Aperture Aberration Caused by an Off-Axis Feed Defocusing in Generalized Classical Axially-Symmetric Dual-Reflector Antennas

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Abstract— **The present work investigates the aberration caused by small off-axis feed displacements in classical axially-symmetric dual-reflector antennas. Previous works have dealt with the design of such antennas via geometrical optics (GO), enabling a general and closed-form analysis of the GO aperture field in terms of the antenna geometry and feed illumination. Here, a first-order treatment of aberrations is added to the previous formulation, permitting an approximate analysis of the feed-displacement effects upon the antenna far-zone radiation. Geometrical conditions for the aberration reduction are derived, together with a closed-form estimate for the direction of maximum radiation.**

Keywords— **Reflector antennas, aberrations, aperture radiation, feed defocusing.**

I. Introduction

Dual-reflector antennas are largely employed in high-performance communication systems. Axially-symmetric arrangements are of easier implementation relative to their offset counterparts. However, the blockage offered by the subreflector and its supporting struts preclude the attainment of the highest-possible gain for a certain aperture geometry. The blockage mechanisms can be minimized by reducing the main-reflector scattering toward the subreflector. This can be accomplished by either shaping the reflector surfaces or applying alternative reflector arrangements where the axially-symmetric reflectors are generated by axially-displaced conic sections [1]–[6]. Recently, closed-form equations based on geometrical-optics (GO) principles were derived for the design and for the GO aperture-field distribution of these classical axially-symmetric dual-reflector antennas [3],[4]. It was verified that they can be classified into a complete set of four different families, namely Axially-Displaced Cassegrain (ADC), Gregorian (ADG), Ellipse (ADE), and Hyperbola (ADH), whose generating curves are outlined in Figs. 1–4, respectively. The classical Cassegrain and Gregorian configurations are particular cases of the ADC and ADG, respectively [3].

From the above closed-form expressions, parametric studies were conducted to determine the radiation efficiencies of such antennas [5], enabling general insights for the accomplishment of high-gain configurations. The formulation was further employed on the investigation of the first-order aberration caused by small axial feed displacements [6]. Although the GO-based analysis neglects diffraction effects, the derived closed-form equations constitute useful design tools, specially for reflectors with large electrical dimensions.

In the present work the formulation in [6] is extended to off-axis feed displacements, which typically occur whenever feed arrays are employed. The study is conducted in a generalized way, covering all different kinds of classical axially-symmetric dual-reflector antennas. In Sect. II, the aperture aberration function is defined, adopting a procedure based on [7] and only accounting for off-axis displacements. The geometrical conditions for minimizing the aperture aberration are derived in Sect. III. The direction of maximum radiation, which depends on the off-axis feed displacement, is discussed in Sect. IV. The work is concluded by Sect. V.

II. The Aperture Aberration Function for Off-Axis Feed Displacements

To accomplish the objectives of the work, a brief comment on the geometry of the present antennas is in order. The geometry of such antennas is uniquely determined from five input parameters: the main-reflector diameter D_M, the subreflector diameter D_S, the blockage diameter D_B, the subreflector edge-angle θ_E, and the constant path length ℓ_o from the primary antenna focus (at the origin) to the aperture plane (at the plane $z = 0$) [3]. The basic geometries of the ADC, ADG, ADE, and ADH generating curves, together with some relevant parameters, are depicted in Figs. 1–4, respectively. Note that the three-dimensional surfaces are yield by spinning the corresponding generating conic (a parabola for the main-reflector and an ellipse or hyperbola for the subreflector) around the symmetry axis (z-axis). Under a GO perspective, the subreflector blockage is avoided when $D_B \geq D_S$. The classical Cassegrain and Gregorian antennas are obtained from the ADC and ADG, respectively, by setting $D_B = 0$ [3]. Finally, it is important to note that positive (negative) angular values correspond to counterclockwise (clockwise) angles in the $y = 0$ plane of Figs. 1–4 [3].

The present antennas are specified (under a GO perspective) to provide a uniform phase distribution over the lit portion of the aperture whenever the feed phase center is at the primary focus. When the feed is displaced from

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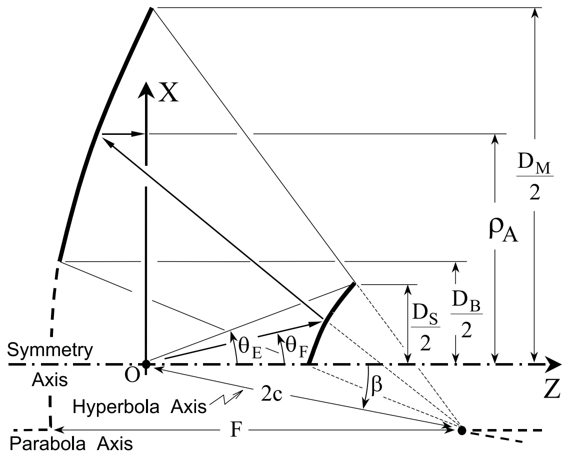

Fig. 1. Geometry of the ADC generating conics.

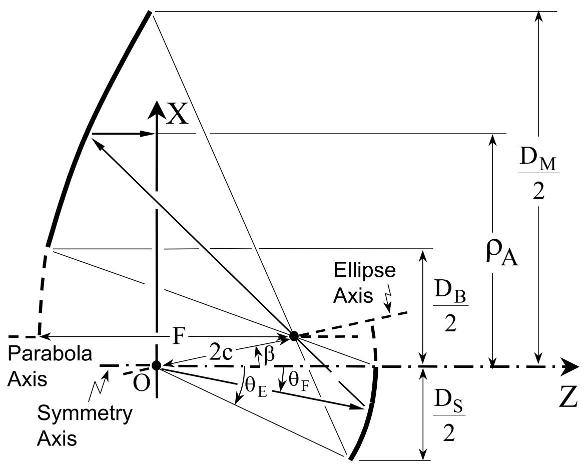

Fig. 2. Geometry of the ADG generating conics.

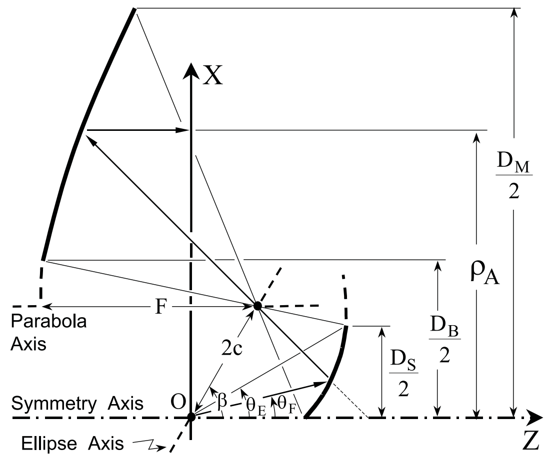

Fig. 3. Geometry of the ADE generating conics.

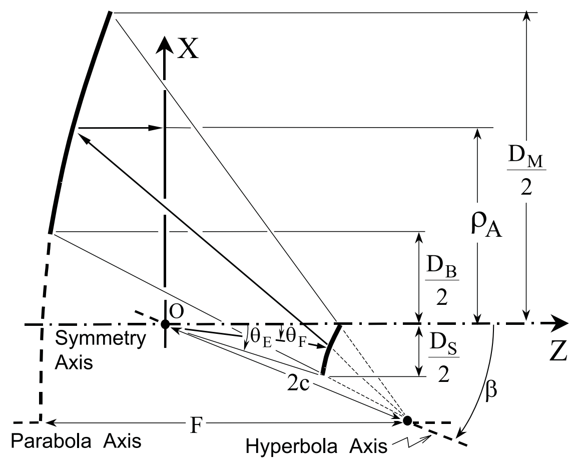

Fig. 4. Geometry of the ADH generating conics.

the focus, the aperture field is perturbed. In practical applications, for displacements smaller than the free-space wavelength (λ_o) the perturbation upon the aperture amplitude and polarization distributions can be ignored and the phase variation becomes the only concern. The phase of the aperture field is proportional to the path length (V) from the feed phase center to the aperture, along the corresponding ray path. Furthermore, V can be described by means of a first-order treatment based on [7]:

$$V \approx \ell_o - [(x_o \cos\phi_F + y_o \sin\phi_F)\sin\theta_F + z_o \cos\theta_F]\,, \quad (1)$$

where x_o, y_o, and z_o are the Cartesian coordinates locating the feed phase center and θ_F and ϕ_F are the usual spherical coordinates representing the feed-ray direction. To better understand the effects of the feed displacement upon the antenna radiation characteristics, it is appropriate to describe V in terms of the aperture polar coordinates ρ_A and ϕ_A (see Figs. 1–4). That means determining the relation between θ_F,ϕ_F (input feed-ray direction) and ρ_A,ϕ_A (output aperture-point location), which is discussed below. Furthermore, the present work is only concerned with off-axis displacements (i.e., $z_0 = 0$). A comprehensive study of the aperture aberration caused by axial displacements and the associated effects can be found in [6].

The relation between θ_F,ϕ_F and ρ_A,ϕ_A is obtained with the help of [3]:

$$\tan(\theta_F/2) = (2\,A_3 - A_1\,\rho_A)/(2\,A_4 - A_2\,\rho_A) \qquad (2)$$

and

$$|\phi_F - \phi_A| = \begin{cases} 0\,, & \text{for the ADC and ADE}\,, \\ \pi\,, & \text{for the ADG and ADH}\,, \end{cases} \qquad (3)$$

where

$$\begin{aligned} A_1 &= 1 - e\,\cos\beta\,, & (4)\\ A_2 &= e\,\sin\beta\,, & (5)\\ A_3 &= [c\,(1 - e\,\cos\beta) + e\,F]\sin\beta\,, & (6)\\ A_4 &= F\,(1 + e\,\cos\beta) + c\,e\,\sin^2\beta\,, & (7) \end{aligned}$$

F is the focal length of the main-reflector generating parabola, e and $2c$ are the eccentricity and the inter-focal distance of the subreflector generating ellipse/hyperbola, respectively, and β is the angle between the axes of the generating conics (see Figs. 1–4). All the above parameters are uniquely determined from the five input parameters [3].

To explicitly obtain V as a function of ρ_A and ϕ_A, we first apply (3) into (1), with $z_o = 0$, to obtain

$$V \approx \ell_o - \epsilon\,(x_o \cos\phi_A + y_o \sin\phi_A)\sin\theta_F\,, \qquad (8)$$

where $\epsilon = 1$ for the ADC and ADE, and $\epsilon = -1$ for the ADG and ADH. From (2), after a simple algebra, one gets

$$\sin\theta_F = \frac{2(2A_4 - A_2\rho_A)(2A_3 - A_1\rho_A)}{(2A_4 - A_2\rho_A)^2 + (2A_3 - A_1\rho_A)^2}\,. \qquad (9)$$

With the help of (3.6.22) of [8], (9) can be rewritten as

$$\sin\theta_F = \sum_{n=0}^{\infty} C_n\,\rho_A^n\,, \qquad (10)$$

where

$$\begin{aligned} C_0 &= \delta_0\,, & (11)\\ C_1 &= \delta_1 - \gamma_1\,C_0\,, & (12)\\ C_2 &= \delta_2 - \gamma_1\,C_1 - \gamma_2\,C_0\,, & (13)\\ C_n &= -\gamma_1\,C_{n-1} - \gamma_2\,C_{n-2}\,, \quad \text{for } n \geq 3\,, & (14) \end{aligned}$$

with

$$\begin{aligned} \gamma_0 &= 4(A_3^2 + A_4^2)\,, & (15)\\ \gamma_1 &= -4(A_1\,A_3 + A_2\,A_4)/\gamma_0\,, & (16)\\ \gamma_2 &= (A_1^2 + A_2^2)/\gamma_0\,, & (17)\\ \delta_0 &= 8\,A_3\,A_4/\gamma_0\,, & (18)\\ \delta_1 &= -4(A_1\,A_4 + A_2\,A_3)/\gamma_0\,, & (19)\\ \delta_2 &= 2\,A_1\,A_2/\gamma_0\,. & (20) \end{aligned}$$

Finally, (10) is applied into (8) to describe V as

$$V \approx \ell_o + (\alpha_p\,x_A + \beta_p\,y_A) + \Phi(\rho_A, \phi_A)\,, \qquad (21)$$

where

$$x_A = \rho_A\,\cos\phi_A \quad \text{and} \quad y_A = \rho_A\,\sin\phi_A \qquad (22)$$

are the Cartesian coordinates of the aperture point,

$$\alpha_p = -\epsilon\,C_1\,x_o \quad \text{and} \quad \beta_p = -\epsilon\,C_1\,y_o \qquad (23)$$

are the direction cosines of the aperture principal ray (which corresponds to the direction of maximum radiation for an aberration-free aperture) [7], and

$$\begin{aligned} \Phi(\rho_A, \phi_A) &= -\,\epsilon\,(x_o\,\cos\phi_A + y_o\,\sin\phi_A)\\ &\times \left(C_0 + \sum_{n=2}^{\infty} C_n\,\rho_A^n\right) \end{aligned} \qquad (24)$$

is the aperture aberration function, describing the nonlinear variation of V.

It is important to note from (24) that the classical axially-symmetric dual-reflector antennas present aberration terms that are not encountered in the usual axially-symmetric optical systems [9]. This is due to the discontinuity of the surface curvature at the subreflector vertex (see Figs. 1–4), which transforms the feed principal ray ($\theta_F = 0$) into a cylindrical locus of output rays (with $\rho_A = D_B/2$ for the ADC and ADG and $\rho_A = D_M/2$ for the ADE and ADH, whenever $x_o = y_o = z_o = 0$). So, the one-to-one correspondence between input feed rays and output aperture rays fails for the principal ray. As a consequence, the bilinear transformation derived in [7] can not be applied for the present antennas (the only exceptions are the classical Cassegrain and Gregorian, which are discussed below).

For the sake of completeness, it is interesting to observe that the present formulation is also applicable to the classical Cassegrain and Gregorian antennas, obtained from the ADC and ADG, respectively, by setting $D_B = 0$ [3]. For such antennas, $\beta = 0$ and, from (5) and (6), $A_2 = A_3 = 0$. From (11)–(20) it can be readily shown that, under these circumstances, $C_n = 0$ for even n-values. Consequently, tilt (C_1) and coma (C_3) are the only primary aberrations caused by an off-axis feed displacement [7].

III. Conditions for Reduced Aberrations

It is quite evident from (1) that the aperture aberration is less pronounced for antenna geometries with smaller $|\theta_E|$-angles (see Figs. 1–4), as it directly implies on smaller variations of the feed angle θ_F. This can be clearly observed from the coefficients C_n. It can be shown from (11)–(20) that

$$C_n \propto (A_1\,A_4 - A_2\,A_3)/(A_3^2 + A_4^2)^{n+1}, \quad \text{for } n \geq 1\,. \qquad (25)$$

So, in order to minimize the aberration and, consequently, the aperture-efficiency degradation caused by the off-axis feed phase-center displacement, one should attempt (in principle) to minimize the numerator and/or to maximize the denominator of (25). Parametric studies were conducted and demonstrated that the maximization of the denominator (which occurs whenever $|e| \to \infty$) does not imply on the minimum possible aberration for practical configurations. On the other hand, from (4)–(7) it is readily shown that the numerator of (25) is equal to

$$A_1\,A_4 - A_2\,A_3 = F\,(1 - e^2)\,, \qquad (26)$$

indicating that the aberration is minimized as $|e| \to 1$. The concern with the sign of e is justified, as, for the ADC and ADH, the generating hyperbola can be either convex ($e > 1$) or concave ($e < -1$) [3]. The condition $|e| = 1$ is never met in practice, as the subreflector generating conic tends to a parabola and, consequently, $2c \to \infty$. However, $|e| \to 1$ as the distance between the antenna primary focus and the subreflector vertex is increased and, consequently,

$|\theta_E| \to 0$ (see Figs. 1–4). Although not discussed here, it can be shown that the same condition holds for axial feed displacements.

IV. Direction of Maximum Radiation

The previous formulation is now applied for the estimation of the main-beam tilt caused by an off-axis feed displacement. Assuming that such displacement is sufficiently small, the beam tilt is basically provoked by the phase variation of the aperture field. So, in order to accomplish a simple and closed-form equation, suited for design purposes, it is assumed that the amplitude and polarization of the aperture field are both uniform. This approximation does not yields severe errors, as far as the feed displacement is kept sufficiently small compared to λ_o and the antenna designed for maximum gain [5]. So, the quasi linearly-polarized far-zone field can be approximately evaluated from the diffraction integral

$$I = \int_0^{2\pi} \int_0^{\frac{D_M}{2}} e^{-jk_o[V - \rho_A \sin\theta \cos(\phi_A - \phi)]} \rho_A \, d\rho_A \, d\phi_A \,, \tag{27}$$

for observations around the antenna bore-sight. In (27), $k_o = 2\pi/\lambda_o$, θ and ϕ are the spherical coordinates of the far-zone direction, and the lower limit of the ρ_A-integral is assumed equal to zero as, for high-gain configurations, D_B and $D_S \leq 0.1\, D_M$ [5]. The maximum radiation direction is obtained from [7]

$$\frac{\partial |I|^2}{\partial \theta} = \frac{\partial |I|^2}{\partial \phi} = 0 \,. \tag{28}$$

To solve (28) in closed form, the exponential term in (27) is first expanded as in [7]. After some algebra, it can be shown that (28) implies on

$$\sin\theta_\infty \, e^{j\,\phi_\infty} = -\,(x_o + j\,y_o)/f_o \,, \tag{29}$$

where θ_∞ and ϕ_∞ are the spherical coordinates of the maximum radiation direction and f_o is given by

$$f_o \approx \frac{D_M}{8\epsilon} \left[\sum_{n=0}^{\infty} \frac{C_n}{3+n} \left(\frac{D_M}{2} \right)^n \right]^{-1} . \tag{30}$$

Note that, following the discussion in [7], f_o could be interpreted as an equivalent focal distance for the generalized classical axially-symmetric dual-reflector antennas. However, there is no possible equivalent paraboloid for the present reflector arrangements, as the wavefront leaves the subreflector with different principal curvatures (i.e., as an astigmatic tube of rays). Furthermore, for the classical Cassegrain and Gregorian antennas, it can be shown that (30) reduces to

$$f_o \approx F\, |(e+1)/(e-1)| \,, \tag{31}$$

as expected.

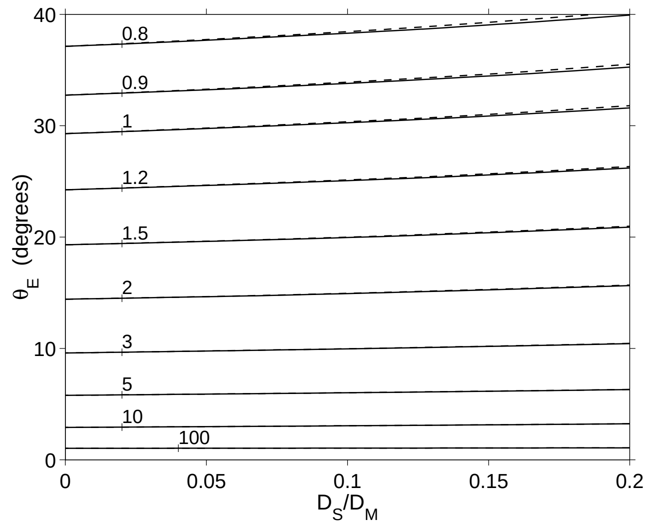

Fig. 5. ADC f_o/D_M variation with D_S/D_M ($D_B = D_S$) and θ_E: $\ell_o/D_M = 0.5$ (solid lines) and 1 (dashed lines).

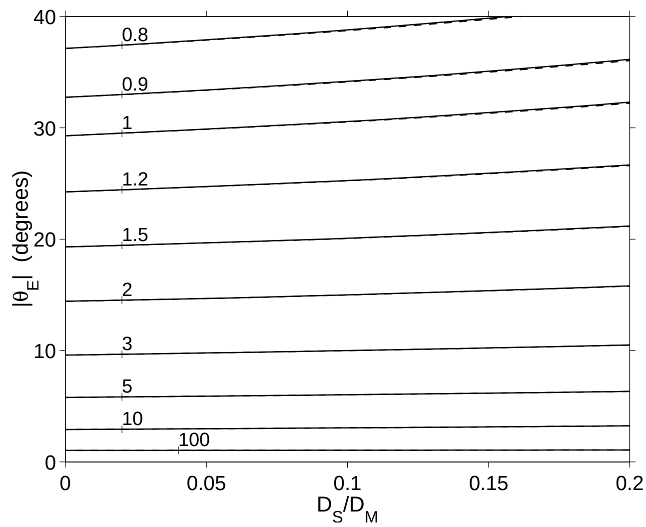

Fig. 6. ADG f_o/D_M variation with D_S/D_M ($D_B = D_S$) and $|\theta_E|$: $\ell_o/D_M = 0.7$ (solid lines) and 1 (dashed lines).

The values of f_o/D_M for several geometries are depicted in Figs. 5–8, for the ADC, ADG, ADE, and ADH, respectively. Here $D_B = D_S$, as generally desired for such antennas. The remaining input parameters were varied in the following manner: $0 < D_S/D_M \leq 0.2$, $|\theta_E| \leq 40°$, and $0.5 \leq \ell_o/D_M \leq 1$. The ADG and ADH geometries investigated in Figs. 6 and 8 were judiciously chosen, such that no blockage mechanisms are present [3]. The results show that f_o does not heavily depend on D_S nor on ℓ_o. Also, the behavior of f_o is similar for the ADC and ADG antennas. The same can be said between the ADE and ADH, at least for $|\theta_E| \leq 12°$ and $\ell_o = D_M$. Also, for a certain edge angle $|\theta_E|$, the ADE and ADH present a larger f_o than the ADC and ADG.

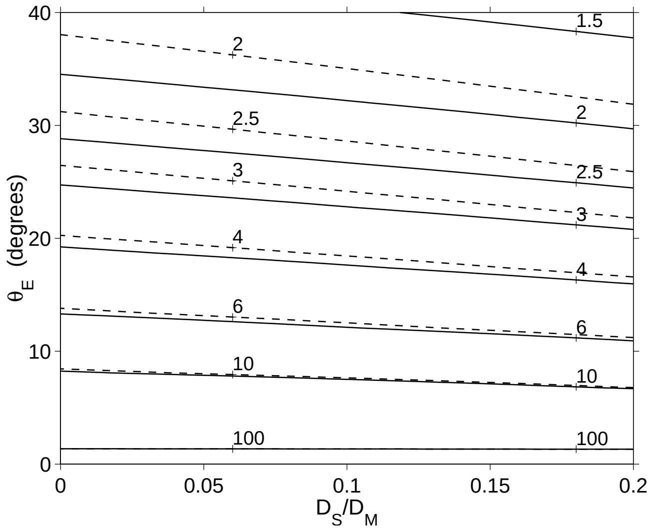

Fig. 7. ADE f_o/D_M variation with D_S/D_M ($D_B = D_S$) and θ_E: $\ell_o/D_M = 0.5$ (solid lines) and 1 (dashed lines).

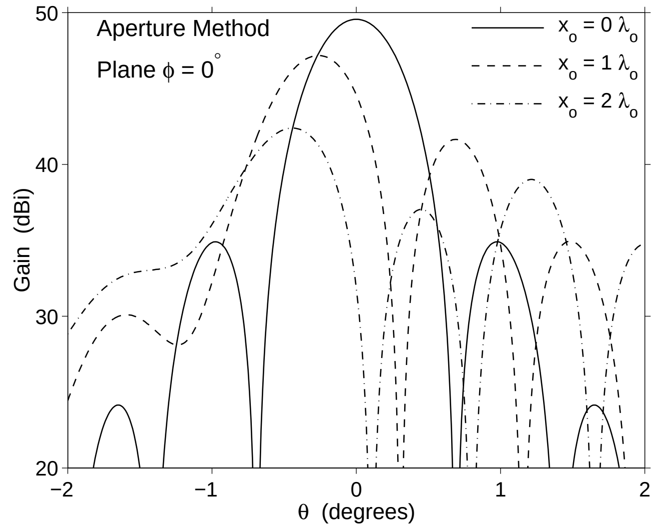

Fig. 9. ADE far-zone patterns (aperture method) for some x_o-values: $D_M = \ell_o = 100\lambda_o$, $D_S = D_B = 10\lambda_o$, and $\theta_E = 30°$.

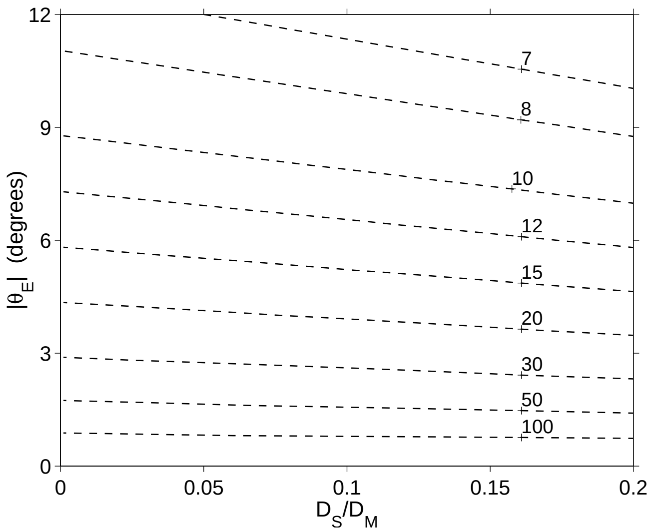

Fig. 8. ADH f_o/D_M variation with D_S/D_M ($D_B = D_S$) and $|\theta_E|$: $\ell_o/D_M = 1$ (dashed lines).

The present theory was employed to investigate the main-beam tilt caused by the variation of x_o ($y_o = 0$) on an ADE antenna with $D_M = \ell_o = 100\lambda_o$, $D_S = D_B = 10\lambda_o$, and $\theta_E = 30°$, fed by a raised-cosine model with a 20 dB edge taper [5]. The far-zone patterns were obtained from the integration of the GO aperture field (aperture method), with the phase distribution corrected by (1). The pattern cuts at the plane $\phi = 0°$ are shown in Fig. 9. From this figure, $\theta_\infty = -0.26°$ and $-0.44°$ for $x_o = 1\lambda_o$ and $2\lambda_o$, respectively. The estimates of (29) provide $\theta_\infty = -0.24°$ and $-0.48°$, respectively. It was verified that the estimate improves for smaller $|\theta_E|$-angles, as the aberration becomes less pronounced.

V. Conclusions

This work presented a first-order treatment of aberrations provoked by small off-axis feed displacements in generalized classical axially-symmetric dual-reflector antennas. The classical Cassegrain and Gregorian antennas are included as particular cases. The theory provides insights for the minimization of such aberrations, being particularly useful for applications involving feed arrays. Also, a closed-form estimate for the direction of maximum radiation, associated to the main-beam tilt caused by the feed displacement, was derived and applied.

References

[1] Yu. A. Yerukhimovich, "Analysis of Two-Mirror Antennas of a General Type," Telecommunications and Radio Engineering, Part 2, **27**, No. 11, pp. 97–103, 1972.

[2] Yu. A. Yerukhimovich and A. Ya. Miroshnichenko, "Development of Double-Reflector Antennas with a Displaced Focal Axis," Telecommunications and Radio Engineering, Part 2, **30**, No. 9, pp. 90–94, 1975.

[3] F. J. S. Moreira, "Design and Rigorous Analysis of Generalized Axially-Symmetric Dual-Reflector Antennas," Ph.D. Dissertation, University of Southern California, Aug. 1997.

[4] F. J. S. Moreira and A. Prata, Jr., "Generalized Classical Axially-Symmetric Dual-Reflector Antennas," 1997 IEEE AP-S International Symp. Digest, Montreal, Canada, pp. 1402–1405, July 1997.

[5] F. J. S. Moreira and A. Prata, Jr., "Maximum-Efficiency Generalized Classical Axially-Symmetric Dual-Reflector Antennas," 10th International Symp. on Antennas (JINA 98), Nice, France, pp. 594–597, Nov. 1998.

[6] F. J. S. Moreira, "Efficiency Degradation Caused by an Axial Feed Defocusing in Generalized Classical Axially-Symmetric Dual-Reflector Antennas," 1999 SBMO/IEEE Internat. Microwave and Optoelectronics Conference (IMOC 99), Rio de Janeiro, Brazil, pp. 112–116, Aug. 1999.

[7] C. Dragone, "A First-Order Treatment of Aberrations in Cassegrainian and Gregorian Antennas," IEEE Trans. on Antennas and Propagat., **AP-30**, No. 3, pp. 331–339, May 1982.

[8] M. Abramowitz and I. A. Stegun (eds.), *Handbook of Mathematical Functions*, Dover Publications, Inc., New York.

[9] M. Born and E. Wolf, *Principles of Optics*, 6th ed., Pergamon Press, Oxford, 1991.