Main Reflector Synthesis of Omnidirectional Reflector Antennas Using Elliptical Sections

T. V. B. Faria and F. J. S. Moreira
GAPTEM – Department of Electronic Engineering
UFMG, Belo Horizonte/MG, 31270-901, Brazil
{tcharlesdefaria, fernandomoreira}@ufmg.br

***Abstract*—The present paper aims to extended a formalism to synthesize the main reflector of an omnidirectional axis-displaced Cassegrain (OADC) through two case studies. The numerical synthesis is based on the consecutive concatenation of elliptical sections. Furthermore, a power distribution pattern it specified on the reflector cylindrical aperture to produce, according to Geometrical Optics, a sectoral coverage. The shaping results are validated using method-of-moments (MoM) analysis.**

Keywords—Elliptical sections; geometrical optics; main reflector; OADC; sectoral coverage.

I. Introduction

Single and dual-reflector antennas are typically employed at microwave and millimeter-wave frequencies [1]-[7]. When such antennas are circularly symmetric, their generatrices can be specified by conic sections or shaped for a desired radiation pattern. In the present work, the synthesis of omnidirectional reflector antennas aims sectoral coverage, controlling the radiation pattern in the elevation plane.

Single omnidirectional reflectors were shaped in [2] to provide high performance. Although simpler, single reflectors have very large diameters compared to their dual-reflector counterparts. Thus, some works have dealt with the shaping of dual-reflector antennas for omnidirectional coverage by solving ordinary differential equations derived from Geometrical Optics (GO) principles [3]-[6].

However, there are some difficulties associated with the approaches cited above, particularly with the linearization of the differential equations used in the shaping process. In order to avoid these difficulties, new techniques based on the concatenation of local conic sections have been investigated by some authors [7]-[11]. Such numerical techniques have show promising results against traditional approaches [7]. Another factor that differentiates this technique form those based differential equations is the formulation, which is less complex [7], [11].

In [8]-[10] the authors used the concepts of GO and the concatenations of ellipsoids to shaped an offset antenna with a single reflector. While in [11], the technique developed in [8] was investigated in the design of a parabolic reflector. The present work adapts the formalism developed and presented in [8]-[11] to shape the main reflector of dual-reflector antennas for omnidirectional coverage. In the present investigation, the main reflector of an OADC configuration is shaped by concatenating elliptical sections (see Fig. 1) to obtain, according to GO, a specific power distribution over the cylindrical aperture of the main reflector. The synthesis of the main reflector is suited to provide a uniform radiation pattern in a specified sector.

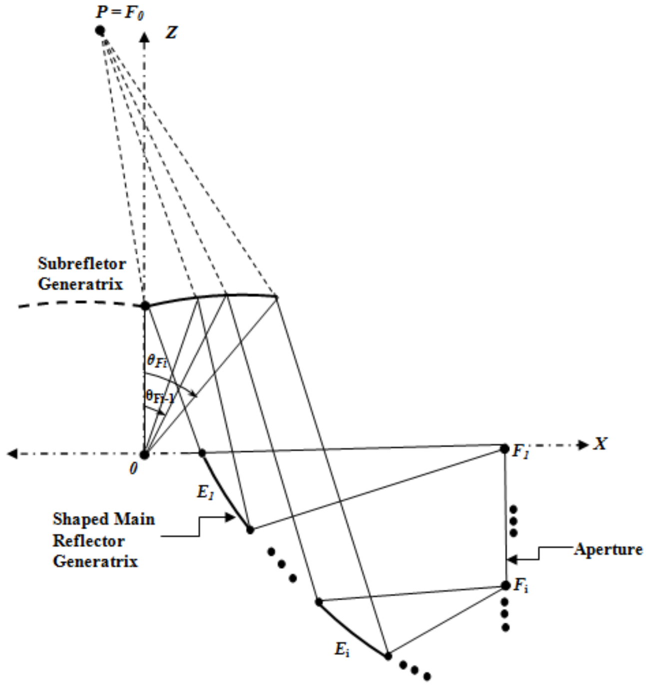

Fig. 1. OADC antenna with shaped main reflector.

II. Shaping Procedure

The basic idea of the synthesis process is to obtain the generatrix which represents the OADC main reflector by concatenating elliptical sections, while maintaining the classic subreflector (see Fig. 1). In this case, the subreflector is generated by a hyperbola with a displaced axis.

Initially, the parameters of such configuration are determined using the procedure of [1]: the interfocal distance ($2c$), axial tilt angle (β) of the hyperbola, eccentricity ($e > 1$), the edge angle ($\theta_E > 0$, for the OADC) and the rim diameter (D_S) for the subreflector. Following the concepts in [1], five geometrical parameters are necessary to determine the above

978-1-4673-9492-5/15/$31.00 ©2015 IEEE

parameters: the width (W_A) of the antenna cylindrical aperture, the main-reflector diameter (D_M) and diameter of the main-reflector central hole (D_B), the distance (V_S) between the principal focus *0* and the subreflector vertex *Q*, and the *z*-coordinate of point *B*.

The subreflector generating hyperbola of such antennas has two foci, one being at the origin (*0*) and the other at point *P*, according Fig. 1. The main reflector is generated by the combination of various elliptical sections (E_i, with i = 1, …, *N*). In this paper, it was considered that the foci of an ellipse *i* are F_0, located at point *P* and common to all *N* ellipses, and F_i located at a predefined point at the cylindrical aperture. Consequently, one must predefine the desired power distribution over the main reflector cylindrical aperture (see Fig. 1) before the synthesis is commenced.

According to GO principles, the present dual-reflector system is illuminated by a point source with center phase at the origin (*0*) and circularly symmetric radiation pattern $I_F(\theta_F)$, where θ_F describes the feed-ray direction with respect to the *z*-axis. All rays emitted by the feed suffer the first reflection at the subreflector and, afterwards, are reflected toward focus *P* (F_0), the focus common to all *N* ellipses. Afterwards, rays are reflected by ellipse *i* and reach the other focus (F_i) at the main-reflector cylindrical aperture (see Fig. 1).

A. Feed Model

In this work, the feed is a coaxial horn excited by its fundamental TEM mode, with internal and external radii R_i and R_e at the horn's aperture, respectively. The electric and magnetic fields radiated by the coaxial horn are described as:

$$\vec{E}_F(r_F) = \left[F(\theta_F)\hat{\theta}_F + P(\theta_F)\hat{\phi}_F\right]\frac{e^{jkr_F}}{r_F} \qquad (1)$$

$$\vec{H}_F(\vec{r}_F) \approx \left[\frac{\hat{r}_F \times \vec{E}_F(\vec{r}_F)}{\eta}\right], \qquad (2)$$

where r_F, θ_F and ϕ_F are the usual spherical coordinates, functions $F(\theta_F)$ and $P(\theta_F)$ represent the radiation provided by the coaxial horn, and $\eta \approx 120\,\pi\Omega$ for the free space. Applying the equivalence principle at the horn's aperture, one gets [12]:

$$F(\theta_F) = \left[\frac{J_0(k\,R_i \sin\theta_F) - J_0(k\,R_e \sin\theta_F)}{\sin\theta_F}\right] \qquad (3)$$

$$P(\theta_F) = 0, \qquad (4)$$

where $J_0(x)$ is the 0-th order Bessel function. Figure 2 shows the radiation pattern of $F(\theta_F)$, with R_i = 0,4 λ and R_e = 0,9 λ.

B. Mathematical Aspects

The present numerical synthesis discretizes the main reflector cylindrical aperture in *N* equidistant points in the half-plane $\phi = 0^0$, as show in Fig. 1. These points correspond to the foci F_i of the *N* ellipses that describe the shaped generatrix of the main reflector. The foci F_i are located at the cylindrical aperture with coordinates (see Fig 3):

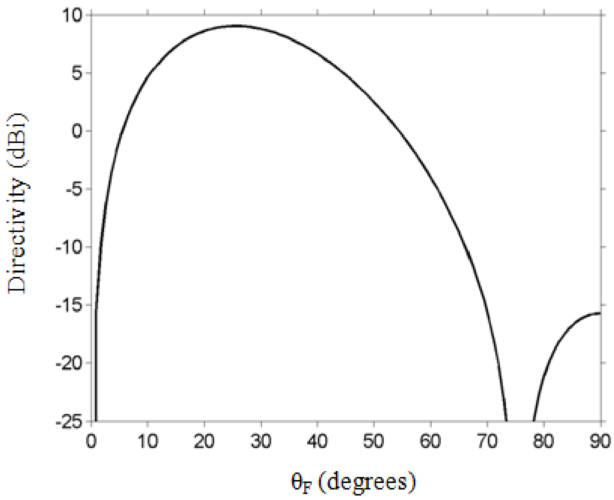

Fig. 2. Radiation pattern of the coaxial horn assuming TEM mode.

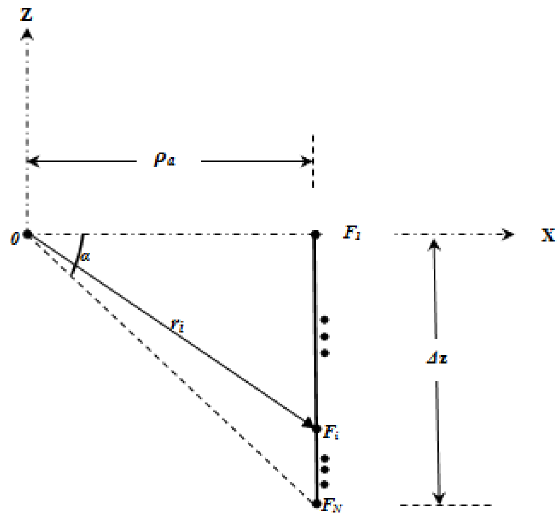

Fig. 3. Distribution of F_i in the cylindrical aperture.

$$x_{a_i} = \rho_a, \qquad (5)$$

$$z_{a_i} = i\left(\frac{-\Delta z}{N}\right), \qquad (6)$$

where ρ_a is the radius of the aperture cylinder, Δz is the width of the cylindrical aperture (see Fig. 3), i = 1, …, *N*, and *N* is the number of ellipses used to represent the shaped main reflector generatrix.

At each foci F_i at the cylindrical aperture, the conservation of energy imposes that the feed power between angles θ_{Fi-1} and θ_{Fi} must be equal to the power that arrives at F_i for the *i-th* ellipse:

$$\int_{\theta_{F_{i-1}}}^{\theta_{F_i}} I_F(\theta_F) r_F^2 \sin\theta_F \, d\theta_F = F P_i , \qquad (7)$$

where $I_F(\theta_F)$ is the feed power density, P_i is the power corresponding to foci F_i and F is a normalization factor given by:

$$F = \left(\sum_{i=1}^{N} P_i\right)^{-1} \int_0^{\theta_E} I_F(\theta_F) r_F^2 \sin\theta_F \, d\theta_F , \qquad (8)$$

where θ_E is the edge angle of the subreflector. Through (7) one obtains the directions of the rays departing from the feed (at origin 0), with angles θ_{Fi}, according to Fig. 1. The iterative algorithm begins $i = 0$ with $\theta_{F0} = 0$. For the last step, θ_{Fi} equals θ_E.

In this study, the main reflector OADC is shaped to provide a sectoral radiation pattern, according to GO. This means a uniform radiation pattern in the following sector:

$$90^o < \theta < 90^o + \alpha , \qquad (9)$$

where the angle α is illustrated in Fig. 3. To generate a uniform radiation pattern in (9) the wave front reflected by the main reflector must be spherical. In this case, to simulate a spherical wavefront, the power distribution P_i at each foci F_i is specified as:

$$P_i \propto \left(\frac{1}{r_i}\right)^2 , \qquad (10)$$

with r_i defined as shown in Fig. 3, with the help of (5) and (6).

In order to specify the ellipses that will represent the shape main reflector, it is necessary to determine their geometric parameters (see Fig. 4) and set the first point of the main reflector with coordinates:

$$x_{m_0} = \frac{D_B}{2}$$
$$z_{m_0} = 0 . \qquad (11)$$

Using the equation of a conic, the directions of the rays reflected by the subreflector (θ_{Pi}) are obtained:

$$\tan\left(\frac{\theta_{F_i} - \beta}{2}\right) = \left(\frac{e+1}{e-1}\right) \cot\left(\frac{\theta_{P_i} - \beta}{2}\right) , \qquad (12)$$

where β is the tilt angle of the axis of the hyperbola that generates the subreflector and e is its eccentricity of the hyperbola. The distance between point P (F_0) and F_i corresponds to the interfocal distance ($2c_i$) of the ellipse i (see Fig. 4):

$$2c_i = \sqrt{\left(x_P - x_{a_i}\right)^2 + \left(z_P - z_{a_i}\right)^2} . \qquad (13)$$

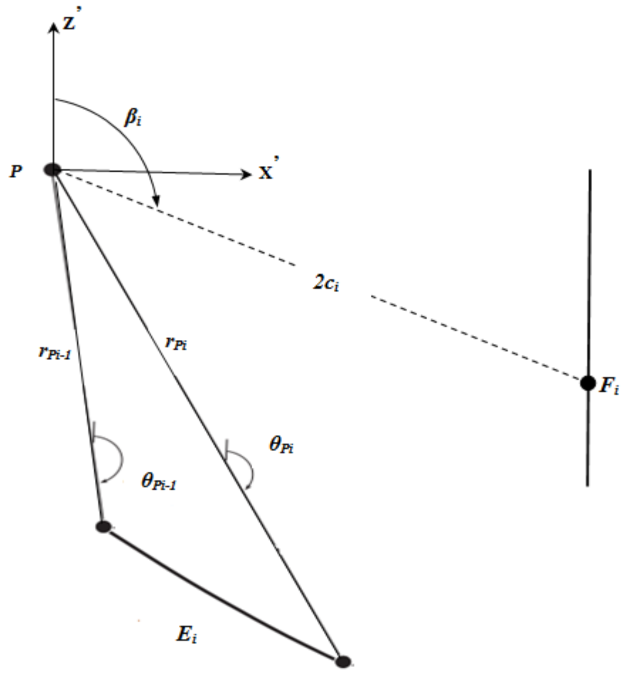

Fig. 4. Ellipse section i and its parameters.

where x_P and z_P are the coordinates of point P [1]. The angles β_i of the ellipses' axis are (see Fig. 4):

$$\tan\beta_i = \frac{x_P - x_{a_i}}{z_P - z_{a_i}} . \qquad (14)$$

Finally, from the polar equation of an ellipse,

$$r_{P_{i-1}} = \frac{c_i\left(\varepsilon_i - 1/\varepsilon_i\right)}{\varepsilon_i \cos\left(\beta_i - \theta_{P_{i-1}}\right) - 1} , \qquad (15)$$

one obtains the values of the eccentricities ε_i of the ellipses, with $0 < \varepsilon_i < 1$. In (15), r_{Pi-1} is known from previous step, where, for $i = 0$:

$$r_{P_0} = \sqrt{\left(x_P - x_{m_0}\right)^2 + \left(z_P - z_{m_0}\right)^2} . \qquad (16)$$

With parameters c_i, β_i, θ_{Pi}, and ε_i calculated, r_{Pi} is finally obtained form (15). Consequently, the coordinates that describe the shaped main reflector generatrix are:

$$x_{m_i} = x'_{m_i} + x_P$$
$$z_{m_i} = z'_{m_i} + z_P , \qquad (17)$$

for $i = 1, ..., N$, where

$$x'_{m_i} = r_{P_i} \sin\theta_{P_i}$$
$$z'_{m_i} = r_{P_i} \cos\theta_{P_i} . \qquad (18)$$

III. Case Studies

To evaluate the optical synthesis procedure developed and presented in Section II, we explore two case studies where an OADC main reflector is shaped to generate a uniform radiation pattern defined in the sector (9). For this, it was specified a power pattern (10) distributed over the foci F_i to simulate a spherical wavefront and produce such sectoral coverage.

In the two case studies ($\alpha = 15^0$ and $\alpha = 30^0$) the synthesis is started by a classic OADC. Such classic antenna is obtained using the formulation presented in [1] with the following initial parameters: $W_A = 7\ \lambda$; $D_M = 34.6\ \lambda$; $D_B = 2\ \lambda$; $V_S = 4.69\ \lambda$ and $z_B = 0$. Using such parameters, the following parameters describing the hyperbolic generatrix of the subreflector were found: $2c = 15.84\ \lambda$; $\beta = -8.5^0$; $e = 2.4239$; $\theta_E = 55.1^0$ and $D_S = 12.6\ \lambda$.

For the case studies proposed here, $N = 200$ ellipses were used to represent the shaped main reflectors, with $\rho_a = 2{,}000\ \lambda$, $R_i = 0.4\ \lambda$ and $R_e = 0.9\ \lambda$. Figure 5 shows the generatrices describing the classic and shaped main reflectors for $\alpha = 15^0$. Figure 6 shows the corresponding radiation pattern, obtained from a full-wave MoM analysis. From Fig. 5 one observes that the dimensions of the shaped reflector increased compared to the classic main reflector. Considering the radiation pattern of the shaped reflector, as illustrated in Fig. 6, it provides an almost uniform radiation in the sector $90^0 < \theta < 105^0$, as desired.

Figures 7 and 8 present the reflectors' geometry and MoM radiation patterns for $\alpha = 30^0$. From Fig. 7 one observes that there is a significant increase in the reflector dimensions, in order to distribute the radiated energy in the sector $90^0 < \theta < 120^0$. From Fig. 8 one observes severe oscillations in the radiation pattern, probably caused by diffraction effects that are not accounted for by the GO shaping procedure.

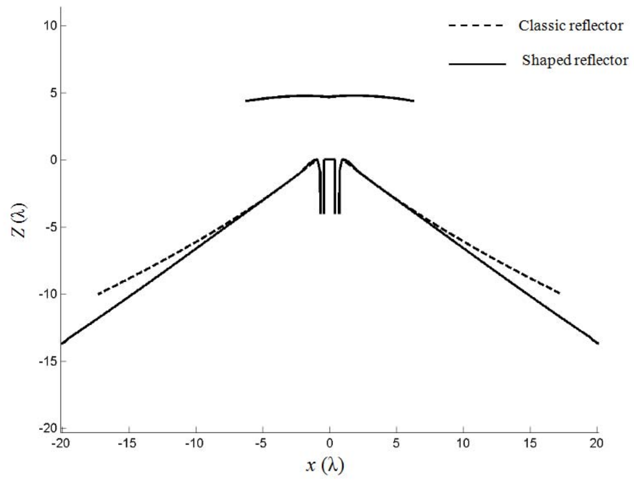

Fig. 5. Clasasic and shaped reflectors for $\alpha = 15^0$.

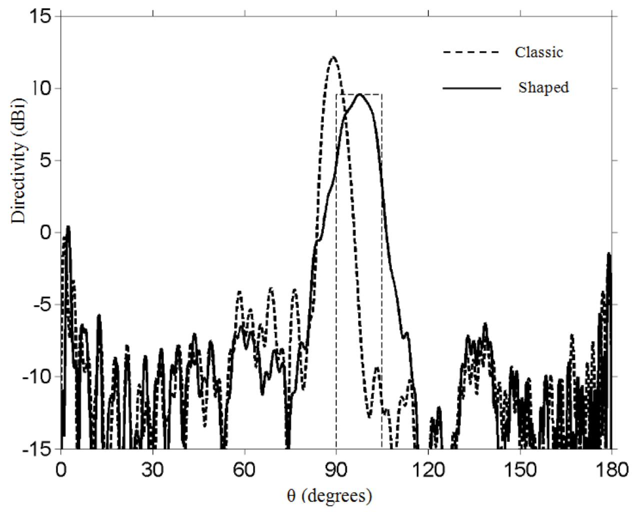

Fig. 6. Radiation pattern for $\alpha = 15^0$.

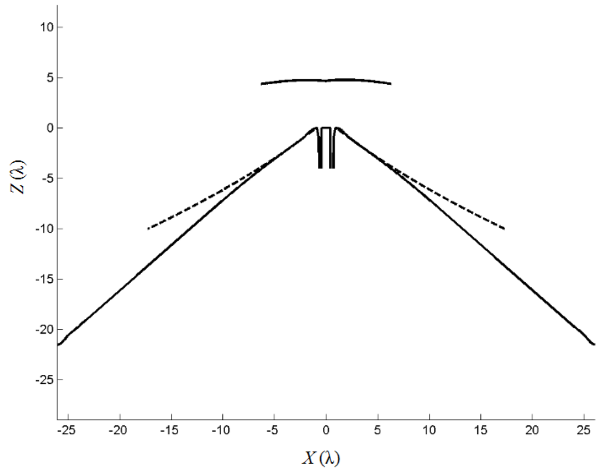

Fig. 7. Clasasic and shaped reflectors for $\alpha = 30^0$.

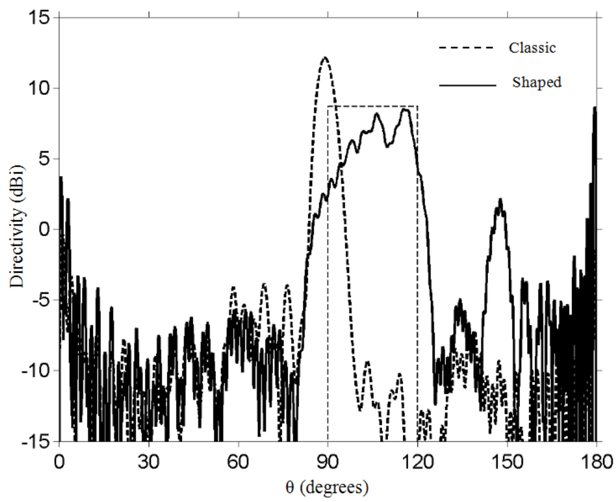

Fig. 8. Radiation pattern for $\alpha = 30^0$.

IV. CONCLUSIONS

This work presented a technique adapted from [8]-[11] to synthesize the main reflector of an OADC antenna. The technique is based on the concatenation of elliptical sections to represent such reflector. The synthesis was performed to produce a power distribution over the main-reflector cylindrical aperture to provide a specific sectoral coverage. The designs were further simulated by the MoM technique to illustrate the usefulness of the proposed design procedure.

ACKNOWLEDGMENT

This work was partially supported by CAPES and CNPq.

REFERENCES

[1] F. J. S. Moreira and J. R. Bergmann, "Classical axis-displaced dual-reflector antennas for omnidirectional coverage," IEEE Trans. Antennas and Propagation, vol. 53, no. 9, pp. 2799-2808, September 2005.

[2] J. R. Bergmann, F. J. V. Hasselmann and M. G. Branco, "A single-reflector design for omnidirectional coverage", Microwave Opt. Tech. Lett., vol, 24, no. 6, pp. 426-429, March, 2000.

[3] A. Norris and W. Waddoup, "A millimetric wave omnidirectional atenna with prescribed elevation shaping," in Proc. ICAP 4th Int. Conf. Antennas and Propagation, 1985, pp. 141-145.

[4] A. Pino, A. Acuña and J. Lopez, "An omnidirectional dual-shaped reflector antenna", Microw. Op. Tech. Lett., vol. 27,pp. 371-374, 2000.

[5] F. J. S. moreira, A. Prata, Jr. And J. R. Bergmann, "GO shaping of omnidirectional dual-reflector antennas for a prescribed equi-phase aperture field distribution," IEEE Trans. Antennas and Propagation, vol. 35, no. 1, pp. 99-106, January 2007.

[6] F. J. S. Moreira, A. Prata, Jr. and J. R. Bergmann, "GO shaping of omnidirectional dual-reflector antennas for maximum gain," in. Proc. Int. Microwave and Optoeletronics Conference (IMOC 03), Foz do Iguaçu, PR, Brazil, pp. 199-204, September 2003.

[7] F. J. S. Moreira and J. R. Bergmann, "Shaping axis-symmetric dual-reflector antennas by combining conic sections," IEEE Trans. Antennas and Propagation, vol. 59, no. 3, pp. 1042-1046, March 2011.

[8] V. I. Oliker, "A rigorous method for synthesis of offset shaped reflector antennas," Computing Letters, vol. 2, no. 1-2, pp. 29-49, March 2006.

[9] V. I. Oliker and S. A. Kochengin, "Determination of reflector surfaces form near-field scattering data II", Numerical Solutions, vol. 79, no. 4, pp. 553-568, 1998.

[10] S. Kochengin and V. I. Oliker, "Computational algorithms for constructiong reflectors," Computing and Visualization in Science,vol. 6, pp. 15-21, 2006.

[11] T. V. B. Faria and F. J. S. Moreira, "Modelagem de antenas refletoras circularmente simétrics através da concatenação de seções de elipses", in Proc. MOMAG 2014,Curitiba, PR, Brasil, pp. 85-89, 2014.

[12] R. F. Harrington, Time-Harmonic Eletromagnetic Field, McGraw-Hill, New York, 1961.