EMFIE and MEFIE Formulations for Numerical Solution of Single or Composities Bodies of Revolution

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Abstract — **In this paper, we present two new accurate formulations, EMFIE (electric-magnetic field integral equation) and MEFIE (magnetic-electric field integral equation), for the solution of the electromagnetic scattering by dielectric and composites bodies of revolution. The new formulations are genereted by the standard electric and magnetic field surfaces integral equations. We also report numerical results are comparated with analitical solution to demostrate the accuracy and capability of the proposed formulation.**

Key-Words **— Electric and magnetic field integral equations, eletromgnetic sacattering by bodies of revolution, Method of moments.**

I. Introduction

Electromagnetic scattering from conducting, dielectric and composite bodies is an important and challenging problem in the field of computacionnal electromagnetics. Analitical solutions are available for only very limited geometries such as spheres, discs, cylinders, etc. For bodies having an arbitrary shape, one has to resort to some approximate numerical technique. A variety of approaches have been developed to study this propblem, which include the method of momentos (MoM), the finite element method (FEM), and the finite-difference time-domain (FDTD) method. When the bodies are homogeneous, MoM is perefed because the problem can be formulated in terms of surface integrals over the conducting and dielectric surfaces [1]-[3]. In special for bodies of revolution (BOR´s) the problem is formulated in terms of integrals over generatrixes [4]-[6]. For perfectly conducting (PEC) BOR´s the problem has been exhaustively studied and the most acurrate formulations are EFIE (electric field integral equation) and CFIE (combined field integral equation that is a linear combination of EFIE and MFIE - magnetic field integral equation) for open and closed conductiong bodies, respectively [5]. For dielectric BOR´s, many combinations of EFIE and MFIE have been investigated [6]-[8]. Some of these are the EFIE, MFIE, CFIE, PMCHWT, and Müller integral equations [6]-[8]. Such solutions (or proper combinations of them) have also been used in the analysis of scattering by composite BOR's [8]-[12]. For bodies with regions of different materials, as them show in the Fig. 1 d) and 1 e), the most used. formulation is PMCHWT [12]. For layered bodies, as them show in the Fig. 1 b) and 1 c), many formulations have been evalueted [8],[9].

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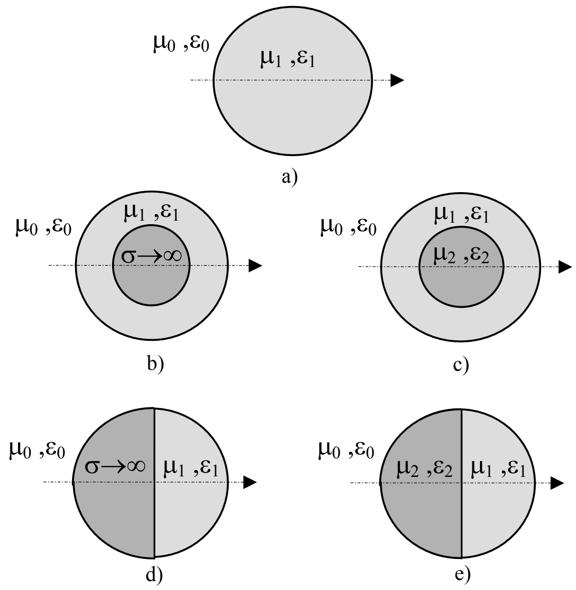

Fig. 1. Generic axially bodies of revolution. a) Dielectric body. b) Coated PEC body. c) Coated dielectric body. d) Body with dielectric and conducting regions e) Body with two dielectric regions

The reported investigations are generally conducted for electrically small BOR's (i.e., with dimensions of the order of the wavelength) [6]-[12]. In the present work we present two new accurate formulations for solution of the electromagnetic scattering by dielectric and composites bodies, as those shown in Fig. 1. For that, the plane-wave scattering from dielectric and composite spheres of different electrical sizes and different relative permittivity (ε_r) values are adopted as study case and the results are compared with analytical data, Mie-series [13]. The study shows that the new formulations are accurate for scattering analysis by different type of BOR´s.

II. Problem Formulation

For a homogeneous body, as that shown in Fig. 1, with permittivity ε_1 and permeability μ_1, immersed in an infinite and homogeneous medium with permittivity ε_0 and permeability μ_0, the equivalence principle can be applied to establish a set of four integral equations (the EFIE and MFIE, for fields inside and outside the body) to solve for the electric (**E**) and magnetic (**H**) fields in terms of equivalent electric (**J**) and magnetic (**M**) surface currents [6]. Assuming that the sources of the incident field ($\mathbf{E}^{inc}$, $\mathbf{H}^{inc}$) are outside the body, the integral equations for the tangential field components can be represented as [6]:

$$[\eta_0 \mathbf{L}_0(\mathbf{J}) + \mathbf{K}_0(\mathbf{M})]_{\tan} = \mathbf{E}_{\tan}^{inc}, \tag{1}$$

$$[\mathbf{L}_0(\mathbf{M}) - \eta_0 \mathbf{K}_0(\mathbf{J})]_{\tan} = \mathbf{H}_{\tan}^{inc}, \tag{2}$$

$$[\eta_1 \mathbf{L}_1(\mathbf{J}) + \mathbf{K}_1(\mathbf{M})]_{\tan} = 0, \tag{3}$$

$$[\mathbf{L}_1(\mathbf{M}) - \eta_1 \mathbf{K}_1(\mathbf{J})]_{\tan} = 0, \tag{4}$$

where, applying the index i to represent the interior (i = 1) and exterior (i = 0) regions, $\eta_i = \sqrt{\mu_i/\varepsilon_i}$ and the integral operators $\mathbf{L}_i$ and $\mathbf{K}_i$ are

$$\mathbf{L}_i(\mathbf{X}) = \frac{j}{k_i}\int_{s'}\left[k_i^2\,\mathbf{X}(\mathbf{r}')\,G_i(\mathbf{r},\mathbf{r}') - \nabla'\,\mathbf{X}(\mathbf{r}')\nabla' G_i(\mathbf{r},\mathbf{r}')\right] ds', \tag{5}$$

$$\mathbf{K}_i(\mathbf{X}) = \upsilon_i\,\hat{\boldsymbol{n}} \times \frac{\mathbf{X}(\mathbf{r})}{2} + \int_{s'} \mathbf{X}(\mathbf{r}') \times \nabla' G_i(\mathbf{r},\mathbf{r}')\, ds', \tag{6}$$

where S denotes the surface of the BOR, $\hat{\boldsymbol{n}}$ is the unit vector normal to the BOR surface, **X** is either the equivalent electric or magnetic current on S, υi is a constant that values 1 if the field point resides on the outer surface and values –1 if the field point resides on the inner surface, $k_i = \omega\sqrt{\mu_i\varepsilon_i}$ and

$$G_i(\mathbf{r},\mathbf{r}') = \frac{e^{-jk_i|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}, \tag{7}$$

is the free space Green's function of region i.

III. New Integral equation Formulations

In (1)-(4) there are four integral equations and two unknowns (**J** and **M**). So, it is possible to develop a number of different combinations among (1)-(4) to solve for **J** and **M**.

A. PEC bodies

For PEC bodies, the EFIE (1), with **M** = 0, is the choice for open shells [5]. For closed-surface bodies, the CFIE avoids spurious resonances and, consequently, provides stable numerical solutions [5]. The CFIE is a linear combination of (1) and (2) with **M** = 0:

$$\alpha'\,\mathrm{EFIE}_0 + \beta'\,\mathrm{MFIE}_0, \tag{8}$$

where EFIE_0 and MFIE_0 represent (1) and (2), respectively, and α' and β' are the linear-combination weights [5].

B. Homogeneous dielectric bodies

For dielectric bodies, (1)-(4) can be combined in many different ways to yield two integral equations suited to determine **J** and **M** [5]-[9]. The commonly adopted combinations are:

i) EFIE: using the EFIE's (1) and (3).

ii) MFIE: using the MFIE's (2) and (4).

iii) CFIE: combining (1) with (2) and (3) with (4) to obtain two new integral equations, that shown in (8) and [6]:

$$\alpha'\,\mathrm{EFIE}_1 + \beta'\,\mathrm{MFIE}_1, \tag{9}$$

where EFIE_1 and MFIE_1 represent (3) and (4), respectively.

Another way to reduce the number of equations is to combine the (1)-(4) in the following way

$$\mathrm{EFIE}_0 + \alpha\,\mathrm{EFIE}_1, \tag{10}$$

$$\mathrm{MFIE}_0 + \beta\,\mathrm{MFIE}_1. \tag{11}$$

iv) PMCHWT: a particular case of the (10) and (11) with linear-combination weights $\alpha = \beta = -1$ [6]-[9].

v) Müller formulation: a special case of (10) and (11) with linear-combination weights $\alpha = \varepsilon_1/\varepsilon_0$ and $\beta = \mu_1/\mu_0$ [6]-[9].

vi) New Formulation EMFIE (electric-magnetic field integral equation): using EFIE_0, (1), and MFIE_1, (4).

vii) New Formulation MEFIE (magnetic-electric field integral equation): using MFIE_0, (2), and EFIE_1, (3).

C. Composite bodies

For composite bodies (i.e., bodies composed of PEC and homogeneous dielectric regions), PEC surfaces are analyzed by (8), with β' = 0 for open PEC shells, while the interfaces between dielectric regions can be treated by any combination cited in Sect. III-B, in principle.

IV. MoM Solution

The scattering from the geometries depicted in Fig. 1 can be analyzed by the integral equations cited in Sects. II and III, which are evaluated by the well-known MoM technique, appropriately suited to the analysis of BOR's [3]-[9]. For that, the surface currents **J** and **M** must be described in terms of basis functions along the BOR generatrix, with azimuthal variations described in terms of Fourier harmonics [4]. In order to improve the accuracy and convergence of the MoM solution, the basis functions must be adequately represented. In this work, that is accomplished by employing the versatile triangular basis functions (TBF) to represent the spatial variations along the BOR generatrix [4]. The TBF is defined over two consecutive segments that describe the BOR generatrix, as shown in Fig 2. The unknown currents **J** and **M** are described as [4]:

$$\mathbf{X}(\mathbf{r}') = \sum_{n=-\infty}^{\infty}\left[\sum_{j=1}^{Nt} I_{jn}^{Xt}\,\frac{T_j^t(t')}{\rho'}\,\hat{\mathbf{t}}' + \sum_{j=1}^{N\phi} I_{jn}^{X\phi}\,\frac{T_j^{\phi}(t')}{\rho'}\,\hat{\boldsymbol{\phi}}'\right] e^{jn\phi'}, \tag{12}$$

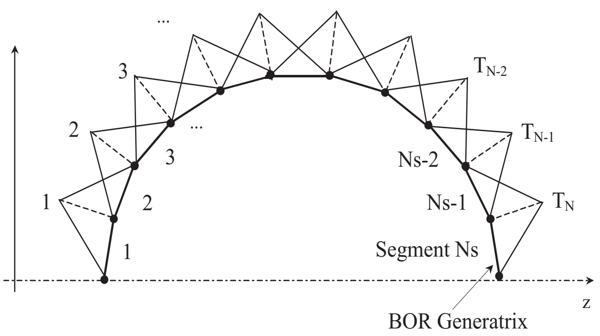

Fig. 2. Triangular basis functions (TBF).

where $\mathbf{X}$ represents either $\mathbf{J}$ or $\mathbf{M}$, $\hat{\mathbf{t}}'$ and $\hat{\boldsymbol{\phi}}'$ are unit directions tangential to the surface S at the source point $\mathbf{r}'$ (with corresponding variables t' and ϕ'), $T_j^t(t')$ and $T_j^\phi(t')$ are TBF´s , and I_{jn}^{Xt} and $I_{jn}^{X\phi}$ are the unknowns coefficients of $\mathbf{X}$. In (12), the term $e^{jn\phi'}$ corresponds to the Fourier expansion in ϕ' and the division by ρ' prevents singularity problems at the symmetry axis [4]. All the integrals appearing in the computation of the V- and Z-matrices of the MoM analysis were evaluated using Gauss-Legendre quadrature, with the self-term singularities properly removed [4].

V. Case Studies

In order to demonstrate the efficiency of proposed formulations, the plane-wave scattering from differently sized dielectric and composite spheres were analyzed. The sphere generatrix is described by straight segments [4]. Using the Mie-series solution [13] numerical results was analyzed using the following mean relative error

$$E_{MR}(\%) = \frac{E_{RJt} + E_{RJ\phi} + E_{RMt} + E_{RM\phi}}{4}, \tag{13}$$

where the mean relative error E_{MR} takes into account the mean relative errors of the tangential $\hat{\mathbf{t}}$ and $\hat{\boldsymbol{\phi}}$ components of $\mathbf{J}$ and $\mathbf{M}$ (E_{RX}), according to:

$$E_{RX}(\%) = 100 \left[\frac{\left| \left| X^{MoM} \right| - \left| X^{Mie} \right| \right|}{\left| X^{Mie} \right|_{max}} \right]_{max}, \tag{14}$$

where X^{MoM} and X^{Mie} denotes the numerical MoM and the analytical Mie-series solutions, respectively.

For a dielectric sphere, as that shown in Fig. 1 a), having radius, R = 0.5 λ_o, where λ_o is the wavelength in vacuum (which surrounds the spheres), and ε_r = 25, computed electric and magnetic current in $\hat{\mathbf{t}}'$ direction are ploted in Fig 3 as functions of surface coordinate $S(\lambda_o)$, which value 0 on the front of the sphere to $2\pi R$ on the back of the sphere. In this case were used 71 TBF to represent the current spatial variation along the BOR generatrix. The agreement between numerical and analitical results is excellent and E_{MR}, is egual to 0.88 % and 0.99 % for EMFIE and MEFIE, respectively, and equal to 0.825 % for classical Müller formulation. For a dielectric sphere with R = 10 λ_o and ε_r = 10, computed electric and magnetic current in $\hat{\mathbf{t}}'$ direction as functions of $S(\lambda_o)$ are presented in Fig 4. In this case were used 785 TBF to represent the current spatial variation. The agreement between numerical and analytical results is excellent and E_{MR}, is equal to 0.592 % and 1.05 % for EMFIE and MEFIE, respectively, and equal to 0.80 % for classical Müller formulation.

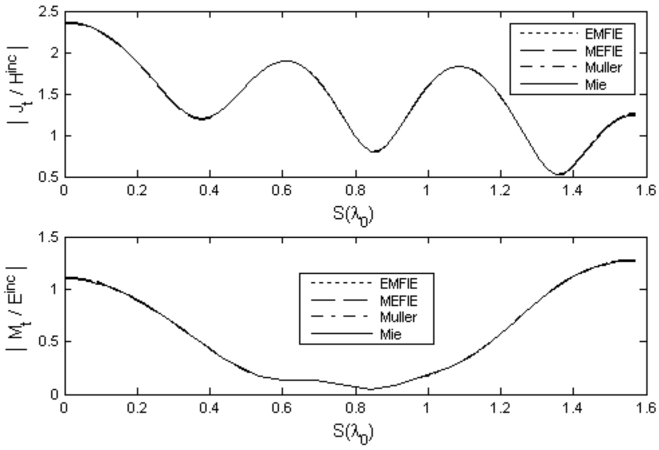

Fig. 3. Dielectric sphere of radius equal to 0.5 λ_o , ε_r = 25. a) Electric current J_t.b) Magnetic current M_t.

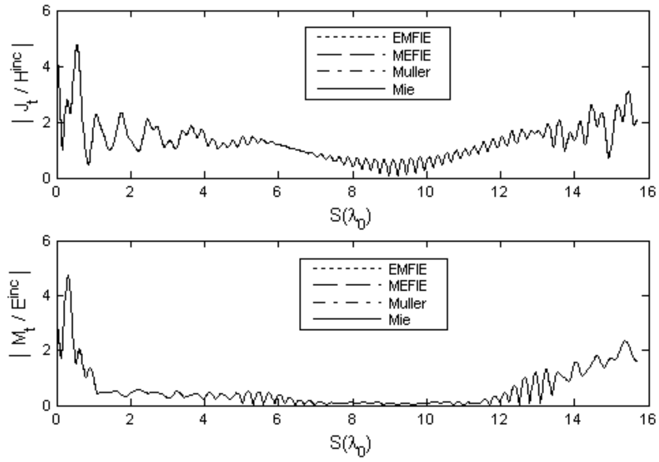

Fig. 4. Dielectric sphere of radius equal to 10 λ_o , ε_r = 10. a) Electric current J_t. b) Magnetic current M_t.

Next we studied the accuracy of the solution to scattering by conducting coated spheres as that shown in Figs. 1 b) and 5. In the first case a = 5 λ_o, b = 0.55 λ_o, ε_r = 10, and were used 79 TBF to represent the current spatial variation in each BOR generatrix. Fig. 6 shows computed electric and magnetic current in $\hat{\mathbf{t}}'$ direction in the dielectric external surface as functions of $S(\lambda_o)$. The agreement between numerical and analitical results is excellentand E_{MR}, is egual to 0.56 % and 0.58 % for EMFIE and MEFIE, respectively, and equal to0.589 % for classical Müller formulation. For a = 5.0 λ_o, b = 5.5 λ_o and ε_r = 10 the computed electric and magnetic current in $\hat{\mathbf{t}}'$ direction in the dielectric external

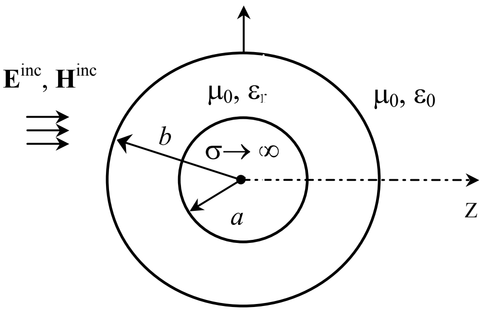

Fig. 5. Coated sphere.

surface as functions of $S(\lambda_o)$ are presented in Fig. 7. In this case were used 471 TBF to represent the current spatial variation in each BOR generatrix. The agreement between numerical and analitical results is excellent and E_{MR}, is egual to to 0.44 % and 1.25 % for EMFIE and MEFIE, respectively, and equal to 0.478 % for classical Müller formulation.

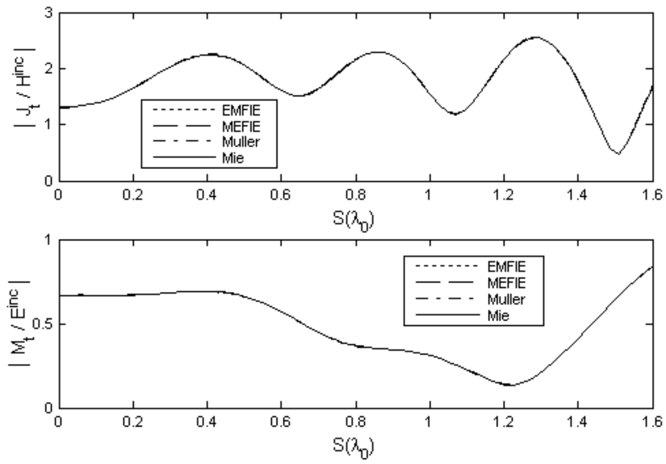

Fig. 6. PEC coated sphere $a = 0.5\ \lambda_o$, $b = 0.55\ \lambda_o$ and $\varepsilon_r = 10$. a) Electric current J_t. b) Magnetic current M_t.

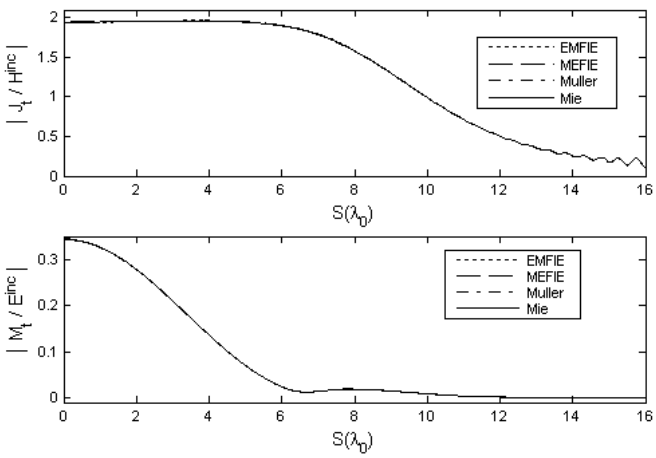

Fig. 7. PEC coated sphere $a = 5.0\ \lambda_o$, $b = 5.5\ \lambda_o$ and $\varepsilon_r = 10$. a) Electric current J_t. b) Magnetic current M_t.

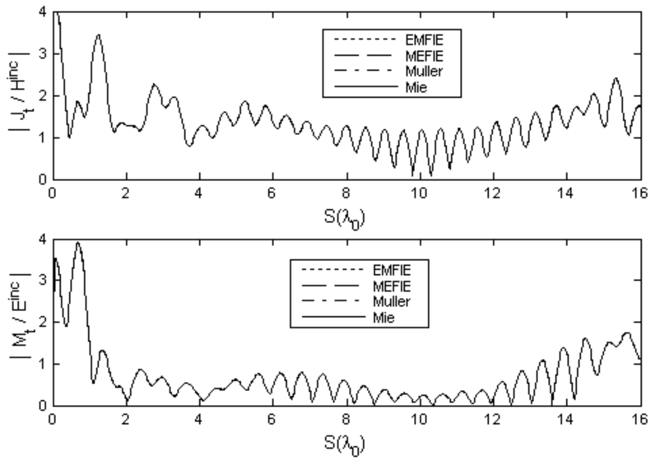

Fig. 8. Dielectric coated sphere $a = 5.0\ \lambda_o$ with and $\varepsilon_r = 9$ and $b = 5.5\ \lambda_o$ with $\varepsilon_r = 10$. a) Electric current J_t. b) Magnetic current M_t.

In the Fig. 5 considering internal sphere dielectric, $a = 5.0\ \lambda_o$ with ε_r e= 9 and $b = 5.5\ \lambda_o$ with $\varepsilon_r = 10$ the computed electric and magnetic current in $\hat{\mathbf{t}}'$ direction in the dielectric external surface as functions of $S(\lambda_o)$ are presented in Fig. 8. In this case it were used 550 TBF to represent the current spatial variation. The agreement between numerical and analytical results is excellent and E_{MR}, is equal to 0.98 % and 0.81 % for EMFIE and MEFIE, respectively, and equal to 0.256 % for classical Müller formulation.

For a bisected sphere, as that shown in Figs. 1 d) and 9, with $a = 20\ \lambda_o$, $\varepsilon_{r1} = 4$ and $\varepsilon_{r2} = 8$ the external electric and magnetic surface current in $\hat{\mathbf{t}}'$ direction as functions of $S(\lambda_o)$ are presented in Fig. 10. In this case it were used 942 TBF to represent the external surface current spatial variation along the external BOR generatrix. The agreement between numerical solutions obtained by new and classical PMCHWT formulations is not so good as in the previous tests, but acceptable. The E_{MR} in relation to classical PMCHWT formulation is egual to 6.21 % and 3.12 % for EMFIE and MEFIE, respectively.

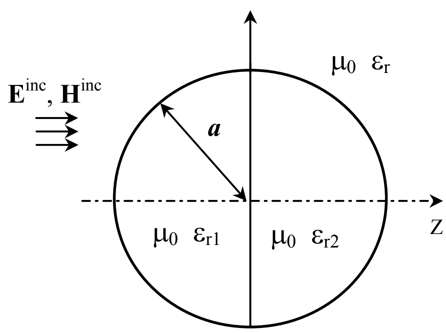

Fig. 9. Bisected sphere.

Finally, Fig. 11 shows computed external surface electric and magnetic current in $\hat{\mathbf{t}}'$ direction as functions of $S(\lambda_o)$ for bisected sphere shown in the Fig. 9, with $a = 20\ \lambda_o$ and ε_{r1} =

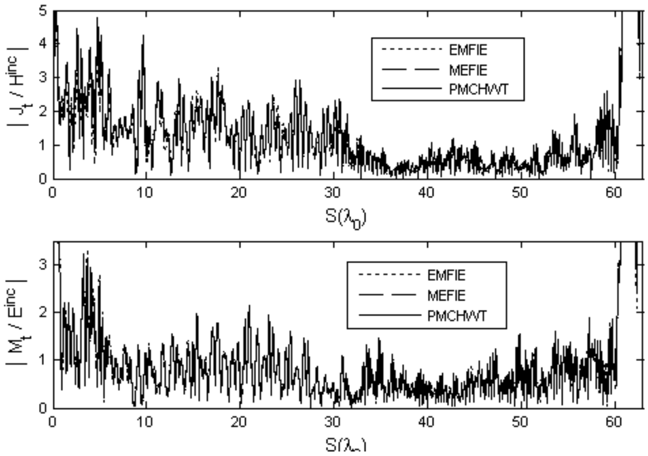

Fig. 10. Electric and magnetic current in extenal surface of bisected Sphere. a) Electric current J_t. b) Magnetic current M_t.

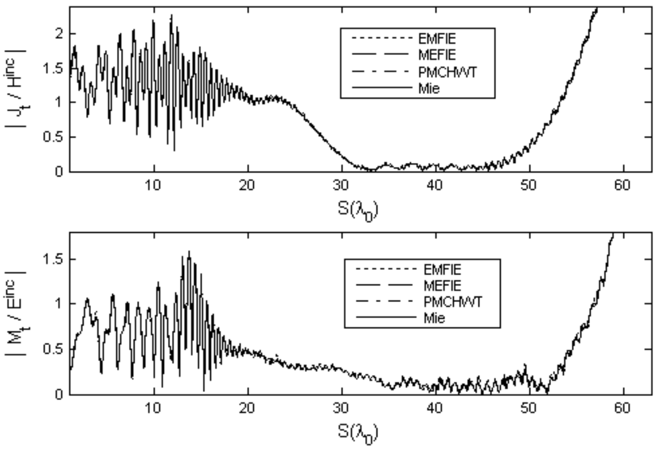

Fig. 11. Electric and magnetic current in extenal surface of bisected Sphere. a) Electric current J_t. b) Magnetic current M_t.

ε_{r2} =4. The homogeneous sphere results are obtained from Mie series. Since both scatters are physically identical, this case provides an excellent test of the accuracy. In this case was used 942 TBF to represent the external surface current spatial variation. The agreement between numerical results is excellent and E_{MR} is equal to 0.602 % and 1.122 % for EMFIE and MEFIE, respectively, and equal to 0.478 % for classical PMCHWT formulation.

VI. Conclusions

In the present work we presented two new formulations for solution of the electromagnetic scattering by dielectric and composites bodies of revolution. The equivalent current representation adopted was the versatile triangular basis functions for $\hat{\mathbf{t}}'$ and $\hat{\boldsymbol{\phi}}'$ directions. By analysis of the plane-wave scattering from dielectric and composite spheres of different electrical sizes and different relative permittivity values was demosntrated the new formulations accuracy. The test results indicated that both new formulations are accurate, but EMFIE seems be better than MEFIE in the most of cases.

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