Determining the Optimal Number of Basis Functions in the MoM Solutions for Bodies of Revolution

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Abstract — **This work investigates the minimum number of basis functions necessary to attain numerical convergence in the method of moment (MoM) analysis of electromagnetic scattering by conducting, dielectric and composites bodies of revolution. The analysis is based on the CFIE and Müller formulations for conductor and dielectrics surfaces, respectively. Empirical formulas are derived for the optimal number of basis functions, based on the analysis of electrically small and large spherical bodies.**

Palavras-chaves **— Electric and magnetic field integral equations, eletromgnetic sacattering by bodies of revolution, Method of moments.**

I. Introduction

One of the most efficient numerical techniques to analyze the scattering by homogeneous bodies is the surface integral equation numericaly evaluated by the MoM [1]-[6]. In special for bodies of revolution (BOR´s), the problem is formulated in terms of integrals over generatrixes and reduces the computational effort significatly. The MoM has been used successfully to compute the scattering by dielectric and conducting bodies of revolution (BOR's) [6]-[9]. For simple homogeneous media, proper combinations of surface electric and magnetic field integral equations (EFIE and MFIE, respectively) have proved to be highly appropriate. Depending on the electromagnetic and geometrical features of the BOR's, certain combinations may yield ill-conditioned numerical analysis. But, the combined field formulation (CFIE) and Müller formulation are recognized to be the most efficient ones for closed perfect electric conductor (PEC) [8] and dielectric [9]-[10] BOR's, respectively. For composite bodies the EFIE and CFIE for conducting surfaces together with Müller or PMCHWT for dielectric surfaces are the most used and efficient formulations [11]-[15].

Although the electromagnetic scattering from conducting, dielectric and composite bodies, as those shown in Fig. 1, has been studied successfully using MoM [1]-[15], only small geometries and small relative permittivity (ε_r) values are evaluated, and a optimal number of basis functions to evaluated the problem is not indicated.

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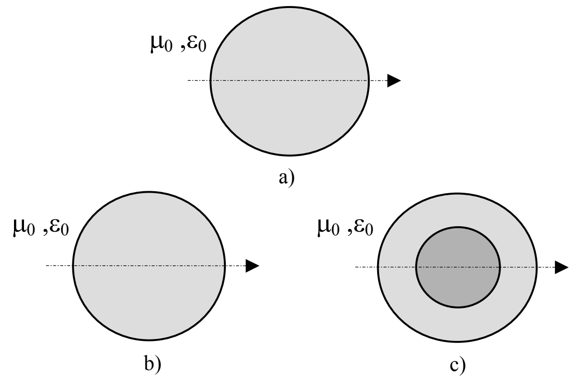

Fig. 1. Generic axially homogeneous bodies of revolution. a) PEC body. b) Dielectric body. c) Coated PEC body.

In this work we adopt the versatile triangular basis functions (TBF) to represent the current spatial variation along the BOR generatrix. Closed conducting and dielectric surfaces are analyzed by CFIE and Müller formulation, respectively. Then, for several different dielectric, conducting and composite spheres, we obtain the minimum number of segments (i.e., basis functions) necessary to attain a desired accuracy for the current representation. The study leads to empirical formulas that provide the optimal (minimum) number of segments (ONS) in terms of the sphere relative permittivity (for dielectrics) and electrical radius. Such formulas may serve as basis for the choice of the number of basis functions in complex BOR geometries.

II. Formulation

For a homogeneous body, as that shown in Fig. 1, with permittivity ε_1 and permeability μ_1, immersed in an infinite and homogeneous medium with permittivity ε_0 and permeability μ_0, the equivalence principle can be applied to establish a set of four integral equations (the EFIE and MFIE, for fields inside and outside the body) to solve for the electric (**E**) and magnetic (**H**) fields in terms of equivalent electric (**J**) and magnetic (**M**) surface currents [6]. Assuming that the sources of the incident field ($\mathbf{E}^{inc}$, $\mathbf{H}^{inc}$) are outside the body, the integral equations for the tangential field components can be represented as [6]:

$$[\eta_0\mathbf{L}_0(\mathbf{J})+\mathbf{K}_0(\mathbf{M})]_{\tan} = \mathbf{E}_{\tan}^{\mathrm{inc}}, \quad (1)$$

$$[\mathbf{L}_0(\mathbf{M})-\eta_0\mathbf{K}_0(\mathbf{J})]_{\tan} = \mathbf{H}_{\tan}^{\mathrm{inc}}, \quad (2)$$

$$[\eta_1\mathbf{L}_1(\mathbf{J})+\mathbf{K}_1(\mathbf{M})]_{\tan} = 0, \quad (3)$$

$$[\mathbf{L}_1(\mathbf{M})-\eta_1\mathbf{K}_1(\mathbf{J})]_{\tan} = 0, \quad (4)$$

where, applying the index i to represent the interior (i = 1) and exterior (i = 0) regions, $\eta_i = \sqrt{\mu_i/\varepsilon_i}$ and the integral operators $\mathbf{L}_i$ and $\mathbf{K}_i$ are

$$\mathbf{L}_i(\mathbf{X}) = \frac{j}{k_i}\int_{s'}\left[k_i^2\,\mathbf{X}(\mathbf{r}')\,\mathrm{G}_i(\mathbf{r},\mathbf{r}') - \nabla'\,\mathbf{X}(\mathbf{r}')\nabla'\mathrm{G}_i(\mathbf{r},\mathbf{r}')\right]\,ds', \quad (5)$$

$$\mathbf{K}_i(\mathbf{X}) = \upsilon_i\,\hat{\boldsymbol{n}} \times \frac{\mathbf{X}(\mathbf{r})}{2} + \int_{s'}\mathbf{X}(\mathbf{r}')\times\nabla'\mathrm{G}_i(\mathbf{r},\mathbf{r}')\,ds', \quad (6)$$

where S denotes the surface of the BOR, $\hat{\boldsymbol{n}}$ is the unit vector normal to the BOR surface, $\mathbf{X}$ is either the equivalent electric or magnetic current on S, υi is a constant that values 1 if the field point resides on the outer surface and values –1 if the field point resides on the inner surface, $k_i = \omega\sqrt{\mu_i\varepsilon_i}$ and

$$\mathrm{G}_i(\mathbf{r},\mathbf{r}') = \frac{e^{-jk_i|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}, \quad (7)$$

is the free space Green's function of region i.

Since in (1)-(4) there are four integral equations and two unknowns (**J** and **M**). So, it is possible to develop a number of different combinations among (1)-(4) to solve for **J** and **M**. For PEC bodies, the EFIE (1), with **M** = 0, is the choice for open shells [5]. For closed-surface bodies, the CFIE avoids spurious resonances and, consequently, provides stable numerical solutions [5]. The CFIE is a linear combination of (1) and (2) with **M** = 0:

$$\alpha'\,\mathrm{EFIE}_0 + \beta'\,\mathrm{MFIE}_0, \quad (8)$$

where α' and β' are linear combination constants [5]. For dielectric surfaces the usual way to reduce the number of equations is to combine them as [6]-[8]:

$$\mathrm{EFIE}_0 + \alpha\ \mathrm{EFIE}_1, \quad (9)$$

$$\mathrm{MFIE}_0 + \beta\ \mathrm{MFIE}_1, \quad (10)$$

where $\alpha = \varepsilon_1/\varepsilon_0$ and $\beta = \mu_1/\mu_0$ is commonly known as the Müller formulation, and $\alpha = -1$ and $\beta = -1$ is commonly known as the PMCHWT formulation.

III. MoM Solution

Equations (8)-(10) provide necessary integral equations for solving the equivalent surface currents for conducting, dielectric and composite BOR´s. Once this equivalent currents are obtained, the field everywhere can be evaluated using integral representation of the field in each region. In order to improve the accuracy and convergence of the numerical analysis, the equivalent current distributions must be adequately represented over the BOR surface. This is achieved by judiciously choosing the shape and quantity of basis functions, according with the expected electromagnetic behavior. In this work we adopt the versatile triangular basis functions (TBF) to represent the current spatial variation along the BOR generatrix [6]. The TBF is defined over two consecutive segments, among those used to describe the generatrix [6], as show Fig 2. The unknown surface currents, **J** and **M**, are expanded

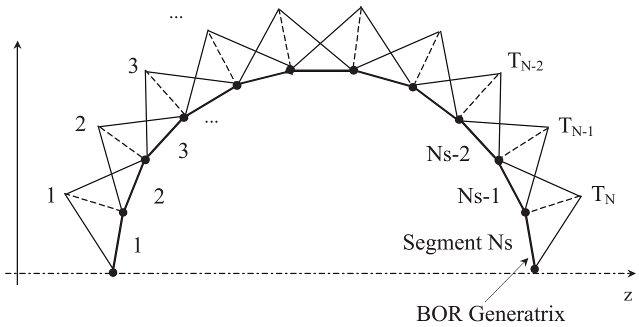

Fig. 2. Triangular basis functions (TBF).

$$\mathbf{X}(\mathbf{r}') = \sum_{n=-\infty}^{\infty}\left[\sum_{j=1}^{\mathrm{Nt}} \mathrm{I}_{jn}^{\mathrm{Xt}}\,\frac{\mathrm{T}_j^{t}(t')}{\rho'}\,\hat{\mathbf{t}}' + \sum_{j=1}^{N\phi}\mathrm{I}_{jn}^{\mathrm{X}\phi}\,\frac{\mathrm{T}_j^{\phi}(t')}{\rho'}\,\hat{\boldsymbol{\phi}}'\right]e^{jn\phi'}, \quad (11)$$

where **X** is either the equivalent electric current or magnetic current, $\hat{\mathbf{t}}$ (generating curve direction) and $\hat{\boldsymbol{\phi}}$ are unit directions in source point $\mathbf{r}'$, t' and ϕ' are the corresponding coordinates locating $\mathbf{r}'$, $T_j^t(t')$ and $T_j^{\phi}(t')$ are TBF, N_t and N_ϕ denote the number of TBF for surface currents **J** and **M**, respectively, and I_{jn}^{Xt} and $I_{jn}^{X\phi}$ are unknown coefficients. In (11) the multiplication for $e^{jn\phi'}$ represents ϕ' variation (Fourier series) and ρ' division prevents singularity problems when ρ' tends to zero.

Using (8)-(10) the integral equations were evaluated by the MoM, together with the Galerkin's method. All integrals appearing in the computation of the Z-matrix terms were evaluated using Legendre-Gauss quadrature, with the self-term singularities properly removed [6].

IV. Numerical Results

By the foregoing formulation the surface currents for several different dielectric, conducting and composites spheres are computed. For all realized tests the spheres are illuminated by a plane wave polarized in $\hat{x}$ direction and propagating in $\hat{z}$ direction, as shown in Fig. 3. Using the Mie-series solution [16] numerical results was analyzed using the following mean relative error

$$\mathrm{E}_{\mathrm{MR}}(\%) = \frac{\mathrm{E}_{\mathrm{RJt}}+\mathrm{E}_{\mathrm{RJ}\phi}+\mathrm{E}_{\mathrm{RMt}}+\mathrm{E}_{\mathrm{RM}\phi}}{4}, \quad (12)$$

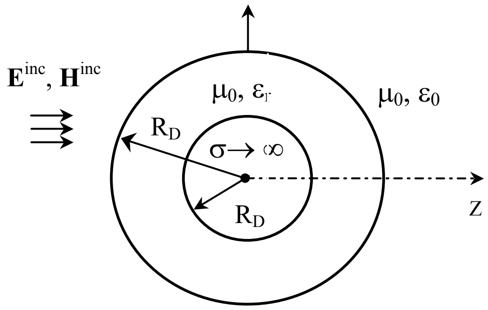

Fig. 3. Spheres to scattering analysis by MoM.

where the mean relative error E_{MR} takes into account the mean relative errors of the tangential $\hat{\mathbf{t}}$ and $\hat{\boldsymbol{\phi}}$ components of **J** and **M** (E_{RX}), according to:

$$E_{RX}(\%) = 100\left[\frac{\left|\left|X^{MoM}\right|-\left|X^{Mie}\right|\right|}{\left|X^{Mie}\right|_{max}}\right]_{max}, \qquad (13)$$

where X^{MoM} and X^{Mie} denotes the numerical MoM and the analytical Mie-series solutions, respectively.

First we examine the scattering from a PEC sphere having a radius $R_C = 0.5\ \lambda_o$, where λ_o is the wavelength in vacuum (which surrounds the spheres). Fig. 4 shows the computed electric current in $\hat{\mathbf{t}}$ and $\hat{\boldsymbol{\phi}}$ directions as functions of surface coordinate $S(\lambda_o)$, which value zero on the front of the sphere and $2\pi R_C$ on the back of the sphere. The agreement between numerical and analytical results is excellent, E_{MR} = 0.935 %. In this case, were used 54 TBF to represent the current spatial variation along of BOR generatrix in each direction, $\hat{\mathbf{t}}$ and $\hat{\boldsymbol{\phi}}$ (i.e. 55 segments to represent the sphere generatrix). Now for a PEC sphere with $R_C = 50\ \lambda_o$ the computed electric current in $\hat{\mathbf{t}}$ and $\hat{\boldsymbol{\phi}}$ directions as functions of $S(\lambda_o)$ are plotted in Fig 5. Again the agreement between numerical and analytical results is excellent, E_{MR} = 0.977 %. In this case were used 2352 (i.e. 2353 segments) TBF to represent the current spatial variation.

For a dielectric sphere having a radius $R_D = 0.5\ \lambda_o$ and ε_r = 100, computed electric and magnetic current in $\hat{\mathbf{t}}$ direction as functions of $S(\lambda_o)$ are plotted in Fig 6. The agreement between numerical and analytical results is excellent, E_{MR} = 0.996 %. In this case were used 124 TBF (i.e. 125 segments) to represent the current spatial variation. For a dielectric sphere with $R_D = 75\ \lambda_o$ and $\varepsilon_r = 1$, computed electric current in $\hat{\mathbf{t}}$ and $\hat{\boldsymbol{\phi}}$ directions as functions of $S(\lambda_o)$ are plotted in Fig 7. Again the agreement between numerical and analytical results is excellent, E_{MR} =1.04 %. In this case were used 3531 TBF (i.e. 3531 segments) to represent the current.

Next we studied the accuracy of the solution to scattering by PEC coated BOR´s, as that presented in Figs. 1 c) and 3. Fig. 8 shows computed electric and magnetic current in $\hat{\mathbf{t}}$ direction in the dielectric external surface of a PEC

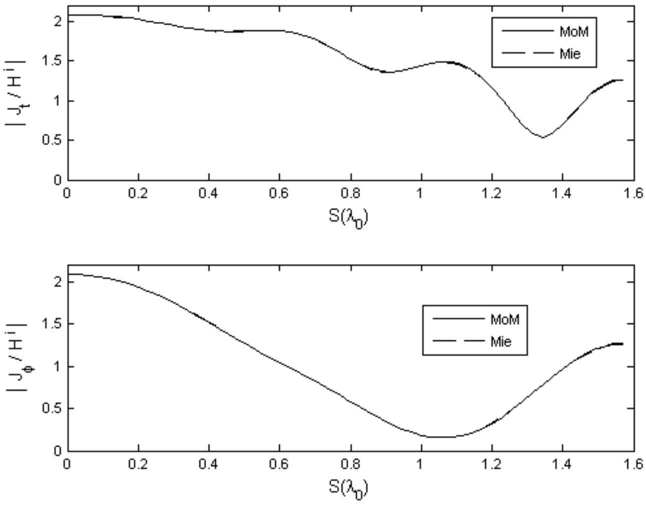

Fig. 4. PEC sphere $R_C = 0.5\ \lambda_o$. a) Electric current J_t. b) Magnetic current M_t.

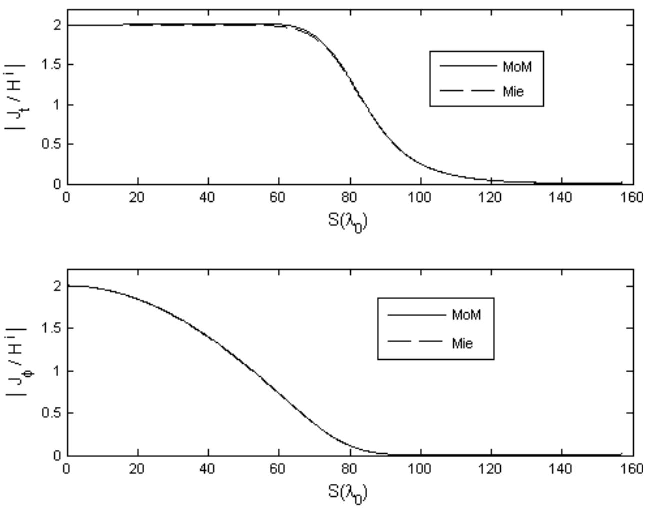

Fig. 5. PEC sphere $R_C = 50\ \lambda_o$. a) Electric current J_t. b) Magnetic current M_t.

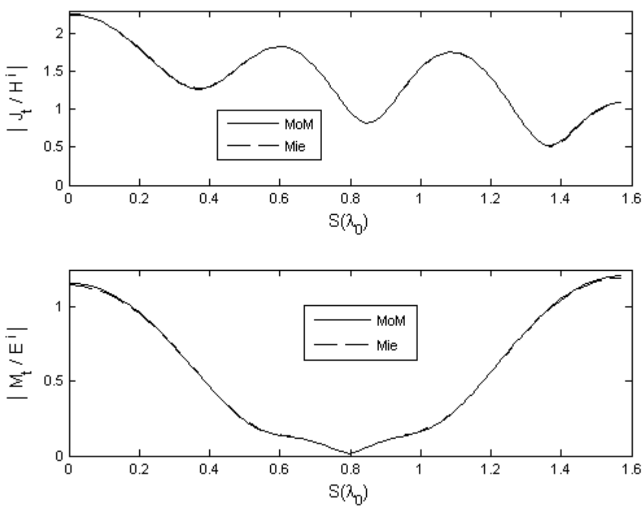

Fig. 6. Dielectric sphere $R_D = 0.5\ \lambda_o\ \varepsilon_r = 100$. a) Electric current J_t. b) Magnetic current M_t.

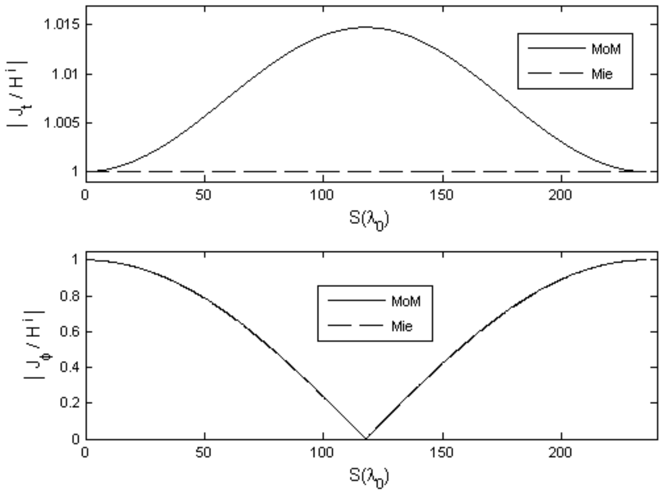

Fig. 7. Dielectric sphere $R_D = 75\ \lambda_O$ $\varepsilon_r = 1$. a) Electric current J_t. b) Electric current J_ϕ.

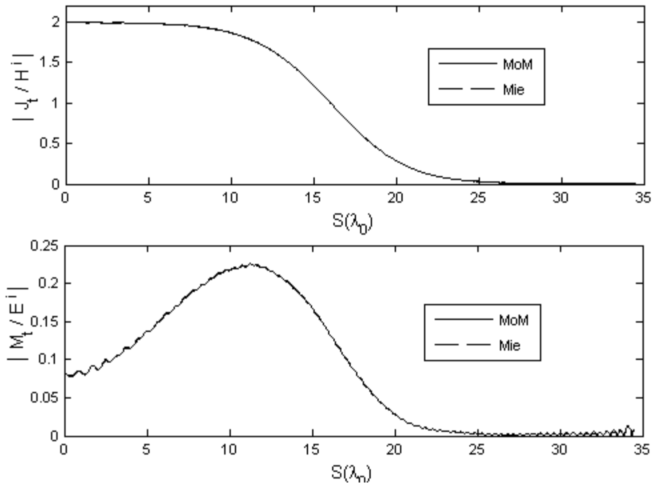

Fig. 9. PEC sphere $R_C = 10\ \lambda_o$ coated with a dielectric layer $R_D = 11\ \lambda_O$, $\varepsilon_r = 20$. a) Electric current J_t. b) Magnetic current M_t.

coated sphere as functions of $S(\lambda_o)$. The radius of PEC surface, $R_C = 5\ \lambda_o$ end for dielectric layer the radius, $R_D = 10\ \lambda_o$ and , $\varepsilon_{r\,=}\ 20$. In this case were used 1099 TBF (i.e. 1100 segments) to represent the current in each surface and direction. The agreement between numerical and analytical results is excellent, E_{MR} = 0.949 %. Finally, Fig. 9 shows computed electric and magnetic current in $\hat{\mathbf{t}}$ direction in the dieletric external surface for a PEC coated sphere as functions of $S(\lambda_o)$ with $R_C = 10\ \lambda_o$, $R_D = 11\ \lambda_o$ and , $\varepsilon_{r\,=}\ 20$. In this case were also used 1099 TBF to represent the current spatial variation The agreement between numerical and analytical results is excellent, E_{MR} = 1.23 %.

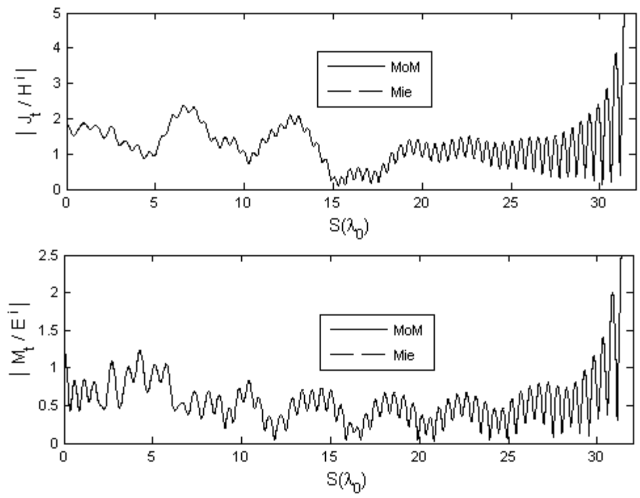

Fig. 8 PEC sphere of $R_C = 5\ \lambda_o$ coated with a dielectric layer $R_D = 10\ \lambda_o$ and $\varepsilon_r = 20$. a) Electric current J_t. b) Magnetic current M_t.

I. Optimum Number of Basis Functions

To obtain the optimum number of segments per λ_0 (i.e., basis functions), ONS per λ_0, the plane-wave scattering by PEC, dielectric and PEC coated spheres was analyzed for several different ε_r and sphere electrical radii ($R(\lambda_o)$). For each computation, the ONS was attained for E_{MR} error smaller than 1.0 %, with respect to the Mie series solutions [16]. Fig. 10 a) shows the ONS per λ_o for PEC spheres as function of $R(\lambda_o)$ and Fig. 10 b) shows the ONS per λ_o for dielectric spheres, which radius R_D varies from 0,5 λ_o to 75 λ_o, as function of ε_r. For PEC spheres it can be observed that as $R(\lambda_o)$ increases (i.e., as the sphere curvature diminishes), lesser segments per λ_o are needed. For dielectric spheres can be observed that ONS per λ_o does not vary with the increase of $R(\lambda_o)$, but as ε_r increases more segments per λ_o are needed. From the numerical data indicated in Fig. 10, empirical formulas for the ONS per λ_o were obtained:

$$\mathrm{ONS}\Big|_{\mathrm{PEC}} = 14.636 + 29.615 e^{-0.8759 R/\lambda_0}, \tag{14}$$

$$\mathrm{ONS}\Big|_{\mathrm{dielectric}} = 0.00024\varepsilon_r^3 - 0.036\varepsilon_r^2 + 1.87\varepsilon_r + 17.71, \tag{15}$$

which provide the curves illustrated in Fig. 10.

The Figs. 11 show the ONS per λ_o for PEC coated spheres, as those presented in Figs. 1 c) and 3. Two cases are considered, in the first, the thickness of the dielectric layer, t_D, is equal to the radius of conducting sphere, $R_C(\lambda_o)$, and in the second case $t_D = 0.1R_C(\lambda_o)$. ONS per λ_o is presented in function of ε_r of dielectric layer. In this case can be observed that as ε_r increases more segments per λ_o are needed and ONS does not vary with the increase of R_C (λ_o). But for all tests realized it was verified that as t_D siminishes lesser segments per λ_o are needed. The smallest value of t_D tested was equal to 0.5 λ_o. It was verified that to attain a desired accuracy it is necessary to use the same number of segments in PEC and dielectric surfaces. ONS per λ_o for PEC coated spheres is bigger that ONS per λ_o for dielectric spheres as a function of t_D. From the numerical data indicated in Fig. 11 empirical formula for the ONS per λ_o was obtained:

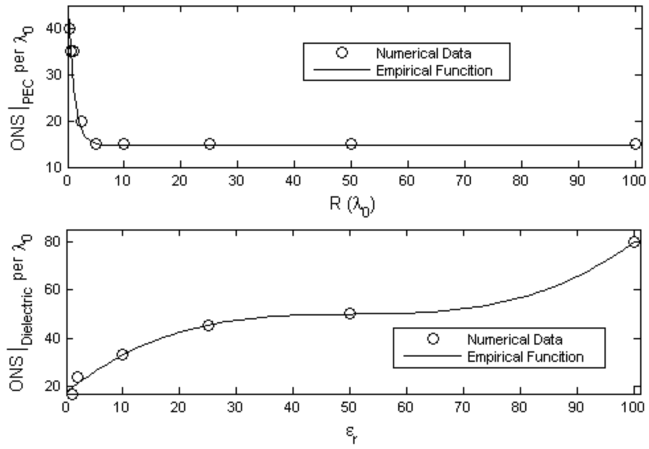

Fig. 10. ONS per λ_o for the PEC and dielectric spheres. a) PEC spheres. b) Dielectric spheres

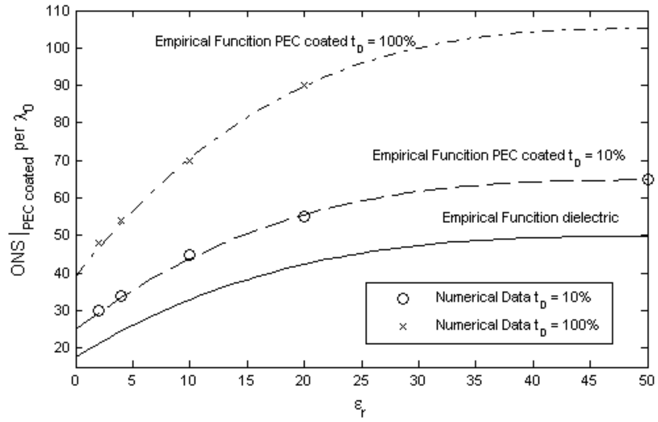

Fig. 11. ONS per λ_o for the PEC coated spheres

$$\mathrm{ONS}\Big|_{\mathrm{PEC\,coated}} = \mathrm{ONS}\Big|_{\mathrm{dielectric}} \left[1.15{+}0.009\, t_D\right] + 3 \qquad (16)$$

II. CONCLUSIONS

In this paper, the scattering by PEC, dielectric and PEC coated bodies was solved. The analysis was based on the CFIE and Müller formulations. Was successfully analyzed electrically small and large spherical bodies, and for dielectric structures was evaluated different relative permittivity values. From the simulations results empirical formulas to derive ONS were found. We verified that for PEC spheres as $R(\lambda_o)$ increases, lesser segments per λ_o are needed. For dielectric spheres can be observed that ONS does not vary with the increase of $R(\lambda_o)$, but as ε_r increases more segments per λ_o are needed. For PEC coated spheres needed and ONS does not vary with the increase of $R(\lambda_o)$ of PEC interface. But for all tests realized it was verified that as thickness of the dielectric layer diminishes lesser segments per λ_o are needed. Was verified that it is necessary to use the same number of segments in PEC interface and dielectric surface.

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