Synthesis and Rigorous Analysis of Omnidirectional Dual-Reflector Antennas with Shaped Main Reflector Described by Local Conic Sections

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Abstract— **This work presents a formulation for shaping the main reflector of an axis-symmetric dual reflector antenna designed to offer an omnidirectional coverage with an arbitrary radiation pattern in the vertical plane. The subreflector is generated by an axis-displaced confocal conic and the main reflector is shaped to achieve a prescribed far-field radiation pattern. The shaping procedure is based on geometrical optics (GO) principles. For validation of the antenna designs, the wotk employs technique by the accurate method of moments technique.**

I. Introduction

Last-mile broadband access has been employed as an alternative wireless technology to attend the growing interest on high-speed Internet access, as it can show lower costs, rapid deployment, and lower maintenance when compared with cable technologies. For the antenna in the hub station, compact single and dual reflector omnidirectional antennas have been considered [1]-[2]. The reflector surfaces are bodies of revolution obtained by rotating the generating curves about a symmetry axis, as illustrated in Fig. 1. To ensure vertical polarization, the dual-reflector antenna designs are fed by the coaxial TEM horn.

In order to provide a uniform distribution power in coverage cell region, a study was presented with a main reflector shaping technique based on the solution of an ordinary differential equation derived from Geometrical Optics (GO) principles [3]. Recently, some works have dealt with a different shaping technique based on the combination of local conic sections [4]-[6]. The present work proposes an extension of the shaping method presented in [6] to omnidirectional axis-displaced ellipse (OADE) dual-reflector antennas using a cosecant square objective function.

Besides shaping radiation pattern in the elevation plane and the reduction of the return loss at the feed input, the design of the main reflector these omnidirectional antennas has to provide compact configurations to make them attractive. However, the design for compact geometries implies on increasing the coupling effects between feed and reflectors. In order to estimate the electromagnetic performance of these antennas, a rigorous analysis is employed, combining the Mode Matching Technique (MMT) to express the fields inside the coaxial horn and the Method of Moments (MoM) to represent the electromagnetic scattering by reflectors [7]-[11].

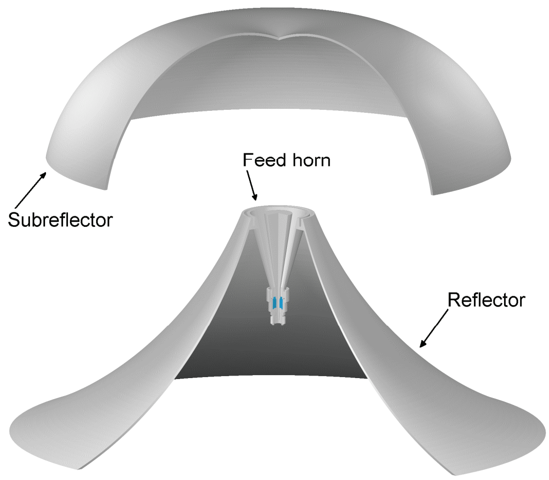

Fig. 1 - Circular symmetric reflector surfaces.

II. Synthesis method

The GO shaping method consists in determining the main reflector surface (the subreflector is generated by a known conic section) which, given a spherical wave front with phase center at the origin (O) and circularly symmetric radiation pattern $G_F(\theta_F)$, provides a desired circularly symmetric radiation pattern $G_A(\theta)$ in the far-field region. The shaped main reflector has its generatrix described by local conic sections sequentially concatenated (see Fig. 2) [6]. The determination of the subreflector generating conic section is discussed in [2]. The subreflector generatrix has two foci, one of them located at the origin (O) and the other at the point P, which defines the subreflector real annular caustic after the generatrix is rotated about the z-axis (see Fig. 2). The subreflector is classical surface and defined a priori, the relation between rays direction (θ_F) departing from feed and reflected by the subreflector (θ_S) is also known from Snell's Law.

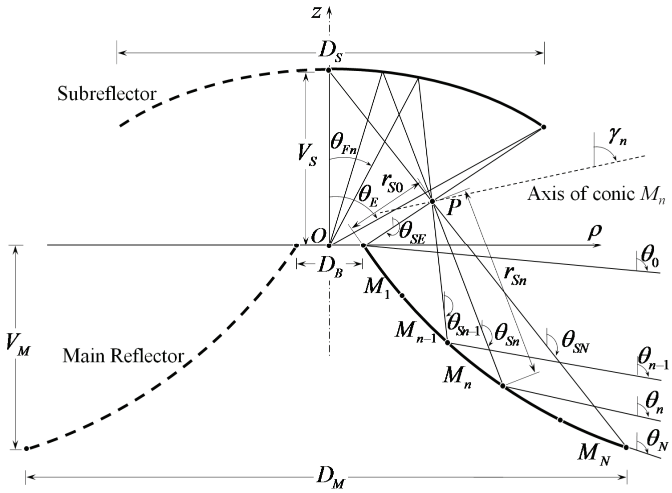

Fig. 2 - Conic sections representing the generatrix shaped main reflector

The main reflector is generated by local conic sections M_n (n = 1,…, N) sequentially concatenated, all of them with one of their focci at P. The axis of M_n passes through P and makes an angle γ_n with the z-direction (see Fig. 2). In order to uniquely determine the main-reflector conic sections M_n, three parameters must be determined for each M_n: the inter-focal distance ($2c_n$), the eccentricity (e_n), and the angle γ_n of the conic axis [6]. So, a set of three equations must be found for each step n of the iterative process. The first equation is given by polar conic equation applied to each one of n conic section M_n. The second equation, which is a relation between ray direction $\theta_S = \theta_{Sn}$ before being reflected by local conic and far-field direction $\theta = \theta_n$, can be obtained by imposing Snell's Law. The last equation is obtained by enforcing Snell's Law to the optical path of the previous step ($\theta_S = \theta_{Sn\text{-}1}$ and $\theta = \theta_{n\text{-}1}$) [6]. So, with a set of three equation one is able to determine the necessary parameters that define each one of local conic sections used to describe the main reflector generatrix.

However, to solve the set of linear equations, one must find the direction θ_n. By enforcing the conservation of energy along the tube of rays departing from O toward the main-reflector far-field region, one obtains the integral form of the mapping equation, which can be solved numerically. The iterative process starts at $n = 0$, with $\theta_{F0} = \theta_{\mathrm{E}}$ (θ_{SE} is obtained from θ_E with the help of the subreflector conic equation), $\theta = \theta_0$, and r_{S0} given by the location of the subreflector caustic P and by the desired diameter D_B of the main-reflector central opening (see Fig. 2).

To illustrate the shaping technique, the feed is assumed to be a TEM coaxial horn, with a radiation pattern described by [3]:

$$G_F(\theta_F) = G_{0F}\left[\frac{J_0(kr_i \sin\theta_F) - J_0(kr_e \sin\theta_F)}{\sin\theta_F}\right]^2 \qquad (1)$$

where G_{0F} is a normalization factor and r_i and r_e are the internal and external radii, respectively (see Fig. 3). In all case studies to be presented, $r_i = 0.45\lambda$ and $r_e = 0.90\lambda$. The cosine-squared objective function that describes radiation pattern of the main reflector in the elevation plane is given by

$$G_A(\theta) = G_{OA} \csc(\theta - \pi/2) \qquad (2)$$

where G_{0A} can be normalized for unit radiated power [3].

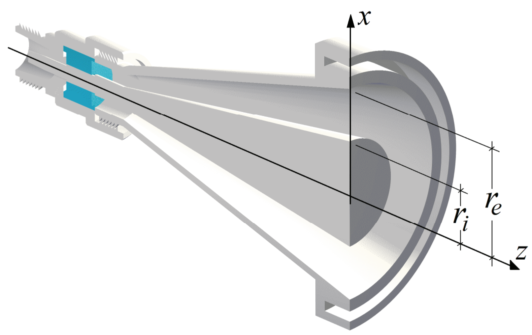

Fig. 3 - Coaxial TEM feed Horn

III. Method of Moment Analysis

In the analysis scheme, the coaxial horn is represented as series of stepped coaxial waveguides sections (see Fig. 4) and the Mode Matching technique is performed by rigorously enforcing the boundary conditions at each step, including the coaxial connector and coupling structure with dielectric loaded. It results in a scattering matrix for the entire transition region that depends on the geometrical parameters and on the wavelength as well as the number of higher modes considered to ensure continuity of the electric and magnetic fields throughout the transition region. Considering the TEM fundamental mode as the only incident mode at connector region C and the circularly symmetric structure, only the TEM and TM are excited throughout the entire transition region, as the TE mode do not couple with the TEM excitation.

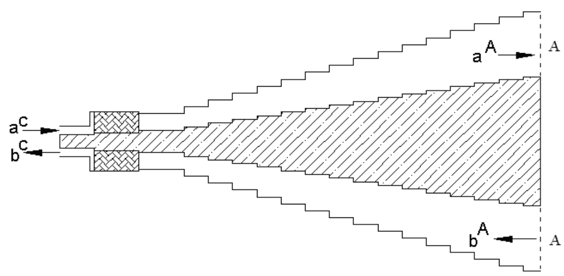

Fig. 4 - Horn represented as series of stepped coaxial waveguides sections

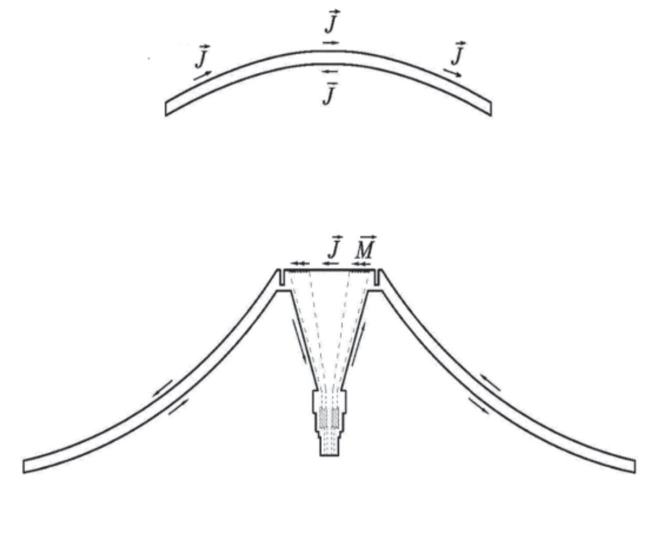

Fig. 5 - HFIE model of the outside surface currents of the feed main reflector

Part of the power of the modes reaching the aperture (region A) is radiated into the space that will induce surface currents on the surface reflectors (see Fig. 5), while the rest is guided back into the horn. The interaction between these fields in the coaxial horn interior region and the reflectors exterior current contributions is accurately analysed by using the EFIE on the exterior surfaces of the coaxial horn. The equivalence currents principle is introduced to formulate the HFIE on the radiating aperture which combines the interior field transition problem with the exterior radiation. The procedure is similar to those described in [7]-[8]. Here, the procedure is adapted to include the reflectors and a coaxial feed horn excited by the TEM mode.

As described in [7], the MoM is employed to solve the EFIE and MFIE in order to calculate the radiate fields and the amplitude of the reflected modes [b^A] into the horn. To apply the above strategy, an equivalent to the actual electromagnetic problem is defined by replacing the feed radiating aperture by a sheet of perfect conductor with an equivalent magnetic current density $\vec{M}$ (see Fig. 5). The magnetic current density radiates into the space and induces an electric surface current $\vec{J}$ on the metallic wall of the equivalent problem. These currents ensure the continuity of the tangential magnetic field across the aperture and zero tangential electric field on the metallic surface at the aperture and reflectors.

To apply the MoM solution, the current densities $\vec{M}$ and $\vec{J}$ are expanded by sub sectional roof-top patch modes and, particularly, over the aperture this functions are converted into eigenmodes of the aperture fields. The application of the MoM analysis technique yields the antenna radiation pattern as well as the antenna return loss at the end of the N-type connector.

IV. Case Study

To illustrate the application of the synthesis and analysis, the main reflector of an omnidirectional axis-displaced ellipse (OADE) configuration was synthesized to radiate, under GO principles, a cosecant square illumination in the elevation plane between $\theta_N = 93°$ to $\theta_0 = 135°$. As $\theta_0 > \theta_N$, the main reflector has a virtual caustic (see Fig. 2). The subreflector generating ellipse was determined from a classical OADE configuration with the following parameters: $V_S = 7.636\lambda$, $D_B = 2.4\lambda$, main-reflector projected diameter $D_M = 17.56\lambda$, focus O at the plane of the main-reflector central opening ($z_B = 0$), main-reflector aperture width $W_A = 7.0\lambda$, and $\gamma = 102°$. These values provide an ellipse with an eccentricity equal to 0.25018, an inter-focal distance $2c = 3.60362\lambda$, and an axis with a tilt angle of 62.41° with respect to the z-axis [3]. As a consequence, the projected diameter of the subreflector $D_S = 14.708\lambda$ and the subreflector edge angle $\theta_E = 54.99°$. These parameters also provide $r_{S0} = 2.60\lambda$ as an initial value for the shaping procedure presented in Sect. II. The generatrices of the subreflector and shaped main reflector are illustrated in Fig. 6.

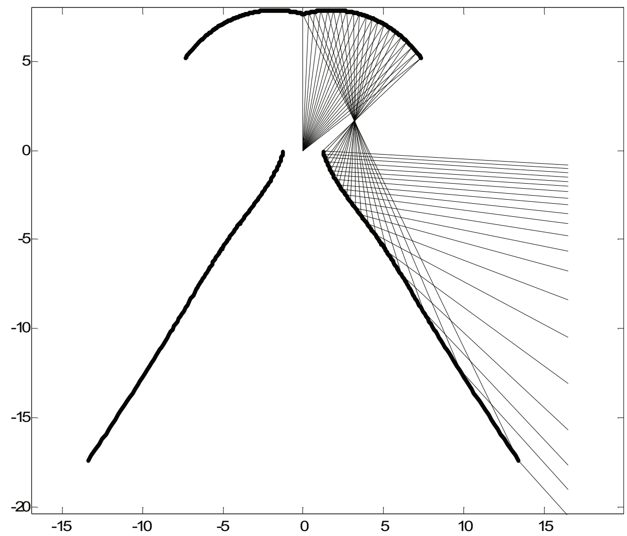

Fig. 6 – Generatrices of a shaped OADE dual-reflector antenna

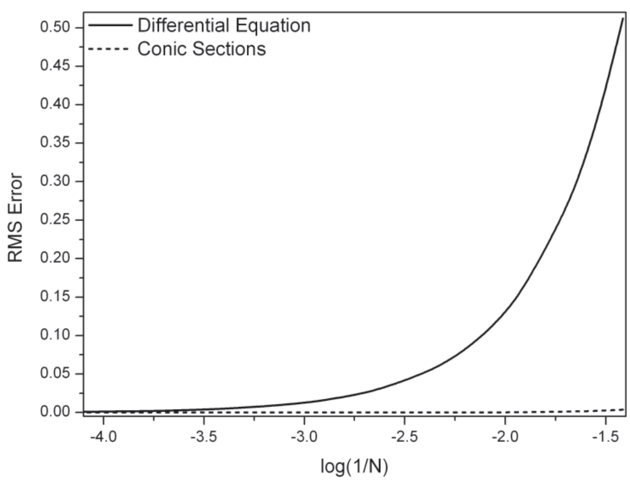

Fig. 7 – Main-reflector RMS error with respect to N

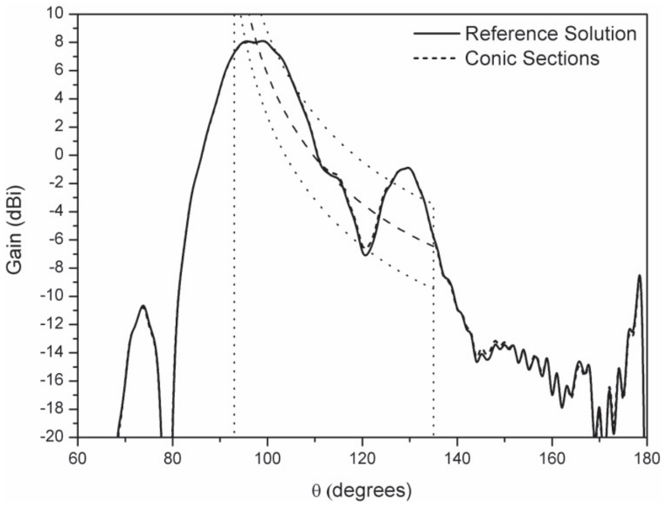

Fig. 8 – MoM radiation patterns in the elevation plane.

When compared with the shaping procedure employed in [2]-[3], the technique presented in Section II is simpler and has a faster numerical convergence. To illustrate the convergence features, the shaped surfaces generated with different number (N) of points were compared with a reference surface generated with a very large number of points. For both shaping technique, Figure 6 shows the main-reflector RMS error (with respect to a reflector shaped with a very large N value) as a function of $\log(1/N)$. The RMS obtained from the procedure presented in Section II by using 25 points to describe the shaped reflector is similar to the RMS yielded by the technique employed in [2]-[3] with 10^4 points.

Figure 8 shows the MoM radiation patterns for the antennas with main reflector synthesized with 25 points and the reference surface, together with the prescribed GO far-field pattern of (2). As we can observe, the radiation pattern show small difference. Figure 9 show the return loss obtained from the MoM analysis for shaped dual-reflector antenna and for the feed horn.

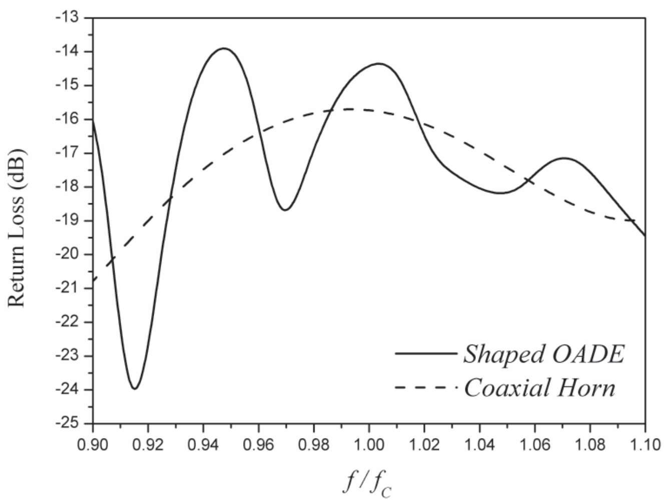

Figure 9 – Antenna return loss

V. Conclusions

This work presented a method for the GO synthesis of omnidirectional dual-reflector antennas. The procedure shapes the main-reflector generatrix, which is represented by local conic sections consecutively concatenated. The subreflector is not shaped and its generatrix is represented by a single conic section. It was shown that the present technique is much more accurate than the one present in [3].

A full-wave analysis was conducted to validate the shaping procedure and measure electromagnetic effects between coaxial horn and the omnidirectional dual-reflector system. The analysis algorithm was applied to calculate the antenna radiation pattern and the antenna return loss of the projected antenna.

Acknowledgment

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