

# Synthesis of Circular Apertures with Blockage

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**Abstract**—This work presents a new method for synthesizing annular apertures with circular symmetry, based on the Method of Stationary Phase. The synthesis is performed in two steps: first, a mapping between the aperture and radiation variables is done, and then, using this, the solution of a differential equation will generate the synthesized field. Two examples of synthesis are presented.

**Index Terms**—aperture, circular, synthesis, stationary

## I. INTRODUCTION

In [1] a formulation for the synthesis of circular apertures,  $f(\rho', \phi')$ , with and without blocking, based on the Method of Stationary Phase, is presented. This synthesis assumes that the amplitude function of the field at the aperture, given by  $A(\rho', \phi')$ , is known and that the phase function over it, given by  $k\psi(\rho', \phi')$ , is desired, where  $k$  is the propagation constant. Despite the circular geometry, the authors chose to work with Cartesian coordinates and synthesize the phase by varying only one of the rectangular variables.

In circular symmetric problems (field at the aperture independent of  $\phi'$ ), such as the design of dual-reflector antennas (see Fig. 1), as explored in [2] and [3], the formulation proposed in [1] cannot be applied.

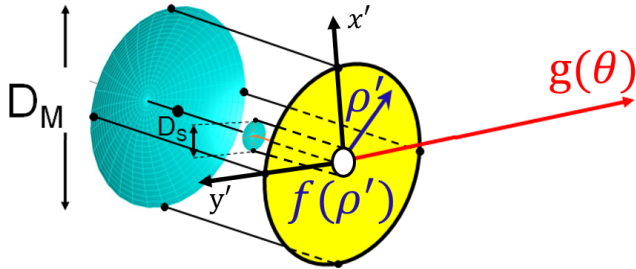


Fig. 1. Dual-reflector antenna aperture.

Thus, for a radiated field in the far-field region without the spherical wave factor,  $g(\theta)$ , and a field at the aperture given by

$$f(\rho') = A(\rho')e^{jk\psi(\rho')} \quad (1)$$

it is desired to find the function  $\psi(\rho')$  which satisfies the radiation integral

$$g(\theta) = \int_b^a A(\rho')e^{jk\psi(\rho')} J_0(k\rho' \sin\theta) \rho' d\rho' \quad (2)$$

with radii  $a = \frac{D_M}{2}$  and  $b = \frac{D_S}{2}$ , according to Fig. 1.

## II. PROBLEM FORMULATION

In (2), the term  $A(\rho')\rho'$  only alters the amplitude of the integrand, and  $e^{jk\psi(\rho')}$  only alters the phase, while  $J_0(k\rho' \sin\theta)$  affects both. Using Hankel functions to approximate the Bessel function, it is possible to separate the integrand into amplitude and phase. Thus, assuming that  $A(\rho')$  is a smooth and known function, that the constant  $k$  is large, and applying the Method of Stationary Phase [4], an asymptotic approximation for (2) is obtained, which increases in accuracy with frequency. In this approximation, the radiation variable,  $\theta$ , and the feed variable,  $\rho'$ , can be separated, obtaining

$$h(u_0) = \frac{\int_{u_i}^{u_o} x|g(x)|^2 dx}{\int_{u_i}^{u_o} x|g(x)|^2 dx} \approx G(\rho'_0) = \frac{\int_b^{\rho'_0} \eta A^2(\eta) d\eta}{\int_b^a \eta A^2(\eta) d\eta} \quad (3)$$

being true at the stationary phase points given by

$$\psi'(\rho'_0) = u_0(\rho'_0) \quad (4)$$

where  $\rho'_0$  is the stationary phase point on the aperture,  $u_0 = \sin(\theta_0)$  for the stationary phase point  $\rho'_0$ ,  $u_i$  is the sine of the smallest desired observation angle and  $u_f$  is the sine of the largest desired observation angle.

Given the functions  $A(\rho')$  and  $g(\theta)$ , one can use (3) to obtain  $h(u_0)$  and  $G(\rho'_0)$ , then map the variable  $u_0$  to each point  $\rho'_0$ , and finally use this relationship to solve the differential equation given by (4) and obtain the phase function  $\psi'(\rho'_0)$ .

## III. RESULTS

In the two examples that follow, apertures with wavelength  $\lambda = 0.06m$ ,  $D_M = 200\lambda$ ,  $D_S = 20\lambda$  will be used, together with a field amplitude at the aperture given by

$$A(\rho') = C_1 \left[ \sin\left(\pi \frac{\rho' - b}{a - b}\right) + 0.01 \right] \quad (5)$$

in order to reduce the effects of secondary lobes as per [5], with  $C_1$  being a normalizing constant.

### A. Flat-Topped Pattern

For a flat-topped radiated field (as used in [6] and [7]), that is, a constant function within an interval  $\theta$ , where  $g(\theta)$  equals

$$g(\theta) = \begin{cases} 1, & 0^\circ \leq \theta < 45^\circ \\ 0, & 45^\circ \leq \theta < 90^\circ \end{cases}$$

the field at the aperture,  $f(\rho')$  (Fig. 2) and the function  $g(\theta)$ , obtained after integrating (2) (Fig. 3), were obtained.

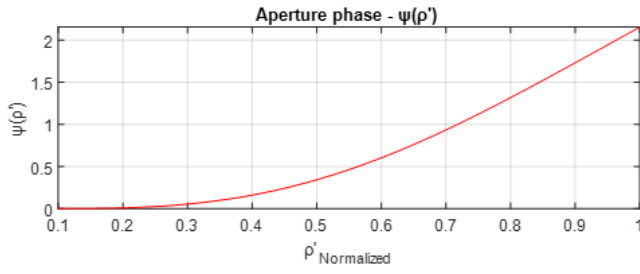


Fig. 2. Field phase at the aperture - Patterns flat-topped.

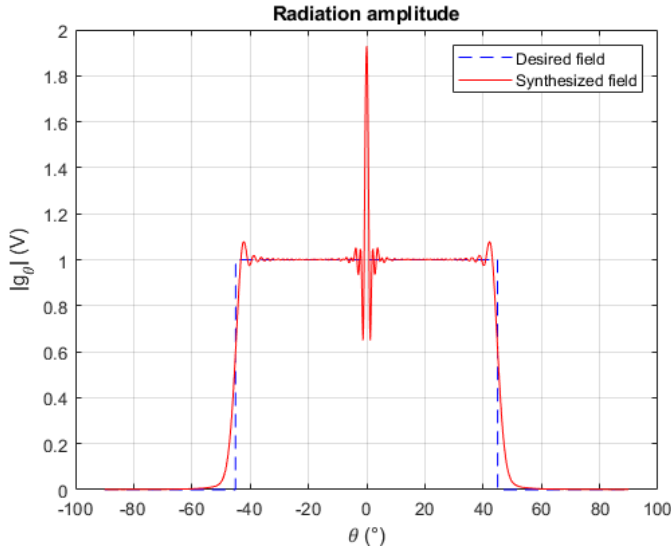


Fig. 3. Modulus of the function  $g(\theta)$  - Patterns flat-topped.

### B. ISOFLUX Pattern

Now, for an ISOFLUX type radiated field (as used in [6] and [3]), that is, a function that illuminates Earth's surface with uniform power density, within an interval  $\theta$ , the field at the aperture,  $f(\rho')$  (Fig. 4) and the function  $g(\theta)$ , obtained after integrating (2) (Fig. 5), were obtained.

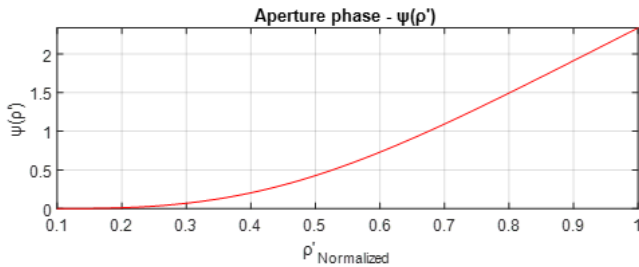


Fig. 4. Field phase at the aperture - Patterns ISOFLUX.

## IV. CONCLUSION

The synthesized apertures yielded excellent results, maintaining small distortions between the desired and synthesized radiated field. A large error was obtained in the vicinity of  $\theta = 0$ , possibly caused by diffraction effects, which are

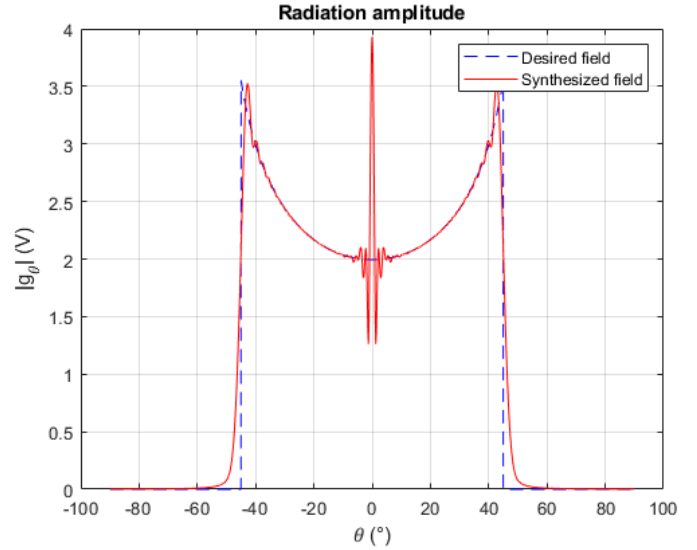


Fig. 5. Modulus of the function  $g(\theta)$  - Patterns ISOFLUX.

not taken into account by the Method of Stationary Phase approximation.

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