# Time-Domain Analysis of a Reflector Antenna Illuminated by a Gaussian Pulse

Cassio G. Rego, Flavio J. V. Hasselmann, and Fernando J. S. Moreira

*Abstract* — **In this work the asymptotic Time-Domain Physical Optics and Time-Domain Uniform Theory of Diffraction are applied to the analysis of a hyperboloidal reflector antenna illuminated by a Gaussian pulse to illustrate their applicability. The results are compared with those obtained by a reference integral-equation formulation, in order to specify the limitations of the time-domain formulations.**

*Index Terms* — **Asymptotic methods, time-domain, reflector antennas.**

## I. Introduction

The recent interest in the use of ultra-wide band (short-pulse) antennas for target identification has motivated the transient analysis of ondulatory electromagnetic phenomena. Also, the analysis of the response of an antenna to pulsed excitations allows one to study its behaviour in a very wide band of frequencies. If this analysis can be made directly in the time domain, a transfer function of the scatterer, or of a radio channel itself, can be obtained by means of its impulsional response.

Some numerical techniques such as the finite-difference time-domain method (FDTD), the iteration method, and the space-time integral-equation method have been applied to the problems of determing the transient response of electromagnetic scatterers. These techniques have inherent difficulties with instability, interpolation errors, and a need of extensive computer memory and CPU time to solve problems involving large scatterers [1]. On the other hand, the use of asymptotic methods, such as the time-domain physical optics (TDPO) and the time-domain uniform theory of diffraction (TD-UTD) is very attractive. The simplicity of the physical optics currents over a smooth surface and the simple ray picture of radiation in the uniform theory of diffraction in the frequency domain are sustained in the time domain, since a time-function excitation may be decomposed into their frequency-domain components [1,2].

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TDPO and TD-UTD are implemented herein to perform the analysis of a hyperboloidal reflector antenna excited by a spherical wave with temporal behaviour in the form of a Gaussian pulse. Results are also compared with those obtained from a Fourier inversion of a reference frequency-domain method of moments (MoM) solution.

## II. The TDPO and TD-UTD Techniques

### A. Formulation of the TDPO

The TDPO technique is derived via a Fourier inversion of the corresponding frequency-domain formulation. For a perfectly conducting body, the frequency-domain PO-induced current distribution over the illuminated surface is

$$\bar{J}_S^{PO}(\bar{r},\omega) = 2\hat{n}\times\bar{H}^{inc}(\bar{r},\omega) \; , \tag{1}$$

where $\hat{n}$ is the surface unit normal at point $\bar{r}$, and $\bar{H}^{inc}(\bar{r},\omega)$ is the incident magnetic field with angular frequency $\omega$. The frequency-domain scattered field is obtained by calculating the integral over the illuminated surface using the free-space Green's function [3]:

$$\bar{E}^{PO}(\bar{r},\omega) = -\frac{j\omega\eta_0}{4\pi rc} e^{-jk_0 r} \iint_{S'} \bar{J}_{st}^{PO}(\bar{r}',\omega) e^{-jk_0|\bar{r}-\bar{r}'|} dS' \; , \tag{2}$$

where $\eta_0$ is the intrinsic free-space impedance, $c$ is the velocity of light in vacuum, $k_0$ is the wave number, the vector $\bar{r}'$ locates the integration point on the scatterer surface, and

$$\bar{J}_{st}^{PO}(\bar{r}',\omega) = \bar{J}_S^{PO}(\bar{r}',\omega) - \left[\bar{J}_S^{PO}(\bar{r}',\omega)\cdot\hat{a}_r\right]\hat{a}_r \; . \tag{3}$$

The scattered field of the TDPO is obtained by performing the inverse Fourier transform over the field expressed in (2):

$$\bar{e}^{TDPO}(\bar{r},t) = -\frac{\eta_0}{4\pi rc} \iint_{S'} \frac{\partial \bar{j}_{st}^{PO}(\bar{r}',\tau)}{\partial\tau} dS' \; , \tag{4}$$

with

$$\bar{j}_{st}^{PO}(\bar{r}',\tau) = \bar{j}_S^{PO}(\bar{r}',\tau) - \left[\bar{j}_S^{PO}(\bar{r}',\tau)\cdot\hat{a}_r\right]\hat{a}_r \; , \tag{5}$$

$$\vec{j}_S^{PO}(\vec{r}',\tau)=2\hat{n}\times\vec{h}^{inc}(\vec{r}',t)\ , \qquad (6)$$

where $\vec{j}_S^{PO}(\vec{r}',\tau)$ is the surface current distribuition in the time domain and $\vec{h}^{inc}(\vec{r}',t)$ is the time-domain magnetic field incident on the surface. The temporal variable $\tau$ is given by

$$\tau=t-\frac{|\vec{r}-\vec{r}'|}{c}-\frac{|\vec{r}'|}{c}\ . \qquad (7)$$

*B. Formulation of the TD-UTD*

The TD-UTD formulation for the field scattered by a perfectly conducting surface with an edge is obtained after a Fourier inversion of corresponding incident, reflected, and diffracted field expressions in the frequency domain, which are expressed, respectively, by

$$\vec{E}^i(\vec{r},\omega)=\vec{E}_0^i(\vec{r},\omega)|A_i(s^i)|\,j^{n_i}\,e^{-jk_0 s^i}\ , \qquad (8)$$

$$\vec{E}^r(\vec{r},\omega)=\vec{E}^i(Q_R,\omega)\bullet\overline{\overline{R}}|A_r(s^r)|\,j^{n_r}\,e^{-jk_0 s^r}\ , \qquad (9)$$

$$\vec{E}^d(\vec{r},\omega)=\vec{E}^i(Q_E,\omega)\bullet\overline{\overline{D}}(\omega)|A_d(s^d)|\,j^{n_d}\,e^{-jk_0 s^d}\ . \qquad (10)$$

In the equations above $A_i(s^i)$, $A_r(s^r)$, and $A_d(s^d)$ are the spread factors for the incident, reflected and diffracted fields, respectively; $n_i$, $n_r$, and $n_d$ are the number of focal points that each ray has traversed; $\overline{\overline{R}}$ is the dyadic reflection coefficient; and $\overline{\overline{D}}(\omega)$ is the dyadic diffraction coefficient [4].

When using ray methods to study transient electromagnetic phenomena, it appears to be more convenient to work with analytic time functions [2]. The analytic function is obtained from a inverse Fourier transform

$$\overset{+}{f}(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}2U(\omega)F(\omega)e^{-j\omega t}d\omega=\frac{1}{\pi}\int_0^{\infty}F(\omega)e^{-j\omega t}d\omega\ , \qquad (11)$$

where $U(\omega)$ is the unit step function and $F(\omega)$ is the Fourier transform of the time function $f(t)$. The function $\overset{+}{f}(t)$ is analytic in the upper half of the $t$-plane defined by Im $t > 0$, which makes the integral in (11) to be convergent. When $\overset{+}{f}(t)$ is evaluated for real time (Im $t$ = 0) it is given by

$$\overset{+}{f}(t)=f(t)+\frac{j}{\pi}\int_{-\infty}^{\infty}\frac{f(\tau)}{t-\tau}d\tau\ . \qquad (12)$$

The use of analytical functions allows one to avoid the complexities that arise in the TD-UTD development when the rays of the frequency-domain uniform theory of diffraction (UTD) pass through caustics.

The inverse Fourier transform expressed in (11) applied over (8), (9), and (10) leads to the TD-UTD formulation [2]

$$\overset{+}{\vec{e}}^{\,TD\text{-}UTD}(\vec{r},t)=\overset{+}{\vec{e}}^{\,i}(\vec{r},t)U_i+\overset{+}{\vec{e}}^{\,r}(\vec{r},t)U_r+\overset{+}{\vec{e}}^{\,d}(\vec{r},t)\ , \qquad (13)$$

where $\overset{+}{\vec{e}}^{\,i}(\vec{r},t)$ is the analytical function for the incident field and

$$\overset{+}{\vec{e}}^{\,r}(\vec{r},t)=\left\{\overset{+}{\vec{e}}^{\,i}(Q_R,\tau_R)-2\left[\overset{+}{\vec{e}}^{\,i}(Q_R,\tau_R)\cdot\hat{n}\right]\hat{n}\right\}|A_r(s^r)|\ , \qquad (14)$$

$$\overset{+}{\vec{e}}^{\,d}(\vec{r},t)=\left\{\left[\overset{+}{d}_s(\tau_D)*\overset{+}{e}^{\,i}_{\beta_0'}(Q_E,\tau_i)\right]\hat{\beta}_0+\left[\overset{+}{d}_h(\tau_D)*\overset{+}{e}^{\,i}_{\phi_0'}(Q_E,\tau_i)\right]\hat{\phi}_0\right\}\times$$
$$\times|A_d(s^d)|\ , \qquad (15)$$

are the analytical functions for the reflected and diffracted fields, respectively. The spatial unit step functions $U_i$ and $U_r$ are one on the lit side of the shadow boundaries and zero otherwise. The symbol * in (15) denotes a time convolution. The TD-UTD diffraction coefficients for a curved screen are expressed by

$$\overset{+}{d}_{s,h}(t)=\frac{-1}{2\sqrt{2\pi}\sin\beta_0}\left[\frac{\overset{+}{f}(x_A,t)}{\cos\left(\frac{\phi-\phi'}{2}\right)}\mp\frac{\overset{+}{f}(x_B,t)}{\cos\left(\frac{\phi+\phi'}{2}\right)}\right]\ , \qquad (16)$$

with

$$x_{A,B}=2L^{i,r}\cos^2\left(\frac{\phi\mp\phi'}{2}\right), \qquad (17)$$

where $L^i$ and $L^r$ are the distance parameters of the diffraction coefficients [4]. The function $\overset{+}{f}(x,t)$ is given by

$$\overset{+}{f}(x,t)=\frac{1}{\pi}\int_0^{\infty}F(x,\omega)e^{-j\omega t}d\omega\ , \qquad (18)$$

where $F(x,\omega)$ collects the frequency dependence of the diffraction coefficients of the UTD and is given by

$$F(x,\omega)=\sqrt{\pi x}\,e^{j\omega x/c}\,erfc\left(\sqrt{\frac{j\omega x}{c}}\right), \qquad (19)$$

where

$$erfc(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-y^2} dy. \tag{20}$$

The temporal variables in (14) and (15) are

$$\tau_R = t - \frac{s^i + s^r}{c}, \tag{21}$$

$$\tau_D = t - \frac{s^d}{c}, \tag{22}$$

$$\tau_i = t - \frac{s^i}{c}, \tag{23}$$

where distances $s^i$, $s^r$ and $s^d$ are represented in Fig. 1.

Geometrical parameters such as specular points coordinates, the spread factors of the scattered fields, the ray-fixed coordinates and vector components, as well as those involved on the determination of the diffraction coefficients, are calculated as in the frequency domain [2].

The asymptotic nature of UTD and PO in the frequency domain makes the TD-UTD and TDPO valid only when the frequencies present in the spectrum of the excitation have wavelengths which are small compared to the dimensions and radii of curvature of the scatterer. Thus, these techniques are accurate only at early observation times, in the neighborhood of the arrival of the first wavefronts [4].

### III. Response of a Hyperboloidal Reflector Antenna to a Gaussian Pulse

Fig. 1 shows the geometry of a hyperboloidal reflector illuminated by an incident spherical wave from the focus, whose temporal behaviour is defined by a Gaussian pulse, given by

$$e_G(t) = 2\exp\left\{-\left[2\left(\frac{c}{a}\right)t\right]^2\right\}, \tag{24}$$

where the coefficient $a$ determines the pulse width (see Fig. 2). Fig. 3 shows the response of the hyperboloid for a pulse with $a = 0.6$ at two observation points, calculated using the TD-UTD and TDPO formulations. These results are compared with a reference solution based on a frequency-domain MoM and transformed into the time domain using an inverse fast Fourier transform algorithm (IFFT). The results show a good agreement between the techniques. In Fig. 3a the observation point is on the lit side of the reflector shadow boundary and the behaviour of the scattered field is the same as observed when frequency-domain techniques are used in the analysis, with the good agreement between TDPO and TD-UTD for the first pulse of the reflector response, which is associated with the reflected field.

For diffracted field components, TD-UTD results appear to be more accurate than those obtained by TDPO since the latter may predict erroneous currents near the reflector rim, thereby influencing the calculation of sidelobe levels. Results in Fig. 3b are calculated at a point very close to the shadow boundary associated with the upper diffraction point, with the first observed pulse corresponding to a typical merge of reflected and first-diffracted fields. Also, since the pulse width is narrower than the reflector diameter, the reflector is only partially illuminated (as illustrated in Fig. 4 for the currents over the reflector at the instant when the pulse maximum impinges upon the reflector rim), which may also explain some discrepancies observed between TD-UTD and those obtained by currents integration methods for the diffracted pulses.

The influence of frequency contents of different pulse waveforms, which may impact on the comparative analysis with asymptotic methods, is presently under investigation.

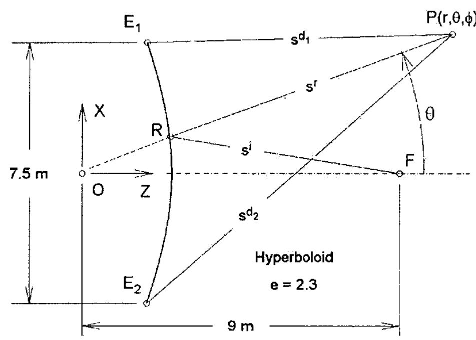


Fig. 1. Geometry of the hyperboloidal reflector.

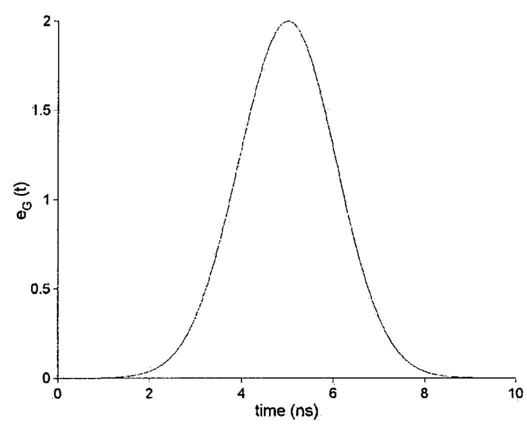


Fig. 2. Temporal behaviour of the incident pulse.

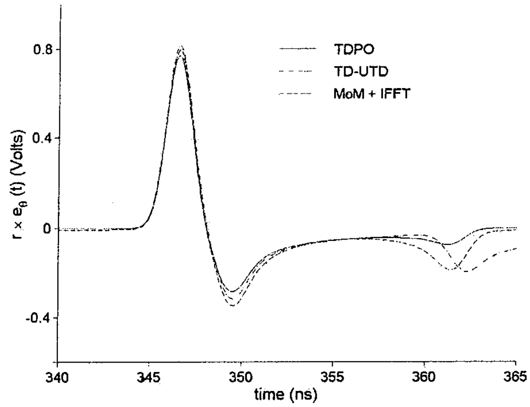


Fig. 3a. Scattered field at r = 100 m, $\theta = 30^0$, and $\phi = 0^0$.

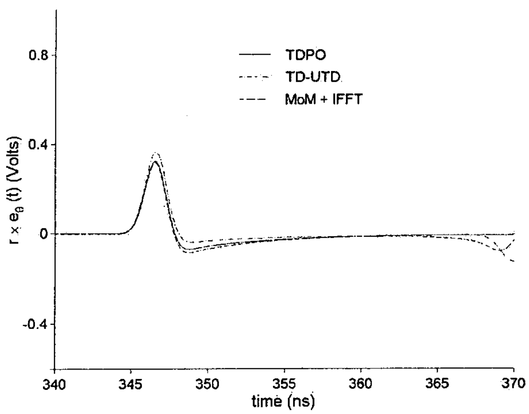


Fig. 3b. Scattered field at r = 100 m, $\theta = 65^0$, and $\phi = 0^0$.

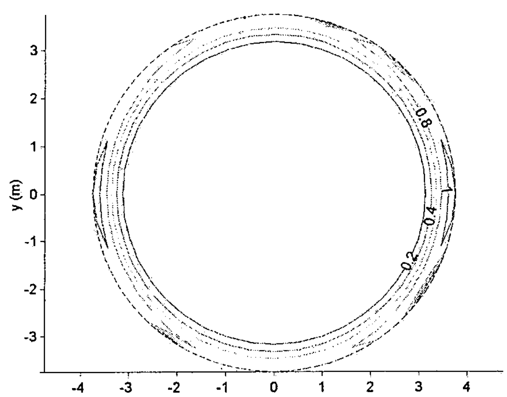


Fig. 4a. TDPO normalized induced currents at the instant when the pulse maximum impinges upon the reflector rim (dashed line).

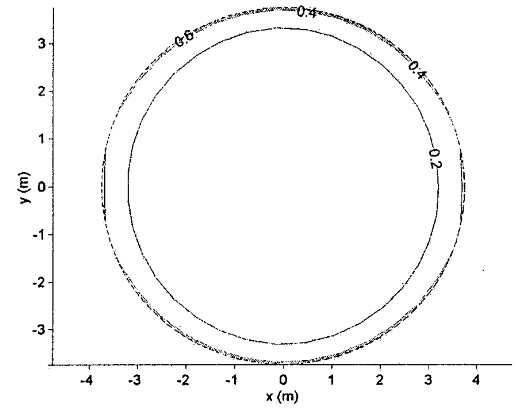


Fig. 4b. MoM + IFFT normalized induced currents at the instant when the pulse maximum impinges upon the reflector rim (dashed line).

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