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A NEW MESHLESS-MOM HYBRID METHOD APPLIED TO THE ANALISYS OF 2D ELECTROMAGNETIC SCATTERING

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Abstract* – This paper presents a new hybrid method to solve electromagnetic scattering. The method combines the Interpolating Element Free Galerkin Meshless Method and the Electric Field Integral Equation evaluated by the Method of Moments. Such technique has the advantages of the mesh-free IEFG while avoiding numerical treatment of the scatter exterior region, as the inclusion of absorbing boundary conditions. The method is applied to the analysis of TMz scattering by an infinite dielectric cylinder. Numerical results are compared with analytical ones to illustrate the usefulness of the proposed technique.

Introduction

The solution of electromagnetic scattering by conducting and dielectric bodies has been extensively dealt in the literature using integral equation and differential equation methods [1]. Integral equation methods, such as the Method of Moments (MoM), can treat unbounded problems very effectively, but they are computationally inefficient when complex inhomogeneities are present. In contrast, inhomogeneities in the media are easily treated by differential equation methods, such as the Finite Element Method (FEM) and Finite Difference Time Domain (FDTD). However, those methods are most suitable for bounded problems. Therefore, as both integral equation techniques and partial differential solutions have advantages and disadvantages, hybrid methods are especially interesting since they can meet the most efficient characteristics of the methods involved. The most common hybrid technique available in the literature combines FEM and MoM [2]-[4]. However FEM is a mesh-based method and the appropriate mesh generation may be a difficult and time-consuming task for complex problems, especially those with media discontinuities, moving boundaries or severe deformations.

In recent years, a new class of differential equation methods –Meshless Methods (MM)–, which does not require a mesh structure, has been developed. In these methods, the solution is obtained using only a cloud of nodes spread throughout the region of interest. These nodes do not have any predefined connection among them. That feature makes MM appropriate to deal with complex geometries and inhomogeneities. As results of several studies, many MM have been proposed, such as the Element Free Galerkin (EFG), Diffusive Element Method (DEM), Reproducing Kernel Particle (RKPM), Smoothed ParticleHydrodynamics (SPH), Moving Particle Semi-implicit (MPS), Partition of Unity, and meshless local Petrov–Galerkin (MLPG) [5].

Among MM, one of the most used is the EFG [6]. It presents good convergence rates and although it requires a background cell to numerically evaluate the integrals present in the weak form of the problem, the integration process is completely independent of the nodes distribution. EFG couples the Moving Least Squares (MLS) [5],[7] approximation with the Galerkin weak form to obtain a domain-based MM. However, the MLS provides shape functions (SF) which do not satisfy the Kronecher delta property; thus, additional techniques are required for enforcing essential boundary

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conditions (EBC) due to the noninterpolating property of the approximation [5]. Nevertheless, it is possible to overcome this restriction adapting the MLS. This procedure is known as Interloping Moving Least Squares Method (IMLS) and is performed using a singular weight function in the SF construction process [8],[9]. Thus, it is possible to obtain EFG SF which satisfy the Kronecker delta property and can interpolate the desire function. Hence, the EBC can be imposed directly into the discrete linear system. Then, this paper, instead of using the MLS, uses IMLS in the EFG formulation which is now called Interpolating Element Free Galerkin method (IEFG).

EFG has proven to be reliable for solving static and quasi-static electromagnetic problems [10]-[12] and also electromagnetic scattering ones [13],[14]. For scattering problems, it is necessary to establish a global boundary located at a certain distance away from the scatter, where an Absorbing Boundary Condition (ABC) has to be defined to enforce the Sommerfeld radiation condition, limiting the numerical region. This work presents a new hybrid technique, IEFG-MoM, which combines the Interpolating Element Free Galerkin meshless method with the Electric Field Integral Equation (EFIE) evaluated by the MOM. Here, the technique is applied in the solution of a 2D electromagnetic scattering problem. The scattering problem is divided in two: an internal problem, where IEFG is applied, and an external problem, where MoM is employed. Both problems are coupled by imposing the continuity of the tangential fields over the scatter surface. Consequently, the size of the problem is reduced as the region outside the scatter is not considered nor a global ABC is defined. As a case study, an infinite dielectric cylinder illuminated by a TM_Z plane wave is analyzed. The IEFG-MoM numerical results are compared with analytical ones to illustrate the accuracy and usefulness of the proposed hybrid technique.

2D Electromagnetic Scattering by IEFG – Internal Problem

The problem under analysis is a z-directed infinitely long dielectric cylinder, with relative electric permeability ε_r and relative magnetic permeability μ_r (μ_r = 1 for dielectrics), surrounded by vacuum (free space), as illustrated in Fig. 1. The cylinder cross section (Ω) is a circle and Γ is its boundary, in which the normal vector is $\boldsymbol{n}$ and the tangential direction is $\boldsymbol{t}$. For a normally incident (ϕ = 180°) TM_Z plane wave, the total electric field, E_z, and equivalent electric surface current, J_z, have only the z-component, which can be determined in the internal problem using IEFG to solve the weak form of a 2D scalar Helmholtz equation [14]:

$$\int_{\Omega}\left[\mu_r^{-1}\nabla W_{IEFG}\cdot\nabla E_z - k_0^2\varepsilon_r W_{IEFG}E_z\right]d\Omega - \int_{\Gamma}\mu_r^{-1}W_{IEFG}\nabla E_z\cdot\boldsymbol{n}\,d\Gamma = 0, \quad (1)$$

where k_0 is the vacuum wave number and W_{IEFG} represents the IEFG test functions. Using Faraday's Law, the last integral in (1) can rewritten as:

$$\int_{\Omega}\left[\nabla W_{IEFG}\cdot(\mu_r^{-1}\nabla E_z) - k_0^2\varepsilon_r W_{IEFG}E_z\right]d\Omega - j\omega\mu_0\int_{\Gamma}W_{IEFG}\cdot J_z\,d\Gamma = 0, \quad (2)$$

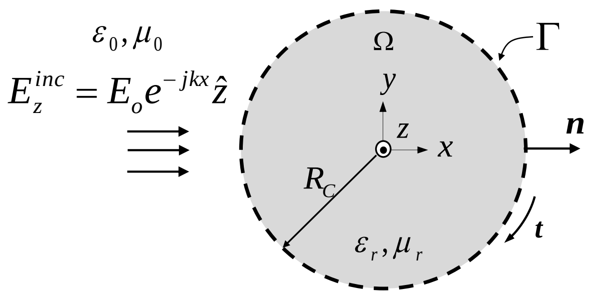

Fig.1 The problem domain

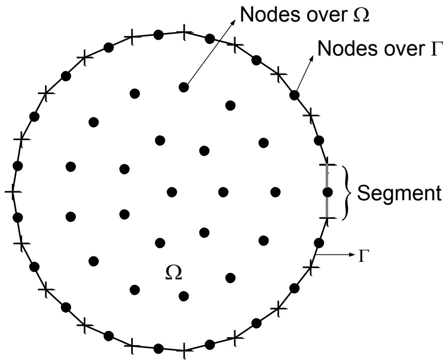

Fig.2 Node distribution

where μ_0 is the vacuum permeability, ω is the angular frequency, and E_z (over Ω) and J_z (over Γ) are unknown quantities to be determined.

In the IEFG meshless approach, a set of N nodes is spread over the problem domain, as illustrated in Fig. 2. Each node, I, is a point $\mathbf{x}_I = (x, y) \in \Omega$ for which a shape function, Φ_I, is associated. So, the unknown functions $E_z(\mathbf{x})$ and $J_z(\mathbf{x})$ can be approximated by shape functions [5]:

$$\mathrm{E}^h(\mathbf{x}) = \sum_{I=1}^{N} \Phi_I(\mathbf{x}) \upsilon^E(\mathbf{x}_I)\,, \tag{3}$$

$$\mathrm{J}^h(\mathbf{x}) = \sum_{I=1}^{N} \Phi_I(\mathbf{x}) \upsilon^J(\mathbf{x}_I)\,, \tag{4}$$

where $\mathbf{x} = (x, y)$, $\upsilon^E(\mathbf{x}_I)$ and $\upsilon^J(\mathbf{x}_I)$ are unknown coefficients. Considering the Galerkin method, IEFG test functions, $\mathrm{W}^h_{IEFG}(\mathbf{x})$, are approached using Φ_I and arbitrary constants, ω_I, as:

$$\mathrm{W}^h_{IEFG}(\mathbf{x}) = \sum_{I=1}^{N} \Phi_I(\mathbf{x}) \omega_I\,. \tag{5}$$

In the IMLS approach developed in this work, $\Phi_I(\mathbf{x})$ are determined by minimizing a weighted discrete L^2 norm, for which the weight function is [9]:

$$\mathrm{W}(r) = 1/(r^n + \beta^n)\,, \tag{6}$$

where β is a constant small enough to ensure no division by zero, n is a constant adjusted to improve the results accuracy and r is the adjustable support radius of the circular influence domain of each node. The IMLS process leads to 2D shape functions, as illustrated in Fig. 2, which degenerates to 1D shape functions, like those shown in Fig. 3 for 19 uniformly spaced nodes distributed along Γ [14]. It can possible to see that 2D and 1D shape functions satisfy the Kronecher delta property.

If the functions $E^h(\mathbf{x})$ and $J^h(\mathbf{x})$ satisfy the boundary conditions on Γ, the Galerkin method leads to a $Z^{\mathrm{IEFG}}(N\times N)$ and a $B^{\mathrm{IEFG}}(N\times N_\Gamma)$ matrixes, where N_Γ is the number of shape functions over Γ and

$$\mathrm{Z}^{\mathrm{IEFG}}_{ij} = \int_\Omega \mu_r^{-1} \nabla\Phi_i \nabla\Phi_j - k_0^2 \varepsilon_r \Phi_i \Phi_j \, d\Omega\,, \tag{7}$$

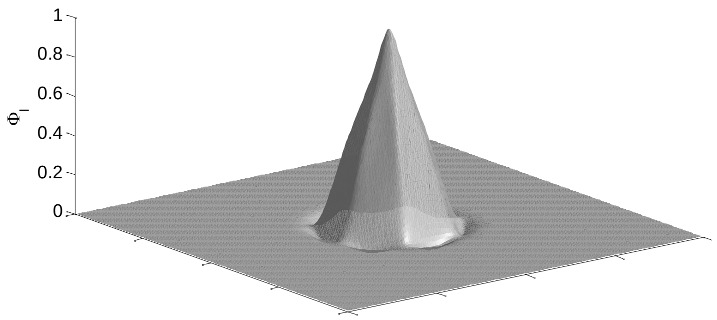

Fig.3 2D IMLS shape function

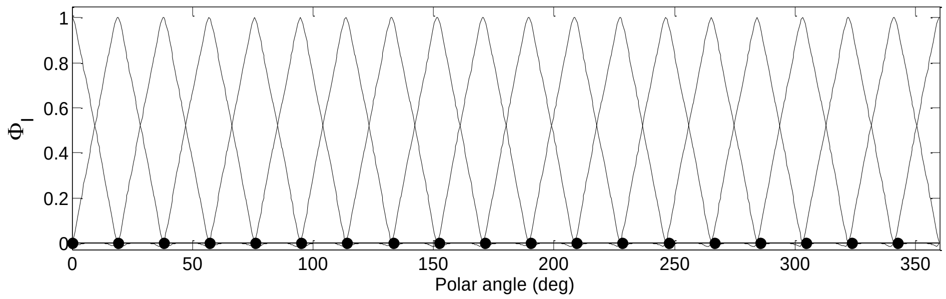

Fig.4 A set of 19 1D IMLS shape functions over Γ

$$\mathrm{B}_{ij}^{\mathrm{IEFG}} = -j\omega\mu_r \int_{\Gamma} \Phi_i \Phi_j \, d\Gamma \,. \tag{8}$$

In this work, $\mathrm{Z}_{ij}^{\mathrm{IEFG}}$ and $\mathrm{B}_{ij}^{\mathrm{IEFG}}$ elements are calculated using integrals evaluated by a two-point Gaussian quadrature, with the help of an auxiliary rectangular cell structure [6]. Therefore, the values of the shape functions and their derivatives are calculated over the set of integration points.

2D Electromagnetic Scattering by MoM – External Problem

For homogeneous scatters, the equivalence principle can be applied to establish a set of integral equations, inside and outside the scatter, in order to solve for the electric and magnetic fields in terms of equivalent electric (**J**) and magnetic (**M**) surface currents. Specifically, the EFIE for the external region is [1]:

$$\mathbf{E}_{tan}^{inc} = \left[\boldsymbol{n} \times 0{,}5\mathbf{M}(\boldsymbol{r}) + \int_{S'} \left\{ j\eta_0 k_0 \left[\mathbf{J}(\boldsymbol{r}')G(\boldsymbol{r},\boldsymbol{r}') - \nabla' \mathbf{J}(\boldsymbol{r}') \nabla' G(r,r') \right] + \mathbf{M}(\boldsymbol{r}') \times \nabla' G(r,r') \right\} dS' \right]_{tan} , \tag{9}$$

where $\mathbf{E}_{tan}^{inc}$ is the incident electric field tangential component, S is the scatter surface, $G(\boldsymbol{r},\boldsymbol{r}')$ is the free-space Green's function, and $\boldsymbol{r}$ and $\boldsymbol{r}'$ locate the observer e source points, respectively. For the problem presented in Fig. 1, where the total electric field and the equivalent electric surface current **J** are both z-oriented, the magnetic current **M** has only the ϕ-component. Besides, there is no variation with coordinate z. So, (9) can be rewritten as [1], [15]:

$$\mathrm{E}_z^{inc} = \boldsymbol{n} \times 0{,}5\mathrm{M}_\phi(\rho) + \frac{1}{4} \int_0^{2\pi} \left\{ \eta_0 k_0 \mathrm{J}_z(\rho') H_0^2(k_0 R) + \mathrm{M}_\phi(\rho') \cos\psi \, H_1^2(k_0 R) \right\} \rho' d\phi' , \tag{10}$$

where ρ and ρ' are the observer and source points, respectively, $H_0^2(k_0R)$ and $H_1^2(k_0R)$ are second kind Hankel's functions and ψ is the angle between $\boldsymbol{n}$, on source point, and the vector $\boldsymbol{R} = |\boldsymbol{r} - \boldsymbol{r}'|$.

In this work, surface currents J_z and M_ϕ are described in terms of pulse basis functions along Γ. Each pulse function is defined over a segment among those used to represent Γ, as shown in Fig. 5. The unknown currents J_z and M_ϕ are described as:

$$J_z(\rho') = \sum_{I=1}^{NP} P_I(t')\upsilon_I^J , \tag{11}$$

$$M_\varphi(\rho') = \sum_{I=1}^{NP} P_I(t')\upsilon_I^M , \tag{12}$$

where $P_I(t')$ is a pulse function, υ^J and υ^M are the unknown coefficients of J_z and M_ϕ, respectively, and *NP* is the number of pulse functions (segments).

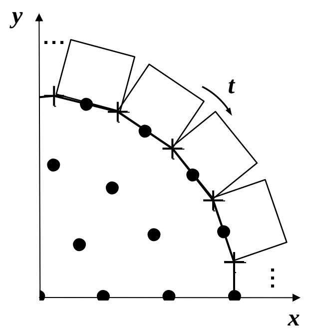

Fig.5 Pulse functions over a quarter section of cylinder cross section

Here, the integral equation (10) is evaluated by the MoM using Galerkin method, i.e., the weight functions W_{MOM} are also pulse functions, leading to matrices $Z^{JMOM}(NP\times NP)$, $Z^{MMOM}(NP\times NP)$, and $V^{MOM}(NP\times 1)$ with elements given by:

$$Z_{ij}^{JMOM} = \frac{\eta_0 k_0}{4}\int_0^{2\pi}\int_0^{2\pi} P_i(t)\,P_j(t')\,\eta_0 k_0\, H_0^2(k_0R)\,\rho\rho' d\phi' d\phi , \tag{13}$$

$$Z_{ij}^{MMOM} = \int_0^{2\pi} P_i(t)\,\boldsymbol{n}\times\frac{M_\phi(\rho)}{2}\rho d\phi + \frac{1}{4}\int_0^{2\pi}\int_0^{2\pi} P_i(t)\,P_j(t')\cos\psi\, H_1^2(k_0R)\,\rho\rho' d\phi' d\phi , \tag{14}$$

$$V_i^{MOM} = \int_0^{2\pi} P_i(t)\,E_z^{inc}\,\rho d\phi . \tag{15}$$

In (13)-(15) the integrals are evaluated by a two-point and a three-point Gaussian quadrature for integrals in ϕ and ϕ', respectively.

IEFG-MoM Hybrid Formulation

The hybrid technique proposed in this work combines the IEFG, applied to the interior problem, and the EFIE solved by MOM for exterior problem. The IEFG and MOM solutions are coupled enforcing the continuity of the tangential electric and magnetic fields at the surface *S* of the scatter. The continuity of the tangential magnetic field is imposed by the Γ line integral in (2), where Jz = $\boldsymbol{n}\times\mathbf{H}$, with **H** being the total magnetic field just outside Γ. The continuity of the tangential electric field is imposed by [2]:

$$\int_\Gamma \mathrm{W}_{MOM}\cdot\left(\mathrm{E}_z-\boldsymbol{n}\times\mathrm{M}_\phi\right)d\Gamma=0\,, \tag{16}$$

where W_{MOM} are the MoM pulse weight functions. To numerically evaluate (14), E_Z and M_ϕ are described by pulse basis functions. So M_ϕ is described like in (12) and E_Z as:

$$\mathrm{E}_z(\rho')=\sum_{I=1}^{NP}P_I(t')\upsilon_I^E\,. \tag{17}$$

Therefore, (16) can be converted in a $\mathrm{H}^\mathrm{E}(NP\times NP)$ and a $\mathrm{H}^\mathrm{M}(NP\times NP)$ matrices [2]:

$$\mathrm{H}_{ij}^\mathrm{E}=\mathrm{H}_{ij}^\mathrm{M}=\int_0^{2\pi}\int_0^{2\pi}P_i(t)\,P_j(t')\;\rho\rho' d\phi' d\phi\,, \tag{18}$$

where all elements are evaluated a one-point Gaussian quadrature for integrals in ϕ and ϕ', respectively. In order to obtain simultaneous solution for unknown E_z over Ω and J_z and M_ϕ over Γ (7), (8), (13)-(15) and (18) should be combined in a single linear system [2]:

$$\cdot\begin{bmatrix}\mathrm{Z}^{\mathrm{IEFG}} & \mathrm{B}^{\mathrm{IEFG}} & 0\\ \mathrm{H}^\mathrm{E} & 0 & \mathrm{H}^M\\ 0 & \mathrm{Z}^{\mathrm{JMOM}} & \mathrm{Z}^{\mathrm{MMOM}}\end{bmatrix}\begin{bmatrix}\upsilon^E\\ \upsilon^J\\ \upsilon^M\end{bmatrix}=\begin{bmatrix}0\\ 0\\ \mathrm{V}^{\mathrm{MOM}}\end{bmatrix} \tag{19}$$

Numerical Results

In order to demonstrate the accuracy of the proposed IEFG-MoM hybrid method, the scattering of a normally incident TMz plane wave from a dielectric homogenous cylinder with radius $0.5\lambda_0$ (λ_0 is the wavelength in vacuum) and $\varepsilon_r = 2$ was analyzed. The analysis was carried out varying number of nodes spread over Ω and Γ from 40 to 2.937 and the IEFG-MoM accuracy was verified by the following error criterion:

$$e(X)=100\sqrt{\left|X-AS\right|^2/\left|AS\right|^2}\,, \tag{20}$$

where *X* represents either E_z, J_z , or M_ϕ numerical results and *AS* represents the corresponding analytical solution.

The IEFG-MoM matrix condition number is shown in Fig. 6 as a function of the number of nodes. For the proposed technique, this is a very important analysis because the zeros in the main diagonal of (19) can lead to high values of condition number and, therefore, to instabilities in the numerical solution. As can be observed from Fig. 6, the condition number increase with the number of nodes, as expected, and it is not significantly great although, what ensure the solution numerical stability.

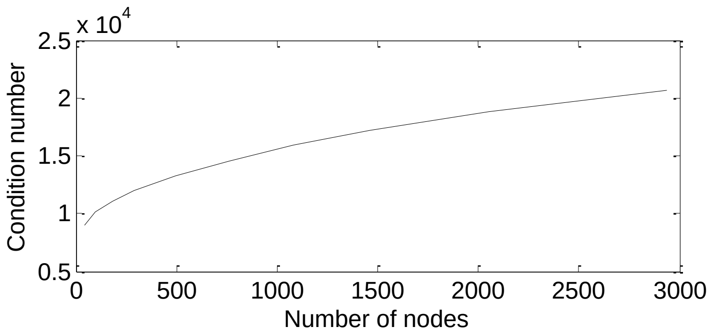

Fig.6 IEFG-MoM matrix condition number

The errors $e(E_z)$, e(Jz), and $e(M_\phi)$ are presented in Fig. 7 as functions of the number of nodes. As can be verified from the figure, the errors decrease as the number of nodes increase. Specifically, for 1.084 nodes (with 114 nodes are spread over Γ) the errors are less than 1% ($e(E_z)$ = 0.86%, e(Jz) = 0.98%, and $e(M_\phi)$ = 0.43%) and the numerical results are presented in Fig. 8 together analytical solution which confirm the proposed hybrid technique accuracy.

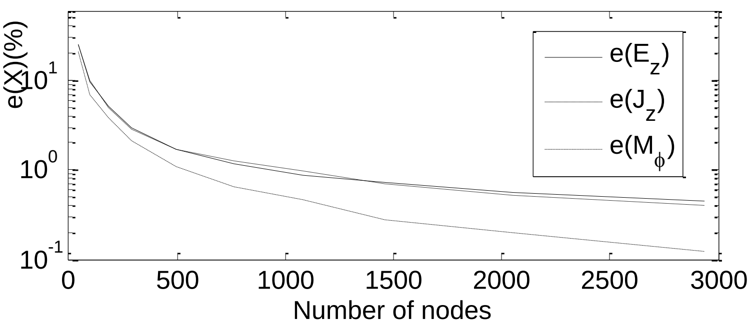

Fig. 7. IEFG-MoM error

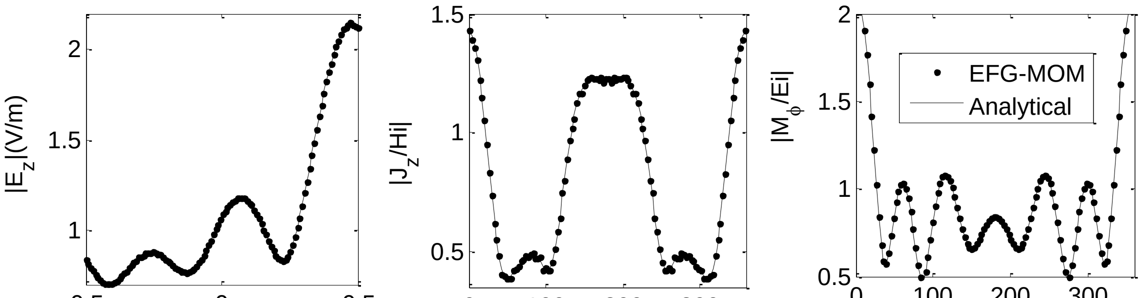

Fig.8.E_z along a horizontal line, L_x, passing through the center of the cylinder over x axis (left) and surface electric (medium) and magnetic (right) currents.

Conclusion

To solve electromagnetic scattering problems hybrid methods have been shown to be especially interesting because they can combineonly the most efficient characteristics of each involvedmethods.The most widelyused hybrid technique combines FEM and MoM. However FEM requires a mesh that for complex problems can be difficult to be generated. This work presents a new proposal for a hybrid technique which successfully combines the IEFG, that does not use a mesh, and the MOM in a very straightforward manner, while uses the both methods full capabilities. By the proposed technique is possible to obtain simultaneously, by a single process, electric field inside the obstacle and electric and magnetic equivalent surface current on the boundary of the obstacle. The validity and accuracy of the solution were confirmed by comparing the numerical results against analytical ones.

Acknowledgment

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