AN EFFICIENT APPROACH FOR THE SYNTHESIS OF SHAPED REFLECTOR

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ABSTRACT

Reflectors antennas for shaped beam applications have been designed using synthesis techniques based on Geometrical Optics and Physical Optics approximations. Optimisation methods have been used to solve the latter, being very demanding on computing. To reduce this demand, we combine in one single goal both techniques in a numerical strategy used to design a single shaped reflector to meet BRASILSAT specifications.

INTRODUCTION

In many spacecraft applications, the antennas radiation patterns have to provide an efficient coverage of the desired area. This requires antennas generating shaped beams such that the pattern contours match the shape of these regions. For a single fixed beam, it is possible to conform the radiation patterns by using a single feed and a shaped reflector.

In order to design these shaped reflectors, some effort has been applied to develop efficient synthesis procedures. When the reflector size does not impose diffractive limitations on the emerging pattern, Geometrical Optics (GO) is usually a good approximation for the calculation of the farfield radiation in the coverage area. Based on GO, Westcott [1] has presented a formulation for the synthesis of a single reflector illuminated by a point-source. The usual numerical solution applied to this formulation yields a set of reflector data points, being convenient an additional adjustment by surface fitting models.

On the other hand, when the design has to deal with roll-off and sidelobe specifications, as well as restrictions on the size of reflector, the diffractive effects have to be incorporated in the synthesis. Associated with optimisation methods, several diffractive techniques have been presented [2-4], relying essentially on efficient numerical codes to evaluate the radiated field. However, whatever the optimisation algorithm used, this is very demanding from a computation viewpoint, as it requires repeated evaluations of the Physical Optics (PO) current integration.

A reduction in the computing time can be achieved by an adequate start of the optimisation procedure. In general, standard surfaces obtained by guess or by simple pre-synthesis analysis are used to initialize the iteration. The critical point of searching for a best start is here overcome by adopting a strategy that employs the reflector obtained from the GO synthesis to initialise the PO iterative scheme. To make the whole algorithm transparent, the same optimisation scheme is used to solve the PO and GO steps. To illustrate this numerical
scheme, it is applied to design a reflector antenna to satisfy BRASILSAT satellite specifications.

REFLECTOR SYNTHESIS PROCEDURE

The numerical procedure adopted here is similar to the one described in Reference [2]. There, the reflector surface is expressed by a polynomial and Fourier series

\[ z(x,y) = a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2 + \sum_{n=1}^{N_x} \sum_{m=1}^{N_y} b_{mn} f_n(x) f_m(y) \]  

(Eq.1)

where \( f(x) = 1, \sin(x), \cos(x), \sin(2x), \text{etc} \) for \( n = 1, 2, 3, 4, \ldots \). For convenience \( x, y \) were normalised to the range \([-\pi/2, \pi/2]\).

Thus, the reflector shape is controlled by the coefficients in the above equation, and these can be varied so that the gain of the antenna within the field of view conforms to that specified at the sample points, to within a prescribed tolerance. The gain \( G^o(u,v) \) of the antenna is calculated using PO currents on the reflector and are compared to the specified gains \( G(u,v) \) through a one side sum of the residues as below

\[ F = \sum_{k=1}^{N_u} \sum_{j=1}^{N_v} \Delta^2_{jk} \quad \Delta_{jk} = \frac{[G^E_{jk} - G^o_{jk}]}{\Delta G} > 0 \]  

(Eq.2)

where \( \Delta G \) is the tolerance. The summation covers Nyquist cells on a regular grid of directions cosines \( (u,v) \) with \( N_u \) and \( N_v \) intervals. Finally, the coefficients \( [a_i, b_{mn}] \) are varied in a search to minimise the above objective function \( F \).

The GO Initial Solution

The usual starting for the iteration is a section of paraboloid or a surface obtained by adjusting the quadratic polynomial to produce an elliptical beam with the approximate dimensions of the desired coverage. As mentioned before, instead of pre-adjusting the coefficients, we took a different approach by initialising the diffractive optimisation with the Geometrical Optics solution for the problem. As presented in Reference [1], the power transformation within a tube of rays is given by a nonlinear partial differential equation of the Monge-Ampere type in terms of \( z(x,y) \). Thus, for a given reflector surface defined by a set of coefficients (Eq.1), it is possible to calculate the farfield density of power \( G^o(u,v) \) by replacing \( z(x,y) \) and its derivatives in the Monge-Ampere equation. In this fashion, a procedure similar to the one described before can be established by replacing \( G^o \) in the objective function. Then, the coefficients \( [a_i, b_{mn}] \) are varied to minimise the sum of the residues of power gain. However, to keep the rays within the desired coverage area during the GO iterative procedure and, consequently, avoid instabilities, the rays in the reflector edge are enforced on the boundary of the coverage area. This is achieved by adding to \( F \) a term obtained from the residues of the boundary equation \( \Omega_m \). Thus, the objective function for the GO step becomes

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\[ F_{00} = \sum_{n=1}^{\text{Nu}} \sum_{m=1}^{\text{Nv}} A_{nj}^2 + W_n \sum_{m=1}^{\text{M}} C_{m} \text{ for } \Delta_{jk} = [G_{jk}^{E} - G_{jk}^{O}] / \Delta_{G} > 0 \] (Eq.3)

for the points \((N_x, N_y)\) in the Nyquist grid and for the M points on the boundary defined by the dashed line in Fig.3. The weight \(W_n\) is empirically found. The convergence of this procedure is found by a slow deformation of the boundary from a circular or elliptical shape towards the desired contour.

**BRASILSAT COVERAGE**

To illustrate the synthesis strategy outlined in the previous section, it has been applied to design the BRASILSAT shaped contours radiation patterns [5]. It requires a 27 dBi within the national boundary as shown by the dashed line in Fig.3.

Following BRASILSAT specifications, a reflector diameter \(D=37\lambda\) was used in the study. As shown in Fig.1, the feed tilt angle was designed to avoid blockage and to minimize, for linear polarisation, the crosspolar peak in the farfield pattern. For this example, it was used a feed model with a \(\cos^4\theta\) pattern, 15 dB attenuation at the reflector edge, and zero crosspolarisation.

For the GO step of the synthesis, it was used a grid with 15 points within the coverage area and 20 on the boundary. The number of coefficients used in the series were \(N_x \times N_y = 5\) that are associated with the number of sampling points. If more freedom is brought into the design by including higher order terms in the Fourier series, they may result in undesired ripples on the surface. The goal for this step was an uniform power distribution (\(\pm 32\).Bi) on the coverage area. For the PO step, only the 15 interior points were used for checking the farfield pattern.

To evaluate the strategy employed here, Fig.2 shows the objective function at each iteration of the PO synthesis by using three types of starting values. The curve A shows iterative sequence by starting with a section of a paraboloid that produces a collimated beam narrower that the desired one. In case B, by using a pre-synthesis ray analysis, the coefficients of the quadratic polynomial were set to generate an elliptical beam with the approximate width of the Brazilian coverage. For the case C, by using the reflector given by the GO synthesis as starting point, only four iterations were necessary to reach the desired goal. The overall computer time involved in Case C is dominated by the PO step since the GO synthesis is faster than one PO iteration.

The radiation pattern given by the initial GO solution is shown in Fig.3. Despite the fact that the 27dBi contour (represented by the 7.1 dB contour) resembles the desired shape, the pattern does not meet the specification near the boundary and has a very nonuniform distribution with a unique peak of 34.4 dBi. Since the desired beam width is not much larger than the minimum spot \((\Delta_{\theta} \approx 1/D)\), the diffractive effects, not included in the GO synthesis, round the beam shape at the edge of the coverage area.
and focus the beam. As shown in Fig.4.a and b, the four PO iterations lead to more even distributions of power over the area, being the peak reduced to 31.1 dBi (represented by 4.1 dB).

COMMENTS

As seen from the Fig.2, the approach brought significant reduction to the computer time of the synthesis. This may become more significant when a dual reflector antenna has to be synthesised since it involve an additional integration over the PO subreflector currents. Regarding the GO synthesis, the scheme is very convenient by furnishing a result compatible with the next step and converging very quickly as only a few points were necessary to check the power distribution. Another point to be raised is the simplicity of the implementation of the GO synthesis easily extended to other configurations.

REFERENCES


This work was supported by the Brazilian Telecommunications Agency under Contract PUC-TELEBRAS 293/88-JDPqD

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Fig. 1 - Reflector geometry.
Fig. 2 - Sequence of iterations for different starting values

+ A  * B  C

Fig. 3 - Copolar radiation pattern for the starting solution C
Contours are referred to the (34.1 dBi) peak in dB.
Fig. 4 - Radiation pattern contours for the synthesised reflector contours refered to the peak (31.1 dBi) in dB.