Multiobjective Optimization
Visualization of Solution Sets in Many-Objective Optimization

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Visualization of Solution Sets

Presentation
Many-objective optimization problem

- Multiobjective optimization problem in which $m > 3$.

$$\min_{\mathbf{x}} \mathbf{f}(\mathbf{x}) \in \mathbb{R}^m, \ \mathbf{x} \in \mathcal{F}$$

$$\mathbf{f} : \mathcal{X} \subset \mathbb{R}^n \mapsto \mathcal{Y} \subset \mathbb{R}^m$$

$$\mathcal{F} = \begin{cases} g_i(\mathbf{x}) \leq 0; & i = 1, \ldots, p \\ h_j(\mathbf{x}) = 0; & j = 1, \ldots, q \\ \mathbf{x} \in \mathcal{X} \end{cases}$$
Popular Visualization Approaches for MaOPs

- In a low-dimensional space with two or three objectives, a scatter plot shows the location, distribution, and shape of the approximated front, where each axis directly represents one objective.

- From a scatter plot, a decision-maker easily makes choices and picks up preferred solutions.

- However, it cannot be extended to high-dimensional objective spaces with more than 3 criteria.
Popular Visualization Approaches for MaOPs

- In order to illustrate the maps:

  - Consider the well-known test problem DTLZ2, and generated results for $m = 2, 3, 5$ objectives.

  - Suppose the solutions have been coloured according to the objective $m$ on which the solution performs “best”.

  - In this way: $1 = \text{red}, 2 = \text{green}, 3 = \text{blue}, 4 = \text{cyan}, 5 = \text{black}$. 
Popular Visualization Approaches for MaOPs

- Scatter plot representation:

(a) 2-objective scatter plot

(b) 3-objective scatter plot
Visualization Based on Parallel Coordinate System:

- For an $m$-dimensional objective space, the respective parallel coordinate system would contain $m$ parallel axes, each of which represents one objective.

- This method can directly show the original objective values of solutions.

- In the objective space, if objectives of interest are positioned adjacent to each other, this method can also provide information about the tradeoff relationships between objectives and contour information of the approximate Pareto front for decision-maker.
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- Visualization Based on Parallel Coordinate System:
  - Parallel coordinates.
Popular Visualization Approaches for MaOPs

- Visualization Based on Parallel Coordinate System:
  - Heatmap.

![Visualization of Solution Sets](image-url)
Popular Visualization Approaches for MaOPs

- Visualization Based on Mapping:
  - This type of methods uses some more sophisticated mapping techniques to perform dimension reduction to the 2-D space.
  - All mapping methods try to preserve Pareto dominance among individuals and retain local distances between each pair of them.
  - They are often computationally expensive and not robust as the mapping heavily depends on the values of the objective vectors in the approximate front.
  - Meanwhile, all kinds of mapping methods cannot avoid losing some important information.
  - Moreover, in this 2-D map, it is still not easy to discover the shape of the approximate front and tradeoffs between objectives.
Popular Visualization Approaches for MaOPs

- Visualization Based on Parallel Coordinate System:
  - Pairwise coordinate plot.
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- Visualization Based on Parallel Coordinate System:
  - PCA map.
  - As the colouring shows, the solutions are overlayed in such a way that is difficult to distinguish among them; moreover, the information from clusters are lost.
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- Visualization Based on Mapping:
  - Self-organizing map.
  - Nearby vectors in the input high-dimensional space are mapped to nearby units (neurons) in SOMs.
Popular Visualization Approaches for MaOPs

- Visualization Based on Parallel Coordinate System:
  - Generative Topographical Mapping (GTM).
  - It is an alternative to the SOM map. Individuals have been broadly clustered into similar groups, but there is an imperfect separation.
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- Visualization Based on Mapping:
  - Neuroscale
  - Preserves the local distances, but segregation into distinct regions is unclear.
Popular Visualization Approaches for MaOPs

- Visualization Based on Mapping:
  - Sammon mapping (preserves the local distances).
Popular Visualization Approaches for MaOPs

- Visualization Based on Mapping:
  - Radial coordinate visualization.
  - Preserves well the distribution of vectors but it cannot distinguish the shape of the approximate Pareto fronts.
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- Visualization Based on Mapping:
  - Isomap.
  - Preserves the intrinsic geometry of solutions when mapping to 2-D space, where each solution is linked only to its closest neighbors.
Some of the visualization techniques can lose information;

Each visualization technique has specific properties;

This way, each one can be useful depending on the criterion considered;

The visualization of solution sets in MaO is an open problem!
References
