Sample Size Required in Error-Rate Measurement on Fading Channels

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This paper presents analytical and numerical results on the sample size required to achieve a specified root mean square (rms) error in estimating error rate for flat fading channels having complex Gaussian statistics. The analysis shows that for the large sample sizes normally used in estimating error rates, k, the required sample size normalized to the required sample size for independent symbol fading, can be expressed in the form $k = 1 + d\beta$ where d is the symbol rate normalized to the Doppler spread of the channel. For a given modem, β is a function of the error probability and the order of diversity. It is shown that if the Doppler spread measure used is proportional to the rms Doppler spread, β will be relatively insensitive to the shape of the Doppler power spectrum.[1] *Numerical results are presented for Lth order diversity reception of binary phase shift keying (PSK), differential PSK, and frequency shift keying (FSK) signals and for five different Doppler power spectra. Ideal maximal ratio combining is assumed for the PSK modem, and square law combining is assumed for the DPSK and FSK modems.*

Keywords— *Error analysis, error rate, fading channels, measurement errors.*

I. Introduction

In attempting to measure the error rate for data transmitted over a live or simulated fading channel, it is important to know how large a transmitted symbol sample size is required to keep the fractional root mean square (rms) measurement error below a specified value. Not infrequently, experimenters use rules of thumb based upon nonfading channels in which the symbol decision variables are independent. The utility of such rules for fading channels is questionable since, unless considerable interleaving is used, the symbol decision variables are statistically dependent. There do not appear to be available analytical or numerical results on error-rate measurement errors for fading channels from which rules of thumb may be obtained. This paper takes a first step toward filling the gap.

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[1] Doppler power spectra with infinite rms Doppler spread exist, such as the spectrum of a first-order Markov process. While numerical results are presented for the first-order Markov process, no general conclusions are proposed for the case of infinite rms Doppler spread.

Analytical and numerical results are obtained on the rms error in estimating the raw (i.e., uncoded) bit error rate in transmission of digital signals over fading channels using diversity combining. It is assumed that the L diversity channels are flat fading, change little over the time duration of a symbol, and fluctuate independently with the same zero mean complex Gaussian statistics. Some general results are presented in Section II. These are specialized in Section III to binary phase shift keying (BPSK) with maximal ratio combining and to binary differential phase shift keying (BDPSK) and frequency shift keying (FSK) with square law combining. Numerical results are presented in Section IV.

II. Analysis

To estimate the error rate, we form the frequency ratio

$$\varepsilon = \frac{1}{MK}\sum_{k=1}^{K} b(k\Delta) \tag{1}$$

where Δ is the time between successive symbols, K is the number of symbols transmitted in the measurement interval $T = K\Delta$, M is the number of bits/symbol (for simplicity M is assumed to be an integer), and $b(k\Delta)$ is a discrete random variable taking on the value 0 when the kth symbol is demodulated correctly and the value m when the kth symbol is demodulated incorrectly with m bit errors.

Due to fading, the random variables $\{b(k\Delta); k = 1, \cdots, K\}$ are correlated. The mean value of ε is given by

$$E[\varepsilon] = \frac{1}{MK}\sum_{k=1}^{K} E[b(k\Delta)] = \frac{E[b]}{M} \equiv p \tag{2}$$

where p is the bit error probability averaged over both the fluctuations of the diversity channels and the additive noise. We consider "frozen channel" averages first, i.e., averages conditioned on the channel complex amplitudes. Thus

$$E[\varepsilon/\mathrm{froz}] = \frac{1}{MK}\sum_{k=1}^{K} E[b(k\Delta)/\mathrm{froz}]. \tag{3}$$

Under the fading channel assumptions and assuming maximal ratio or square law combining, the expectation in the summand of (3) may be expressed in the form

$$E[b(k\Delta)/\text{froz}] = f(\gamma(k\Delta)) \tag{4}$$

where the right-hand side is a function of a composite signal-to-noise ratio (SNR) γ

$$\gamma = \sum_{n=1}^{L} \rho_n \tag{5}$$

in which ρ_n is the energy/bit-to-noise power density ratio (Eb/No) of the nth diversity channel at the time $t = k\Delta$. Using (4) in (3)

$$E[\varepsilon/\text{froz}] = \frac{1}{MK}\sum_{k=1}^{K} f(\gamma(k\Delta)). \tag{6}$$

For BPSK signals and maximal ratio combining

$$f(\gamma) = \frac{1}{2}\text{erfc}(\sqrt{\gamma}) \tag{7}$$

where $\text{erfc}(x)$ is the complementary error function.[2] For BDPSK and square law combining

$$f(\gamma) = \frac{1}{2^{2L-1}} e^{-\gamma} \sum_{n=0}^{L-1} \frac{1}{n!} \sum_{k=0}^{L-1-n} C_k^{2L-1} \gamma^n \tag{8}$$

in which $C_n^m = m!/(m-n)!n!$ is the binomial coefficient.

Thus, the probability of bit error averaged over the channel fluctuations p is given by the integral

$$p = \frac{E[f(\gamma)]}{M} = \frac{1}{M}\int_0^\infty f(\gamma)W(\gamma)\,d\gamma \tag{9}$$

where $W(\gamma)$ is the probability density function (pdf) of the random variable γ. This pdf is the well-known chi-squared distribution of $2L$th order

$$W(\gamma) = \frac{1}{(L-1)!\left(\frac{r}{L}\right)^L} \gamma^{L-1} e^{-\gamma L/r}. \tag{10}$$

r is the average value of γ, i.e., the average Eb/No for the combined diversity channels.

The average (9) can be carried out analytically in many cases. For the BPSK case, the result is[3]

$$p = \left(\frac{1}{2}\left(1 - \sqrt{\frac{r}{L+r}}\right)\right)^L \sum_{k=0}^{L-1} C_k^{L-1+k} \times \left(\frac{1}{2}\left(1 + \sqrt{\frac{r}{L+r}}\right)\right)^k \tag{11}$$

and for the BDPSK case

$$p = \frac{1}{(2+r/L)^L} \sum_{m=0}^{L-1} \left(\frac{1+r/L}{2+r/L}\right)^m C_m^{L-1+m}. \tag{12}$$

[2] The expressions for $f(\gamma)$ in (7) and (8) are special cases of the general results on adaptive multichannel reception first developed by Price [1], [2]. See also Proakis [3, ch. 4] for a recent textbook reference.

[3] Proakis [3] is a recent text for (11) and (12); see Chapter 7.

From (1), the mean squared value of the error-rate estimator is given by the expression

$$E[\varepsilon^2] = \frac{1}{M^2K^2}\sum_{k=1}^{K}\sum_{n=1}^{K} E[b(k\Delta)b(n\Delta)]. \tag{13}$$

The average in the summand of (13) may be carried out first with the channels frozen and then over the channel fluctuations. Note that when the diversity channels are frozen, the variables $\{b(k\Delta); k = 1, \cdots, K\}$ become statistically independent nonzero-mean random variables. This two-step process yields the following expression for σ^2, the variance of the error-rate estimate:

$$\sigma^2 = E\left[\left(\frac{1}{MK}\sum_{k=1}^{K} f(\gamma(k\Delta))\right)^2\right] - p^2 + \frac{1}{M^2K}\left(E[f_2(\gamma)] - E\left[f^2(\gamma)\right]\right) \tag{14}$$

where

$$E\left[b^2(k\Delta)/\text{froz}\right] \equiv f_2(\gamma(k\Delta)). \tag{15}$$

In the particular case of $M = 1, f_2(\gamma) = f(\gamma)$.

Using the pdf of γ [(10)], the last term in (14) is obtained by evaluating the following integrals numerically:

$$E\left[f^2(\gamma)\right] = \int_0^\infty f^2(\gamma)\frac{1}{(L-1)!\left(\frac{r}{L}\right)^L}\gamma^{L-1}e^{-\gamma L/r}\,d\gamma \tag{16}$$

$$E[f_2(\gamma)] = \int_0^\infty f_2(\gamma)\frac{1}{(L-1)!\left(\frac{r}{L}\right)^L}\gamma^{L-1}e^{-\gamma L/r}\,d\gamma. \tag{17}$$

For finite K, we can expand the square in the first term of (14) and separately average the terms in the resulting double summation. The resulting average can be expressed in the convenient form

$$\sigma^2 = \frac{2}{M^2K}\sum_{1}^{K}\left(1 - \frac{k}{K}\right)\left(R(\rho(k\Delta)) - M^2p^2\right) + \frac{1}{M^2K}\left(E[f_2(\gamma)] - E\left[f^2(\gamma)\right]\right) \tag{18}$$

where

$$\begin{aligned} E[f(\gamma(k\Delta))f(\gamma(q\Delta))] &= R(\rho(k\Delta - q\Delta)) \\ &= R(\rho(q\Delta - k\Delta)) \end{aligned} \tag{19}$$

in which $\rho(\tau)$ is the common normalized ($\rho(0) = 1$) time correlation function [4] of the L diversity channels. This function is the Fourier transform of the Doppler power spectrum [4]. $R(\rho(\tau))$ is the autocorrelation function of $f(\gamma(t))$. Note that $R(0) = E[f(\gamma)]^2 = M^2p^2$ and $R(1) = E[f^2(\gamma)]$. It will be evident from the following section that the expectation in (19) is a function of the magnitude of the time correlation function.

The fractional rms error δ may be expressed in the form

$$\delta = \frac{\sigma}{p} = \sqrt{\frac{2}{K}\sum_{1}^{K}\left(1 - \frac{k}{K}\right)\alpha(\rho(k\Delta)) + \delta_i^2} \tag{20}$$

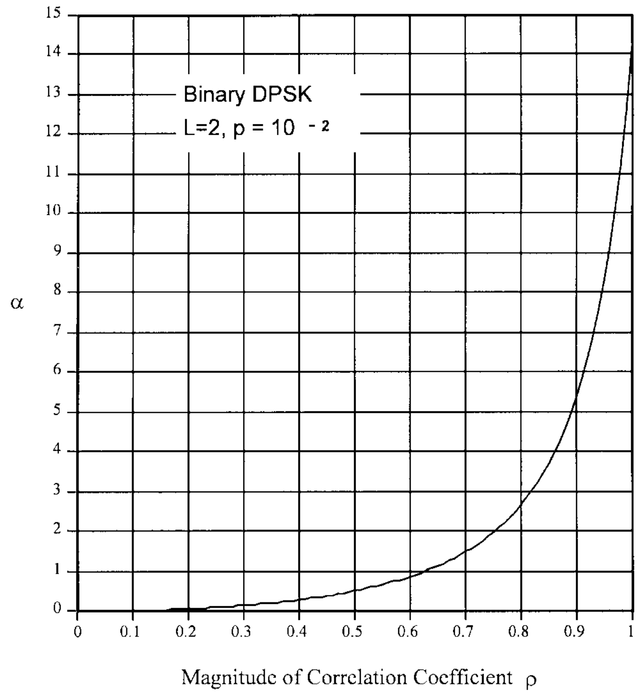

Fig. 1. Plot of $\alpha(\rho)$ for BDPSK modem with L $= 2$ and average error probability of 10^{-2}.

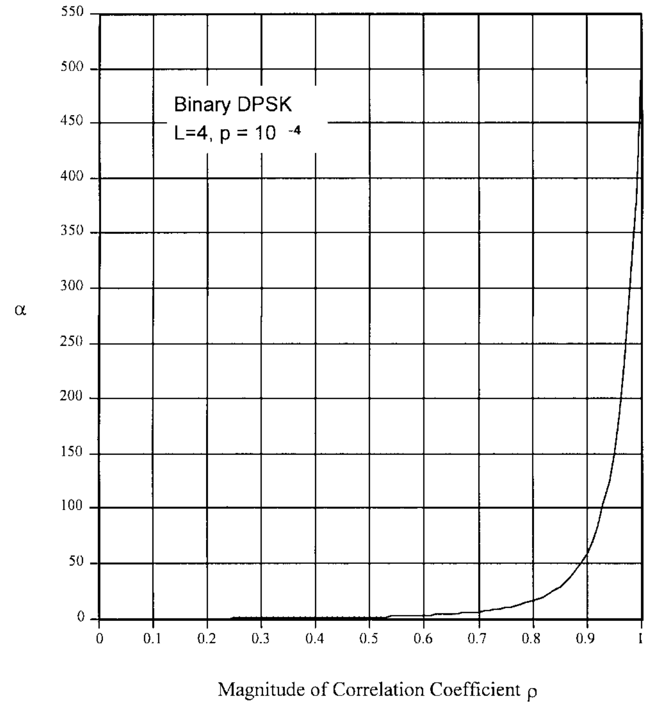

Fig. 2. Plot of $\alpha(\rho)$ for BDPSK modem with L $= 4$ and average error probability of 10^{-4}.

where

$$\delta_i = \frac{1}{Mp}\sqrt{\frac{1}{K}(E[f_2(\gamma)] - E[f^2(\gamma)])} \tag{21}$$

is the fractional error for the independent case and

$$\alpha(\rho(k\Delta)) = \frac{R(\rho(k\Delta)) - M^2p^2}{M^2p^2}. \tag{22}$$

Note that $\alpha(0) = 0$ and $\alpha(1) = (E[f^2(\gamma)] - M^2p^2)/M^2p^2$.

We consider now some approximations to δ. First, we note that while the summation in (20) extends to $k = K$, for most cases of practical interest,[4] K is so large that $\rho(k\Delta)$ and thus $\alpha(\rho(k\Delta))$ will drop to negligible values for k considerably smaller than K. Then $1 - k/K$ may be replaced by one, and the upper limit may be changed to ∞. We call the resulting approximation to δ, δ_1, the *large K approximation*

$$\delta_1 = \sqrt{\frac{2}{K}\sum_{k=1}^{\infty}(\alpha(\rho(k\Delta)) + \delta_i^2}. \tag{23}$$

A further simplification arises when the symbol rate $D = 1/\Delta$ is large compared to the Doppler spread of the channel. In this case, $\rho(k\Delta)$ will change little over a time interval Δ, and the summation in (22) can be approximated by an integral. We call the resulting approximation for δ, δ_2, the *large D approximation*

$$\delta_2 = \sqrt{\frac{2D}{K}\int_0^{\infty}\alpha(\rho(\tau))\,d\tau + \delta_i^2}. \tag{24}$$

[4]Cases in which small fractional rms errors are desired and the average error rate is not inordinately large, e.g., $<.1$.

Some interesting behavior can be predicted by expressing $\rho(\tau)$ in terms of a Doppler spread normalized time correlation function $\rho_0(\tau)$, in which the channel has unit Doppler spread and the measure of Doppler spread is proportional to the rms Doppler spread, i.e.,

$$\rho(\tau) = \rho_0(B\tau) \tag{25}$$

where B is the Doppler spread of the channel. Then

$$\delta_2 = \sqrt{\frac{2d}{K}\int_0^{\infty}\alpha(\rho_0(\tau))\,d\tau + \delta_i^2} \tag{26}$$

where the parameter $d = D/B$ is the symbol rate normalized to the Doppler spread. Note that the integral in (26) is independent of D, K, and B. We will now argue that the integral in (26), and thus the rms measurement error, will be relatively insensitive to the shape of the Doppler power spectrum, provided that the rms Doppler spread is finite and B is proportional to the rms Doppler spread. The argument is based upon the behavior of $\alpha(\rho_0)$, which is zero at $\rho_0 = 0$ and increases monotonically to its maximum value at $\rho_0 = 1$. The numerical results show that most of the value of the integral in (26) is due to the values of $\alpha(\rho_0(\tau))$ near $\rho_0 = 1$ and thus to values of $\rho_0(\tau)$ near $\tau = 0$. It follows that to a first approximation, one may replace $\rho_0(\tau)$ by a Taylor series expansion about $\tau = 0$. Since $\alpha(\rho_0(\tau))$ is a function of $|\rho(\tau)|^2$ only, in the expansion, there is no loss in generality by assuming that the Doppler power spectrum has been normalized so that it's centroid is at zero frequency. Then it is readily shown that the first two terms in a Taylor series expansion of $\rho_0(\tau)$ about $\tau = 0$ is completely specified by the rms Doppler spread.

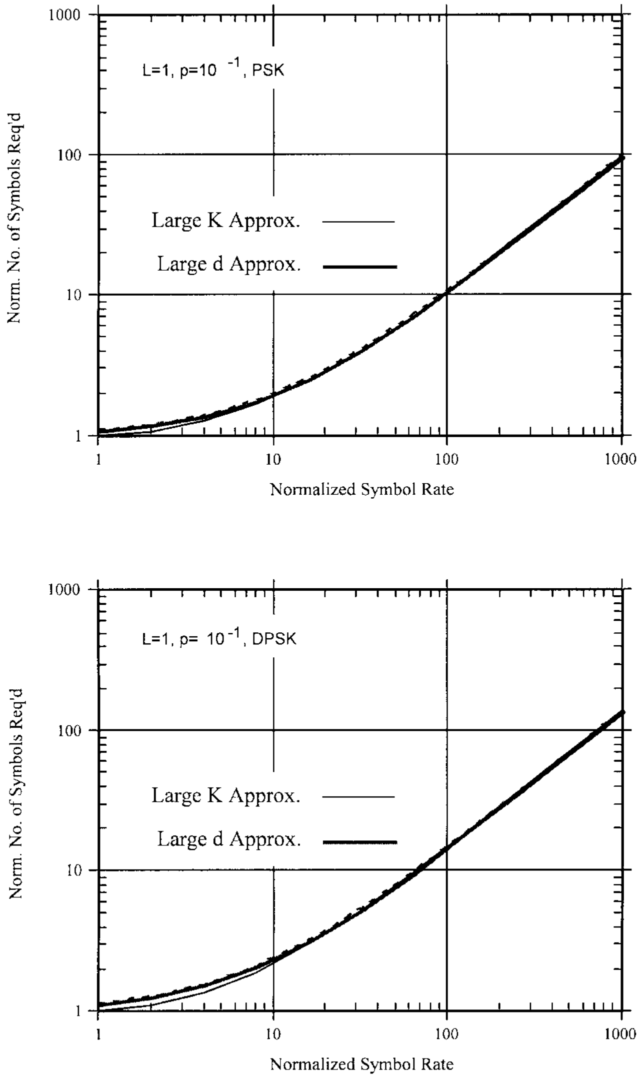

Fig. 3. Comparisons of required sample size for large K and large d approximations for L $= 1$, $p = 10^{-1}$.

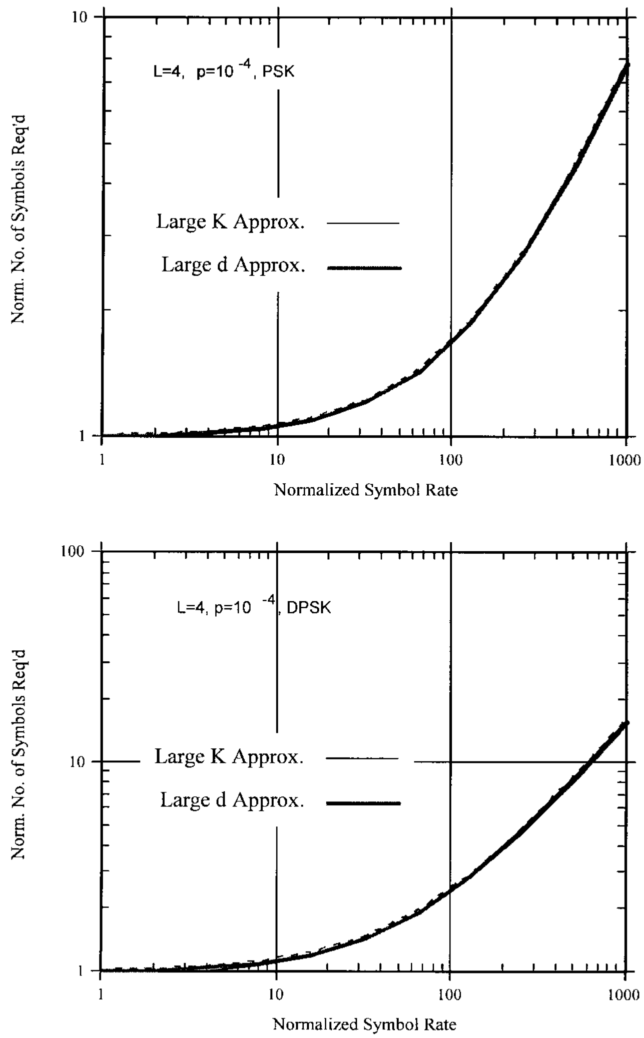

Fig. 4. Comparisons of required sample size for large K and large d approximations for L $= 4$, $p = 10^{-4}$.

Using (21) and (23), we may solve for the required value of K—say, K_2—to achieve a specified value of fractional rms error δ with the large D approximation, as well as the value of K—say, K_i—to achieve the same value of δ for the independent case. If we normalize K_2 with respect to K_i, we find that the resulting value k_2 is given by

$$k_2 = 1 + d\beta(p, L) \tag{27}$$

where the factor

$$\beta(p, L) = \frac{2M^2p^2 \int_0^\infty \alpha(\rho_0(\tau))\, d\tau}{E[f_2(\gamma)] - E[f^2(\gamma)]} \tag{28}$$

is independent of d. From the arguments given above, we conclude that for a given modem, $\beta(p, L)$ is a function of p and L and is relatively insensitive to the shape of the Doppler power spectrum.

For large d, the contribution to the fractional rms error due to independent fading becomes negligible compared to that due to the correlated fading. Then

$$\delta_2 \rightarrow \frac{1}{\sqrt{BT}} \sqrt{2 \int_0^\infty \alpha(\rho_0(\tau))\, d\tau}. \tag{29}$$

Thus, for $d \gg 1$, the fractional rms error is inversely proportional to $\sqrt{BT}$ rather than $\sqrt{K}$, as in the independent case. This result is intuitively satisfying since the product BT is proportional to the number of independent degrees of freedom in the channel fluctuations for a time interval T.

III. Specialization to BPSK, DPSK, and FSK

In the remainder of this paper, we present numerical results for BPSK with maximal ratio combining and BDPSK (and FSK) with square law combining. Numerical evaluations were carried out for the large K and large d approximations. For the range of parameters considered, there were not significant differences between these two approximations, so we consider only the large d approximation and use K instead of K_2 to simplify notation.

Table 1 β for BPSK-Rectangular Spectrum

	Bit Error Probability			
L	10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	.09788	.05866	.02153	.00717
2	.05927	.04688	.02434	.01347
3	.04199	.03399	.01784	.00988
4	.03228	.02596	.01299	.00690

Table 2 β for BDPSK-Rectangular Spectrum

	Bit Error Probability			
L	10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	.13866	.07220	.02574	.00814
2	.10298	.07000	.03684	.01982
3	.08098	.05760	.03120	.01828
4	.07100	.04718	.02560	.01476

For $M = 1$

$$\delta_i^2 = \frac{1-p}{Kp}. \tag{30}$$

After using (30) in (26) and then solving for K, we find

$$K = \frac{1}{\delta^2}\left[2d\int_0^\infty \alpha(\rho_0(\tau))\,d\tau + \frac{1-p}{p}\right]. \tag{31}$$

From (30), the number of symbols required for the independent case is

$$K_i = \frac{1}{\delta^2}\left[\frac{1-p}{p}\right]. \tag{32}$$

We will present results in a form that is independent of δ by normalizing the required number of transmitted samples in (31) to K_i. The normalized value takes the form (27) where

$$\beta = 2\frac{p}{1-p}\int_0^\infty \alpha(\rho_0(\tau))\,d\tau. \tag{33}$$

The autocorrelation function $R(\rho(\tau))$ is given by the double integral

$$R(\rho(\tau)) = \int_0^\infty\int_0^\infty W_L(\gamma_1,\gamma_2,\rho(\tau))f(\gamma_1)f(\gamma_2)\,d\gamma_1\,d\gamma_2 \tag{34}$$

where $f(\gamma)$ is specified in (9) and (11) for the BPSK and DPSK cases and $W_L(\gamma_1,\gamma_2)$ is the joint probability distribution of $\gamma(t)$ and $\gamma(t+\tau)$. This distribution is given by[5]

$$\begin{aligned} W_L(\gamma_1,\gamma_2) = {} & \frac{(\gamma_1,\gamma_2)^{\frac{L-1}{2}}\left(\frac{L}{r}\right)^L}{|\rho|^{L-1}(1-|\rho|^2)(L-1)!} \\ & \times \exp\left(-\frac{\gamma_1}{1-|\rho|^2}\frac{L}{r} - \frac{\gamma_2}{1-|\rho|^2}\frac{L}{r}\right) \\ & \times I_{L-1}\left(\frac{2|\rho|}{1-|\rho|^2}\frac{L}{r}\sqrt{\gamma_1,\gamma_2}\right) \end{aligned} \tag{35}$$

[5]Equation (34) is the generalization of a result of Miller [5] to complex Gaussian variates.

Table 3 β for BPSK-Gaussian Spectrum

	Bit Error Probability			
L	10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	.09789	.05820	.02145	.00717
2	.05972	.04643	.02413	.01342
3	.04245	.03373	.01763	.00980
4	.03271	.02583	.01282	.00683

Table 4 β for BDPSK-Gaussian Spectrum

	Bit Error Probability			
L	10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	.13930	.07290	.02615	.00857
2	.10282	.07085	.03723	.02002
3	.08055	.05777	.03158	.01844
4	.06649	.04771	.02555	.01492

Table 5 β for BPSK-Jakes/Clark Spectrum

	Bit Error Probability			
L	10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	.14574	.06618	.02228	.00725
2	.09722	.05771	.02561	.01358
3	.07136	.04503	.01943	.01002
4	.05598	.03629	.01471	.00708

for zero-mean complex Gaussian channels, where $I_n()$ is the modified Bessel function of the first kind of order n. For nonzero-mean complex Gaussian channels (i.e., Rician channels), the corresponding expression is considerably more complicated (see [5], p. 32), involving an infinite series. With a sufficiently powerful computer, however, it is still useful for obtaining numerical results.

The major numerical difficulties are connected with the accurate computation of

$$\alpha(\rho) = \frac{R(\rho)-p^2}{p^2}. \tag{36}$$

Once $\alpha(\rho)$ is computed, the numerical evaluation of the integral in the definition of β in (33) for different time correlation functions may be readily carried out. Figs. 1 and 2 present calculations of $\alpha(\rho)$ for the PSK modem for $\{L = 2, p = 10^{-2})$ and $(L = 4, p = 10^{-4})$. The concentration of significant values near $\rho = 1$ is evident. The calculations show that this concentration lessens as the error probability increases, regardless of the order of diversity. Thus, one may expect that the effect of Doppler spectrum shape on rms measurement error will become less as the error probability decreases.

IV. Numerical Results

To obtain numerical results, it is necessary to specify the normalized correlation function $\rho_0(\tau)$. We consider first

Table 6 β for BDPSK-Jakes/Clark Spectrum

L	Bit Error Probability 10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	.19821	.08050	.02691	.00864
2	.16150	.08285	.03848	.02011
3	.13234	.07137	.03322	.01854
4	.11206	.06160	.02744	.01505

Table 7 β for BPSK-First Order Markov Spectrum

L	Bit Error Probability 10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	0.03907	0.01351	0.00238	0.00036
2	0.02606	0.01474	0.00463	0.00143
3	0.01893	0.01209	0.00433	0.00152
4	0.01478	0.00987	0.00361	0.00131

Table 8 β for BDPSK First-Order Markov Spectrum

L	Bit Error Probability 10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	0.05315	0.01532	0.00265	0.00037
2	0.04361	0.02024	0.00629	0.00191
3	0.03548	0.01878	0.00672	0.00246
4	0.02986	0.01666	0.00622	0.00246

three different channel time correlation functions corresponding to Doppler power spectra frequently occurring in wireless channel models: the rectangular power spectrum, the Gaussian-shaped power spectrum, and the Jakes/Clark power spectrum. The rms Doppler spreads of each of these spectra have been chosen identically to test the hypothesis that the rms measurement errors should be relatively insensitive to the shape of the Doppler power spectrum, provided they have the same value of rms Doppler spread. A two-sided width of 1 Hz and resultant time correlation function

$$\rho_0(\tau) = \frac{\sin \pi\tau}{\pi\tau} \tag{37}$$

was chosen for the rectangular power spectrum.

The rms Doppler spread of the unit bandwidth rectangular power spectrum is $1/\sqrt{3}$ Hz. For the Gaussian-shaped spectrum, the time correlation function is given by

$$\rho_0(\tau) = \exp\left(-\frac{\pi^2\tau^2}{6}\right) \tag{38}$$

Table 9 β for BPSK Second-Order Markov Spectrum

L	Bit Error Probability 10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	.13107	.06990	.02348	.00746
2	.08150	.05927	.02816	.01463
3	.05816	.04420	.02144	.01112
4	.04491	.03432	.01600	.00798

Table 10 β for BDPSK Second-Order Markov Spectrum

L	Bit Error Probability 10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	.18435	.08563	.02831	.00887
2	.13995	.08781	.04237	.02157
3	.11082	.07357	.03713	.02044
4	.09199	.06176	.03071	.01693

and for the Jakes/Clark spectrum[6] the time correlation function is given by

$$J_0\left(\frac{2\pi t}{\sqrt{6}}\right) \tag{39}$$

where $J_0()$ is the Bessel function of the first kind and order zero. The rms Doppler spreads corresponding to (38) and (39) are $1/\sqrt{3}$ Hz.

In addition to the above spectra, for completeness, we consider time correlation functions corresponding to first- and second-order Markov processes. We present results first for the three time correlation functions described above that are more appropriate for modeling wireless channels.

Calculations were carried out for $L = 1, 2, 3, 4$ and $p = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$. The large d approximation is close to the large K approximation even for small values of d, and it becomes better the higher the order of diversity. As a check, a comparison was made between the large K approximation and the exact result for $\delta = .0316$ (corresponding to fractional error spreads of $\pm.1$ for a ± 3 standard deviation limit). The approximation was found to be negligibly different for all the cases computed. Figs. 3 and 4 present a comparison of the large K and large d approximations for the BPSK and DPSK modems for two cases, $\{p = 10^{-1}, L = 1\}$ and $\{p = 10^{-4}, L = 4\}$, assuming the rectangular Doppler power spectrum. We present computed values of β in Tables 1–6. Tables 1 and 2 apply to the rectangular power spectrum, Tables 3 and 4 to the Gaussian-shaped spectrum, and Tables 5 and 6 to the Jakes/Clark spectrum.

Comparison of Tables 1–6 verifies that the coefficient β and thus the rms measurement error is particularly insensitive to the shapes of the three Doppler spectra at the lower error rates. It is also clear from the tables that for a given order of diversity, the required sample size

[6] The Jakes/Clark Doppler spectrum with rms Doppler $1/\sqrt{3}$ is given by $1/(\pi\sqrt{\frac{1}{6} - f^2}), \quad |f| \leq (1/\sqrt{6})$.

relative to that for the uncorrelated symbol case decreases as the desired error probability decreases. A possible explanation is that with the larger SNR's required for the lower error probabilities, the errors tend to occur due to deeper and sharper fades. These sharper fades suggest more decorrelation between $f(\gamma(t)$ and $f(\gamma(t+\tau))$ for fixed τ.

Another interesting observation is that the required normalized sample size is increasingly smaller as the order of diversity increases with a more noticeable decrease for PSK. For PSK, this behavior may be explained from the fact that with large diversity, the maximal ratio combining used with PSK has the asymptotic property of converting the diversity combined channel into a nonfading additive white Gaussian noise channel. Thus, in the limit of large diversity, the data symbols will become uncorrelated, leading to no increase in normalized sample size.

Last, we consider the first- and second-order Markov processes. The first-order Markov fading process was chosen with the time correlation function

$$\rho_0(\tau) = \exp(-2.14|\tau|). \tag{40}$$

Since the rms Doppler spread for this case is infinite, we arbitrarily chose to set the 3-dB Doppler spread for this case equal to that of the Gaussian process. Tables 7 and 8 present the values of β for this case. A comparison of Tables 1–6 with Tables 7 and 8 shows that the rms error rate measurement errors are substantially less for the same value of normalized data rate in the case of the first order Markov process. This is intuitively satisfying since the slowly decreasing spectral tails of the first order Markov process in comparison with the other processes suggests more degrees of freedom, i.e., independence of fading, in a given time span.

The time correlation function for the second-order Markov process with an rms bandwidth of $1/\sqrt{3}$ is given by

$$\rho_0(\tau) = \left(1 + \frac{\pi|\tau|}{\sqrt{3}}\right)\exp\left(-\frac{\pi|\tau|}{\sqrt{3}}\right). \tag{41}$$

Tables 9 and 10 present the values of β for this case. A comparison of Tables 1–6 with Tables 9 and 10 shows that the values of β are close, as would be expected, since the rms bandwidth was chosen the same.

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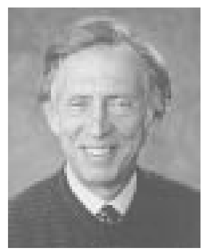

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