# Design of dual-reflector antennas shaped for ISOFLUX radiation patterns

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Abstract—The work presents a geometrical optics (GO) synthesis of a circularly symmetric dual-reflector Cassegrain antenna, shaped to provide a uniform coverage of Earth's surface by means of a ISOFLUX radiation pattern when such antenna is embarked in a low orbit satellite. This is attained by the simultaneous control of the amplitude and phase of the GO field at the main-reflector aperture. A full-wave method of moments (MoM) analysis is further conducted to validate the shaping procedure.

# Keywords—Dual-reflector antenna, ISOFLUX, LEO Satellite.

# I. INTRODUCTION

Digital wireless communication systems require large transmission rates and signal availability over larger areas to provide highly efficient personal communication services [1]. Satellites at Low Earth Orbits (LEO) are relevant alternatives to current tropospheric base stations, as they may provide wide coverage areas over the Earth's globe. Such satellites are approximately H = 2,000 km above sea level, enabling smaller delays when compared to satellites with stationary orbits [2].

To reduce pathloss' differences among ground stations within the coverage cone with angle  $\theta_{FOV}$ , the satellite antenna should ideally radiate a ISOFLUX pattern, providing larger directivities toward the farthest regions, in order to approximately obtain a uniform coverage of the Earth's surface within  $\theta_{FOV}$ , as illustrated in Fig.1. On doing so, the transceivers used on ground stations may have the same antenna gains and amplifiers, regardless of their locations. For base stations outside the coverage area, the satellite antenna should not radiate any power, ideally.

ISOFLUX patterns can be radiated by quadrifilar helixes [3], conical spirals [4], and planar arrays, among other antennas. The use of reflector antennas for this purpose may provide an interesting alternative, due to their inherently larger bandwidth, depending on the feeder [5]. The purpose of this work is to present a GO shaping procedure of circularly symmetric dual-reflector antennas. In order to attain the desired ISOFLUX pattern, the reflectors' shaping must provide the simultaneous control of the amplitude and phase of the GO field over the main-reflector aperture, as illustrated in Fig. 2. This is accomplished by representing the sub- and main-reflector shaped generatrices of the Cassegrain configuration by local conic sections (hyperbolas Sn and ellipses Mn, with n = 1, ..., N, which are consecutively concatenated (Fig. 2). The circularly symmetric reflectors are then obtained by rotating the respective generatrices around a common symmetry axis (the Z axis, as depicted in Fig. 2). The ISOFLUX shaping procedure is validated by a full-wave numerical simulation based on the Method of Moments (MoM) technique.

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Fig. 1. Different paths of the satellite signal (out-of-scale image)



Fig. 2. Shaped generatrices of the sub- and main-reflectors Cassegrain, locally represented by hyperbolas (Sn) and ellipses (Mn) [6]



Fig. 3. Basic parametrs of the confocal conic sections [6]

#### II. PROBLEM FORMULATION

#### A. GO shaping method

In [6] a shaping technique for circularly symmetric dualreflector antennas is presented, using local conic sections to describe the reflector generatrices (Fig. 2). The pair of conics Sn,Mn, together with the feed radiation. stablishes the GO field amplitude and phase at Tn (see Fig. 2), which describes a ring caustic at the main-reflector aperture after the rotation of the generatrices around the Z axis. The technique stands out from other classical design procedures for calculating conic parameters (and, consequently, the reflectors' generatrices) using a set of algebraic equations, instead of numerically evaluating ordinary differential equations.

Equations of (1)-(6), taken from [6], form a set of nonlinear equations, which must be solved for each pair of conics Sn,Mn to obtain their respective parameters, thus locally defining the reflectors' generatrices:

$$l_n = 2c_n/e_n + 2C_n/\epsilon_n \tag{1}$$

$$\frac{2C_n}{\sqrt{(2c_n\cos\beta_n - z_{Tn})^2 + (2c_n\sin\beta_n - \rho_{Tn})^2}}$$
(2)

$$\tan \gamma = (2c_n \sin \beta_n - \rho_{Tn})/(2c_n \cos \beta_n - z_{Tn}) \quad (3)$$

$$r_{Fn-1} = a_n(\eta_F^2 + 1) / [b_n(\eta_F^2 - 1) + 2\eta_F d_n - (\eta_F^2 + 1)]$$
(4)

$$r_{Mn-1}(\sin\theta_{Mn-1} - \sin\theta_{Tn-1}) + (2C_n/\epsilon_n)\sin\theta_{Tn-1} + 2c_n\sin\beta_n = \rho_{Tn}$$
(5)

$$r_{Mn}(\sin\theta_{Mn} - \sin\theta_{Tn}) + (2C_n/\epsilon_n)\sin\theta_{Tn} + 2c_n\sin\beta_n = \rho_{Tn}$$
(6)

Some of these conic parameters are illustrated in Figs. 2 and Fig. 3, and are fully and detailed explained in [6]. Here, it suffices to state that the feed radiated power density  $G_F$  is modeled by the following raised cosine feed model:

$$G_F(\theta_F) = \cos^{2p} \left(\frac{\theta_F}{2}\right) / (2\eta_0 r_F^2) \tag{7}$$

where the exponent p defines the desired feed directivity [6].

#### B. GO aperture field

According to the shaping procedure of [6], the desired aperture field distribution (amplitude and phase) must be specified a *priori*. To obtain the ISOFLUX radiation pattern, the field distribution proposed in [7] for a planar circular array is adopted. The aperture distribution of [7] has a constant amplitude distribution, while the phase distribution is obtained from the solution of the following differential equation:

$$\frac{d\psi(\xi)}{d\xi} = -[2\pi D_M/(2\alpha\lambda)]\tan^{-1}[\xi\tan(\alpha u_0)]$$
(8)

where

$$\alpha = (1/u_0) \sec^{-1}(1/A) \tag{9}$$

$$u_0 = \sin \theta_{FOV} \tag{10}$$

$$\xi = \rho_{T_n} / \rho_{T_N} \tag{11}$$

A is the smallest normalized amplitude of the radiated field within the illumination range of the antenna,  $\theta_{FOV}$  is the coverage cone angle (see Fig. 1), and  $\rho_T$  is the radial coordinate of the aperture point T (Fig. 2). In Fig. 4 the aperture phase distribution for  $\theta_{FOV} = 49.32^{\circ}$  is depicted.



Fig. 4. Aperture phase distribution for a ISOFLUX pattern with  $\theta_{FOV} = 49.32^{\circ}$ 

#### III. RESULTS

The aperture field of [7] was incorporated into the shaping procedure of [6] to design the Cassegrain generatrices. In the case studies investigated below, the following parameters were adopted:  $\lambda = 0.06$  m (i.e., f = 5 GHz), H = 2,000 km, p = 83,  $D_M = 200\lambda$ ,  $D_B = 20\lambda$ , and  $\theta_E = 30^\circ$ . N = 500 pair of conics were used to represent the reflectors' generatrices. Two different coverage cone angles  $\theta_{FOV}$  were considered.

# A. Coverage cone angle $\theta_{FOV} = 49.32^{\circ}$

Figure 5 illustrates the shaped reflectors' generatrices for  $\theta_{FOV} = 49.32^{\circ}$ , obtained using the aperture phase distribution depicted in Fig. 4. The feed phase center is assumed to be at the coordinate system origin.

Figure 6 shows the MoM [8] co- and cx-pol radiation patterns at planes  $\phi = 0^{\circ}$ , 45°, and 90°. The GO ISOFLUX templates, which illustrate the desired objective of the shaping procedure for the co-pol, are also depicted in green. From the figure one observes that the GO design was successfully achieved. With  $\theta_{FOV} = 49.32^{\circ}$ , it is possible to cover a circular ground area with a diameter of 8,000 km, typical for satellite telephony applications [2].

# *B.* Coverage cone angle $\theta_{FOV} = 41.25^{\circ}$

To illustrate the applicability of the proposed design procedure, the Cassegrain reflector antenna was shaped once more, now with  $\theta_{FOV} = 41.25^{\circ}$ . Such antenna allows the coverage of a ground area with a diameter of 4,000 km [2]. Figure 7 shows the shaped generatrices and Fig. 8 presents the MoM radiation patterns, depicting the success of the GO shaping for the present scenario. Comparing the results of Fig. 8 with those of Fig. 6, one observes the narrower beamwidth of the latter radiation pattern, as expected.



Fig. 5. Reflectors' generatrices for a coverage cone angle  $\theta_{FOV} = 49.32^{\circ}$ 



Fig. 6. Radiation pattern for a coverage cone angle  $\theta_{FOV} = 49.32^{\circ}$ 

### IV. CONCLUSION

The GO shaped Cassegrain antennas presented successful results for both cases investigated in this work, demonstrating the applicability of such antenna configuration for radiation of a ISOFLUX patterns. Desirable isolations between co- and cxpol patterns were observed from MoM simulations, indicating good performance for embarked satellite communications.

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Fig. 7. Reflectors' generatrices for a coverage cone angle  $\theta_{FOV} = 41.25^{\circ}$ 



Fig. 8. Radiation pattern for a coverage cone angle  $\theta_{FOV} = 41.25^{\circ}$ 

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