtaining an analytic expression for the observer gains matrix L as a function of machine parameters and operating conditions. This form for L implies in:

$$
\begin{array}{lll}
L_{11} = \frac{a_{13}}{c} = R_s K_5 & L_{12} = \frac{a_{14}}{c} = -R_s K_6 & \\
L_{21} = -L_{12} & L_{22} = L_{11} & L_{31} = \alpha L_{11} \\
L_{32} = \alpha L_{12} & L_{41} = \alpha L_{21} & L_{42} = \alpha L_{22} \\
L_{51} = 0 & L_{52} = 0 &
\end{array}
\tag{33}
$$

With these gains, matrix Φ_f becomes:

$$
\Phi_f = \begin{bmatrix} K_2 & -K_1 & 0 & 0 & L_{13} \\ K_1 & K_2 & 0 & 0 & L_{23} \\ \alpha K_2 & -\alpha K_1 & 0 & 0 & \alpha L_{13} \\ \alpha K_1 & \alpha K_2 & 0 & 0 & \alpha L_{23} \\ 0 & 0 & a_{53} & a_{54} & a_{55} + L_{53} \end{bmatrix} \tag{34}
$$

Three eigenvalues may be assigned through the choice of the remaining parameters L_{13}, L_{23} and L_{53}. Note that the rotor resistance does not influence matrix Φ_f.

The matrix Φ_f eigenvalues are determined through the algebraic expression: $\det(\gamma \mathbf{I} - \Phi_f) = 0$. The expression for parameters L_{13}, L_{23} and L_{53} is given as a linear system of three equations:

$$
\Delta_1 L_{13} + \Delta_2 L_{23} + \Delta_3 L_{53} = \Delta_3(\gamma_i - a_{55}) \tag{35}
$$

in which:

$$
\Delta_1 = \alpha \left[a_{53}(K_2 - \gamma) - a_{54} K_1 \right]
$$

$$
\Delta_2 = \alpha \left[a_{54}(K_2 - \gamma) + a_{53} K_1 \right]
$$

$$
\Delta_3 = \gamma(K_2 - \gamma) + (K_2 \gamma - 1)
$$

and each imposed eigenvalue, γ_1, γ_2 and γ_3, generates one equation. The observer final form is:

$$
\begin{aligned}
\hat{x}_1(k+1) &= K_2 \hat{x}_1 - K_1 \hat{x}_2 + L_{13} \hat{x}_5 + \\
&\quad + K_5 v_{qs} - K_6 v_{ds} - L_{11} i_{qs} - L_{12} i_{ds} - L_{13} \omega_r \\
\hat{x}_2(k+1) &= K_1 \hat{x}_1 - K_2 \hat{x}_2 + L_{23} \hat{x}_5 + \\
&\quad + K_6 v_{qs} + K_5 v_{ds} - L_{21} i_{qs} - L_{22} i_{ds} + L_{23} \omega_r \\
\hat{x}_3(k+1) &= \alpha \hat{x}_1(k+1) - \frac{1}{c} i_{qs}(k+1) \\
\hat{x}_4(k+1) &= \alpha \hat{x}_2(k+1) - \frac{1}{c} i_{ds}(k+1) \\
\hat{x}_5(k+1) &= -g_2 \hat{x}_1 + g_2 \hat{x}_2 + \\
&\quad + (L_{53} + a_{55}) \hat{x}_5 - L_{53} \omega_r
\end{aligned}
\tag{36}
$$

These equations might be implemented in a real time DSP system, involving at each step the solution of equations (22) and then equations (35), which give the time-varying parameters for the dynamic equations (36).

5. State Observer for Unknown Stator Resistance

Using an analogous procedure, one may derive the state observer which rejects the stator resistance disturbances. This derivation is bellow sketched, without comments:

$$
H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}' \tag{37}
$$

$$
(C_1 H)^{-L} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/a \end{bmatrix} \tag{38}
$$

$$
\Phi_1 A = \begin{bmatrix} \frac{a_{31}}{\alpha} & \frac{a_{32}}{\alpha} & \frac{a_{33}}{\alpha} & \frac{a_{34}}{\alpha} & \frac{a_{35}}{\alpha} \\ \frac{a_{41}}{\alpha} & \frac{a_{42}}{\alpha} & \frac{a_{43}}{\alpha} & \frac{a_{44}}{\alpha} & \frac{a_{45}}{\alpha} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \tag{39}
$$

$$
\Phi_1 B u(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}' \tag{40}
$$

$$
\Phi_2 y_1(k+1) = \frac{1}{a} \begin{bmatrix} y_1^1(k+1) \\ y_1^2(k+1) \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{a} \begin{bmatrix} i_{qs}(k+1) \\ i_{ds}(k+1) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{41}
$$

6. Simulation Results

The results presented in figures 1 to 3 refer to acceleration from stall of an induction motor with the following parameters:

> 2.03 HP, 220/380 V, 4 poles, 60 Hz;
> Stator nominal resistance: $R_s = 1.5\Omega$;
> Rotor nominal resistance: $R_r = 1.6\Omega$;
> Stator inductance: $L_{ss} = 109$ mH;
> Rotor inductance: $L_{rr} = 117$ mH;
> Mutual inductance: $M = 98$ mH;
> Inertia: $J = 0.008$ kg.m

The observer eigenvalues are assigned to: $\sigma = \begin{bmatrix} 0 & 0 & 0.6 & 0.65 & 0.7 \end{bmatrix}$. The motor equations are solved through the 4th order Runge-Kutta method, with $100\mu s$ step, and the observer discretization step

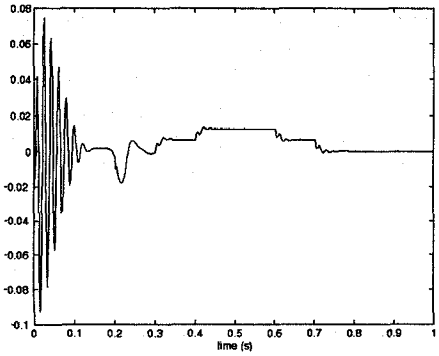

Figure 1: Quadrature-axis rotor flux (λ_{qr}) error, in p.u.

is fixed to be $500\mu s$. Figures 1 and 2 show the errors for states x_1 to x_4 in a per-unit basis, or:

$$e_i = \frac{x_i - \hat{x}_i}{x_{i_{nominal}}} \tag{42}$$

In the above expression, the value of x_i is obtained through the continuous-time model [2]. The discretization error due to the employment of a discrete-time model in the observer derivation is, therefore, at once included in the error expression.

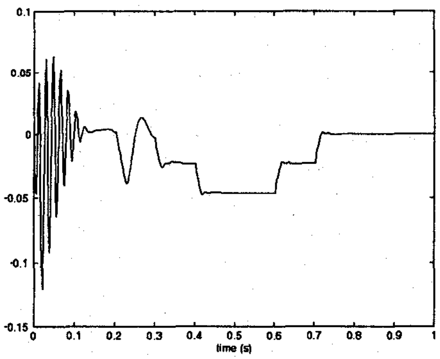

Figure 2: Direct-axis rotor flux (λ_{dr}) error, in p.u.

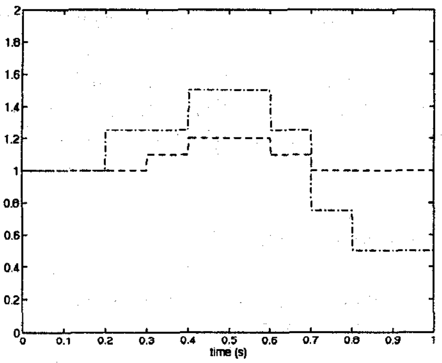

Figure 3: Stator and rotor resistances variation, in p.u.

In order to test the observer robustness against parametric variations, the stator and rotor resistances are varied as shown in figure 3. As would be expected, the observer completely rejects the rotor resistance variations. The error due to a variation of nearly 20% in the stator resistance is about 2% in the estimated variables.

7. Conclusion

An observer design methodology for non-linear uncertain discrete-time systems has been presented. Such methodology has been employed in the estimation of induction motor variables. Two observers have been derived from the general methodology: the first rejecting rotor resistance uncertainties, the second rejecting stator resistance ones. The obtained observers are computationally simple, and thus suitable for real time implementation. Additionally, the observers eigenvalues are arbitrarily assignable. The resulting observer completely decouples the perturbations or uncertainties eventually present in the specified subspaces, what is stronger than an ordinary disturbances attenuation property.

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