Novel UTD Coefficients for Lossy Conducting Wedges

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***Abstract* — This paper presents novel uniform theory of diffraction (UTD) coefficients for the analysis of electromagnetic scattering by lossy conducting wedges. The new heuristic coefficients are based in previously proposed ones that, although presented good results for forward scattering, weren't reciprocal. With minor modifications, the new coefficients treat backscattering correctly, becoming appropriate to handle arbitrary transmitter and receiver positions in urban scenarios, for instance. The diffraction by arbitrary lossy conducting wedges is investigated and the new approach results are compared with accurate Maliuzhinets and method-of-moments analyses.**

***Index Terms* — Heuristic diffraction coefficients, radio propagation, scattering by lossy wedges, Uniform Theory of Diffraction.**

I. Introduction

The Uniform Theory of Diffraction (UTD), developed by Kouyoumjian and Pathak [1], is widely used in the asymptotic analysis of electromagnetic scattering through complex (urban) environments. The UTD coefficients, although being very efficient, were originally suitable only for the evaluation of diffracted fields by perfectly conducting wedges. However, a more realistic radio wave propagation prediction needs to account for lossy obstacles.

Maliuzhinets presented a solution for wedge diffraction by nonperfectly conducting surfaces [2]. However, this solution is not practical for propagation prediction in real (complex) environments, since it uses a special function, which is difficult to calculate for an arbitrary wedge angle.

More practical solutions, based on the UTD coefficients, were heuristically proposed in order to account for diffraction by lossy conducting wedges [3]-[7]. All these coefficients presented good results for some special cases or situations, but not for arbitrary transmitter and receiver positions (i.e., both forward and backscattering). The first coefficients, proposed by Luebbers, took into account losses in the analysis of obstructed VHF and UHF radio links [3]. However, Luebbers' heuristic coefficients presented some problems: they are not accurate deep in the shadow region and they are not reciprocal, as they were derived for forward scattering analysis only. In [4], aiming to turn Luebbers' coefficients into reciprocal ones, Aïdi and Lavergnat heuristically proposed new angular definitions for the Fresnel reflection coefficient presented in Luebbers' formulation. But Aïdi and Lavergnat's coefficients still lack accuracy in some particular regions. In order to improve the performance of Luebbers' coefficients in shadow regions, Holm derived novel heuristic coefficients, based on Fresnel-Kirchhoff theory [5]. Although Holm's coefficients had achieved better results, they still do not obey reciprocity.

Thus, the objective of this work is to obtain reciprocal and efficient heuristic diffraction coefficients suited for lossy wedge scattering and, consequently, radio channel characterizations. The heuristic coefficients presented here are improvements over those of [6] and [7].

In order to estimate the usefulness and applicability of the proposed coefficients, scattering by arbitrary lossy wedges are investigated. The results obtained by the proposed UTD heuristic coefficients are compared against those from Luebbers [3], Holm [4], Aïdi and Lavergnat [5], using Maliuzhinets' solution as reference. Results given by the new coefficients are also compared against the method of moments (MoM) [8] and pathloss measurements in a typical urban scenario [9].

II. Heuristic UTD Coefficients for Lossy Wedges

A. Luebbers' Heuristic Diffraction Coefficients

In [3], Luebbers' soft and hard heuristic diffraction coefficients are described as:

$$D^{s,h} = G_0^{s,h}\left\lfloor D_2 + R_0^{s,h}(\alpha_0)D_4\right\rfloor + G_n^{s,h}\left\lfloor D_1 + R_n^{s,h}(\alpha_n)D_3\right\rfloor, \quad (1)$$

where D_i, for i=1,…,4, are the UTD diffraction coefficients, as defined in [1], and R_0 and R_n are the Fresnel reflection coefficients, relative to 0 ($\phi = 0$) and n ($\phi = n\pi$) wedge faces, respectively (see Fig. 1). The superscripts s and h denote the soft and hard polarizations, respectively, for which the Fresnel reflection coefficients are

$$R^s(\alpha) = \frac{\sin(\alpha) - \sqrt{\hat{\varepsilon}_r - \cos^2(\alpha)}}{\sin(\alpha) + \sqrt{\hat{\varepsilon}_r - \cos^2(\alpha)}},$$

$$R^h(\alpha) = \frac{\hat{\varepsilon}_r\sin(\alpha) - \sqrt{\hat{\varepsilon}_r - \cos^2(\alpha)}}{\hat{\varepsilon}_r\sin(\alpha) + \sqrt{\hat{\varepsilon}_r - \cos^2(\alpha)}}, \quad (2)$$

where $\hat{\varepsilon}_r = \varepsilon_r - j\sigma/(\omega\varepsilon_0)$ is the wedge complex relative permittivity and the incidence angles α_0 and α_n are

$$\alpha_0 = \min(\phi_i, \phi_d), \quad \alpha_n = \min(n\pi - \phi_i, n\pi - \phi_d) \quad (3)$$

where $n\pi$ is the wedge exterior angle, ϕ_i is the direction of the incident wave and ϕ_d is the direction of the diffracted

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wave, both with respect to face 0 (see Fig. 1). The factors $G_0^{s,h}$ and $G_n^{s,h}$, used when grazing incidence occurs, are:

$$G_n^{s,h} = \begin{cases} 1/2, & \phi_i = 0 \\ 1/(1+R_n^{s,h}), & \phi_i = n\pi, \quad (1+R_n^{s,h}) > 0 \\ 1, & \text{otherwise} \end{cases} \tag{4}$$

$$G_0^{s,h} = \begin{cases} 1/2, & \phi_i = n\pi \\ 1/(1+R_0^{s,h}), & \phi_i = 0, \quad (1+R_0^{s,h}) > 0 \\ 1, & \text{otherwise} \end{cases} \tag{5}$$

B. Aïdi and Lavergnat Heuristic Diffraction Coefficients

In [4], new definitions were proposed for α_0 and α_n, in order to make Luebbers' heuristic coefficients reciprocal:

$$\alpha_0 = \alpha_n = \min(\phi_i, \phi_d, n\pi - \phi_i, n\pi - \phi_d), \tag{6}$$

instead of (3). With this simple modification, the diffraction coefficients are now reciprocal, i.e., they are (heuristically) valid independent of the transmitter and receiver positions with respect to wedge faces 0 and n [4]. Case studies indicate that superior performance is achieved, especially in regions close to the wedge faces [4].

C. Holm's Heuristic Diffraction Coefficients

In [5], Holm proposed a new heuristic UTD diffraction coefficient, in line with the heuristic diffraction coefficients proposed by Luebbers in [3], but with better performance in deep shadow regions and valid for wedges with any interior angle. Holm's heuristic coefficients are [5]:

$$D^{s,h} = G_n^{s,h}\left[R_n^{s,h}(\alpha_n)R_0^{s,h}(\alpha_0)D_1 + R_n^{s,h}(\alpha_n)D_3\right] + G_0^{s,h}\left[D_2 + R_0^{s,h}(\alpha_0)D_4\right] \tag{7}$$

where α_0 and α_n are those of (3) and the grazing incidence terms, G_n and G_0, are now simply defined as:

$$\begin{cases} G_0^{s,h} = G_n^{s,h} = 1/2, & \phi_i = 0 \text{ or } \phi_i = n\pi \\ G_0^{s,h} = G_n^{s,h} = 1, & \text{otherwise} \end{cases} \tag{8}$$

Note that in [5] the coefficients D_i are differently numbered with respect to the classical notation in [1], which is the one adopted here.

This formulation was accomplished by initially noticing that the terms D_1 and D_2 are associated with double reflection boundaries, for a wedge with interior angle greater than π: D_1 has a boundary of an incident field first reflected from the 0 face and then from the n face, whereas for D_2 is the contrary. According to [5], for $\phi_i > (n - 1/2)\pi$, the Fresnel reflection coefficients R_0 and R_n should now multiply the term D_2 in (7), instead of D_1. For the case of exterior wedges, where D_1 and D_2 are associated with shadow boundaries, these new coefficients provided better results in shadow regions than the ones presented by Luebbers [3]. For the case studies presented in Sect. III-A, we extended Holm's definitions for all incidence angles as: when $\phi_i < n\pi/2$, Eq. (7) is used; when $\phi_i \geq n\pi/2$, the factor R_0R_n multiplies D_2 instead of D_1 in (7).

D. Novel Heuristic Diffraction Coefficients for Lossy Conducting Wedges

The novel heuristic coefficients were developed aiming their application to coverage predictions in urban scenarios. This means that obstacles with various configurations and finite conductivity surfaces, together with arbitrary transmitter and receiver positions, must be accounted for practical estimates.

The results in [5] indicate that Holm's coefficients provide superior results than Luebbers' ones [3], but they are still not reciprocal. So, for the consideration of arbitrarily located transmitters (sources) and receivers (observers), we propose a modification to Holm's heuristic formulation, in order to make it reciprocal. Such modification is an improvement over that of [6] and [7].

This modification consists in changing the condition for the application of the factor R_0R_n. This factor will now multiply the term D_1 in (7) in the case where $\phi_i < \phi_d$ (forward scattering), whereas backscattering occurs ($\phi_i > \phi_d$), the factor R_0R_n should multiply D_2. This is based on the fact that the terms D_1 and D_2 are reciprocal in relation to the incidence angle (ϕ_i), i.e., $D_1(\phi_i-\delta) = D_2(\phi_i+\delta)$, for any $\delta < \phi_i$. Then, the new heuristic diffraction coefficients are written as:

$$D^{s,h} = G_n^{s,h}\left[W_n^{s,h}D_1 + R_n^{s,h}(\alpha_n)D_3\right] + G_0^{s,h}\left[W_0^{s,h}D_2 + R_0^{s,h}(\alpha_0)D_4\right] \tag{9}$$

where reciprocity imposes that the heuristic terms W_n and W_0 are defined as:

$$W_n^{s,h} = \begin{cases} R_0^{s,h}(\alpha_0)R_n^{s,h}(\alpha_n), & \phi_i < \phi_d \\ 1, & \phi_i \geq \phi_d \end{cases}$$
$$W_0^{s,h} = \begin{cases} 1, & \phi_i < \phi_d \\ R_0^{s,h}(\alpha_0)R_n^{s,h}(\alpha_n), & \phi_i \geq \phi_d \end{cases} \tag{10}$$

Luebbers' angular definitions (3) for the Fresnel reflection coefficients $R_0^{s,h}$ and $R_n^{s,h}$ are the ones used in this novel coefficient, as Holm originally adopted.

E. Slope Diffraction

In the case of double-diffracted rays in consecutive wedges, the use of the first-order diffraction coefficients of (7) will estimate a null field for an observer in the shadow region. To overcome this problem, a second-order term (known as *slope diffraction*) must be considered. The slope diffraction term must be added to the first-order components to yield the total field. The slope diffraction coefficients depend on spatial derivatives of the incident field and of the coefficients in (7). The slope diffraction coefficients are similar to those presented in [5], just observing the new conditions for applying the heuristic terms W_n and W_0 as in (10).

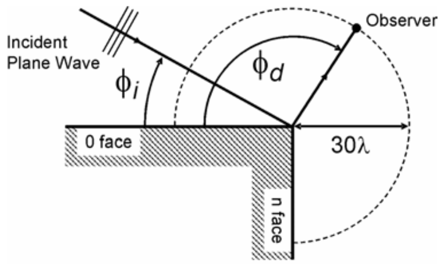

Fig. 1. Right-angle lossy wedge with $\varepsilon_r = 10$ and $\sigma = 0.01$ S/m.

III. CASE STUDIES

A- Comparisons Among Different Heuristic Coefficients

In order to demonstrate the usefulness of the proposed UTD heuristic formulation, several heuristic coefficients (i.e., Luebbers [3], Aïdi and Lavergnat [4], Holm [5], and the novel coefficients proposed in Sect. II-D) are compared to each other in the analysis of a plane-wave scattering by a right-angle lossy wedge with geometrical parameters depicted in Fig. 1. Both TM (soft) and TE (hard) polarizations are considered and an accurate asymptotic diffractive analysis based on Maliuzhinets' coefficients [2] is adopted as reference for the comparative study.

For both TM and TE plane-wave polarizations, three directions of incidence (ϕ_i = π/6, π/2, and 3π/4) are considered. The operation frequency is 1 GHz and the observations are made at a distance of 30 λ from the edge, with the observation direction ϕ_d varying from 0 to 3π/2 (see Fig. 1). The lossy wedge has a relative permittivity $\varepsilon_r = 10$ and a conductivity $\sigma = 0.01$ S/m. In order to account for losses, the Maliuzhinets' coefficients make use of a surface impedance over the wedge faces, meaning that no energy is transmitted to the wedge interior [2].

Figures 2–7 show the relative diffracted electric field amplitude $|E_d|$, with respect to the incident electric field amplitude $|E_i|$, and the absolute error, with respect to the Maliuzhinets' formulation, for the TM and TE polarizations, respectively. The results for both TM and TE polarizations, illustrated in Figs. 2–7, demonstrate that the new soft and hard coefficients of Sect. II-D considerably improves the estimates in all observation regions, with absolute errors smaller than 5 dB, except at some specific points for the TE polarization for ϕ_i = π/6 (Fig. 3) and ϕ_i = π/2 (Fig. 5) where all formulations approximately provide the same error. This constitutes an improvement over the results presented in [6] and [7].

Table 1 shows the mean and the standard deviation of the absolute error for the cases depicted in Figs 2–7. It can be seen that the novel coefficients have superior performance than the previously proposed coefficients [3]-[5].

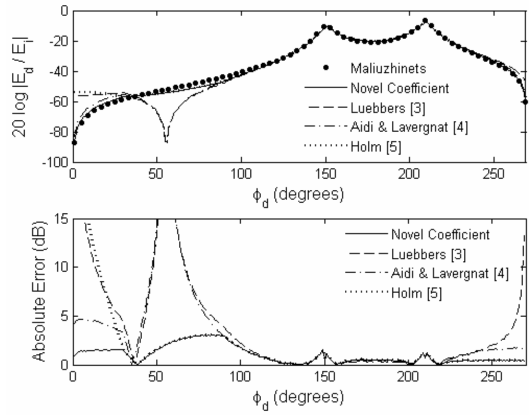

Fig. 2. TM diffracted field around the wedge of Fig. 1 (ϕ_i = π/6).

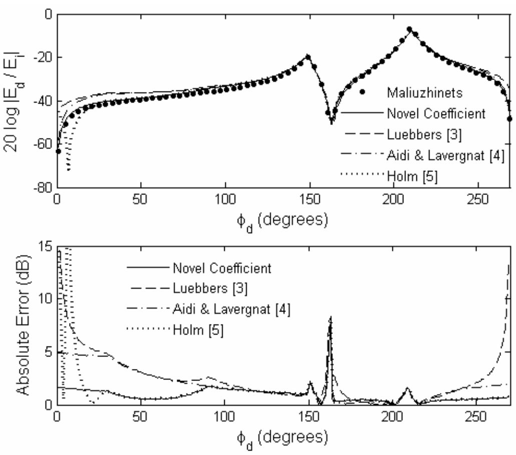

Fig. 3. TE diffracted field around the wedge of Fig. 1 (ϕ_i = π/6).

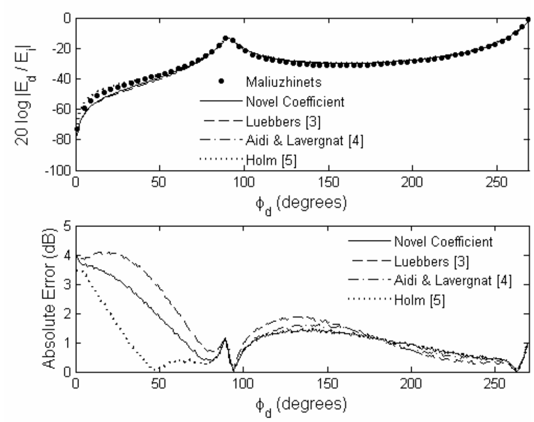

Fig. 4. TM diffracted field around the wedge of Fig. 1 (ϕ_i = π/2).

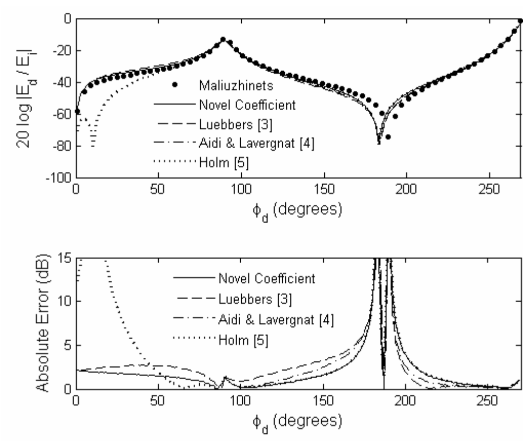

Fig. 5. TE diffracted field around the wedge of Fig. 1 ($\phi_i = \pi/2$).

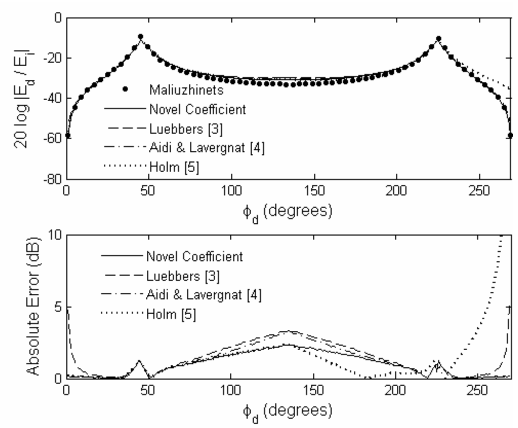

Fig. 6. TM diffracted field around the wedge of Fig. 1 ($\phi_i = 3\pi/4$).

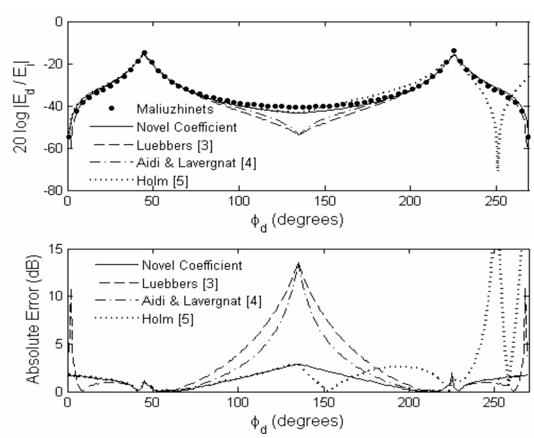

Fig. 7. TE diffracted field around the wedge of Fig. 1 ($\phi_i = 3\pi/4$).

Coefficients	TM Polarization		TE Polarization	
	Mean Error (dB)	Standard Deviation (dB)	Mean Error (dB)	Standard Deviation (dB)
New Coefficient	1.10	0.86	1.36	1.52
Luebbers [3]	2.34	2.77	2.61	2.93
Aïdi et. al. [4]	1.84	2.34	2.06	2.39
Holm [5]	1.53	2.48	2.51	4.29

Table 1. Mean and standard deviation of the absolute error for the cases depicted in Figs 2 – 7.

B- Scattering by Lossy Obstacles with Arbitrary Wedge Angles

The second case study considered here is the scattering by two infinite cylinders, which have cross sections depicted in Fig. 8. The source is an infinite electric line current (for the soft TM polarization), operating at 2.4 GHz. Observations are made along the dotted line illustrated in Fig. 8. Figure 9 shows the total electric field (E_t) normalized with respect to the maximum electric field radiated by the line current incident at the receiver (E_i). The results provided by the novel formulation of Sect. II-D is compared against the numerical estimates obtained by a method-of-moments (MoM) analysis [8] with losses accounted for by means of an impedance boundary condition (IBC) [2]. The results shown in Fig. 9 demonstrate that the proposed heuristic coefficients are appropriate to analyze the scattering by arbitrarily shaped cylinders, as the mean absolute error is not superior to 1 dB.

C- Evaluation of the Novel Heuristic Diffraction Coefficients in an Urban Scenario

The last case study is the scattering analysis through the scenario depicted in Fig. 10, which illustrates a plane view of the downtown core of Ottawa city, Canada. Measured data made at 910 MHz for this scenario are available and used for comparison purposes [9]. In the numerical simulations, the transmitter is a vertical infinitesimal electric dipole (i.e., *soft* polarization with respect to the obstacles' wedges).

In order to handle such urban environment, a quasi-3D ray-tracing algorithm was implemented, based on the image theory [10]. All ray paths with a maximum of 4 reflections and 2 diffractions were considered to track the field from the transmitter (T) to the receivers (located along Laurier St. in Fig. 10). The quasi-3D algorithm considers reflections over ground and the obstacle faces (the transmitter and receiver heights are much smaller than the buildings heights). Losses are considered assuming that $\varepsilon_r = 6$ and $\sigma = 0.5$ S/m for the reflections and diffractions from obstacles (buildings) and $\varepsilon_r = 15$ and $\sigma = 0.05$ S/m for the reflections at ground [9]. Figure 11 shows the path loss at the receiver locations, where the estimates obtained by the novel heuristic formulation of Sect. II-D are compared with the measured data of [9]. The agreement is good considering the lack of information

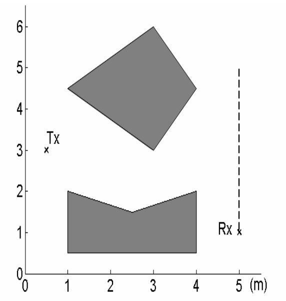

Fig. 8. Environment obstacles with $\varepsilon_r = 6$ and $\sigma = 0.2$ S/m.

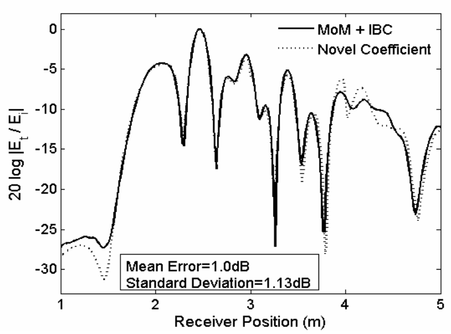

Fig. 9. TM on the environment of Fig. 8.

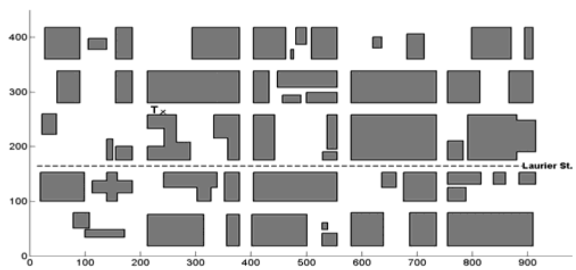

Fig. 10. Plane view of the core region of Ottawa, Canada. Receivers are located along Laurier St.

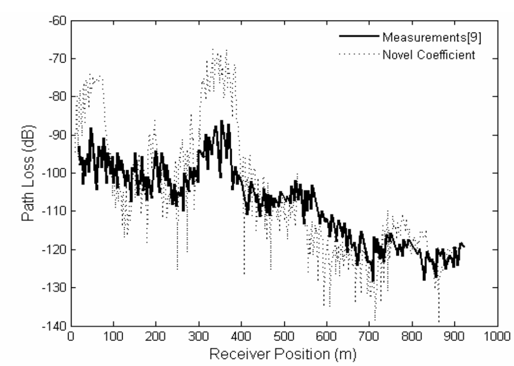

Fig. 11. Path Loss along Laurier Street (Fig. 10).

regarding building data. A possible explanation for the noticeable differences with respect to the measured data (at 0–100m and 300–400m) is the possible presence of trees or other kinds of obstructions, besides the buildings.

IV. CONCLUSION

This work presented novel heuristic UTD coefficients for the analysis of the scattering by lossy wedges. The proposed coefficients are improvements over those of [6] and [7], and provide superior performance than several heuristic UTD coefficients previously proposed in [3]-[5]. Also, the coefficients are reciprocal and, consequently, suited to handle complex scenarios with arbitrarily located sources and observers. To validate the proposed coefficients, three case studies were investigated, where the novel heuristic coefficients results were compared against Maliuzhinets' diffraction coefficients [2], the MoM [8] and measurements [9]. All case studies illustrate and indicate the usefulness and applicability of the novel coefficients for the analysis of lossy wedges.

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