Convergence Analysis of Integral Equations for Characterization of RF Channels

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***Abstract*— The main concern of the present work is to investigate the convergence of the electric and magnetic field integral equations (EFIE and MFIE) adopted in the characterization of radio wave propagation over irregular terrains. The terrain irregularities are smooth, such that back-scattering is neglected and a recursive technique is used to determine the equivalent currents. Due to the near-grazing incidence and the vertical polarization of the incident field, the terrain can be considered an open surface made of a perfect magnetic conductor. This approach leads toward an equivalent problem that, in principle, can not be solved by the EFIE, a fact apparently overlooked in the literature. This may explain why the MFIE provides accurate results with a smaller number of basis functions than the EFIE.**

***Index Terms*— Radio wave propagation, integral equation.**

I. Introduction

Nowadays the overwhelming demand for RF spectrum increases the number of studies involving radio wave coverage prediction. With the evolution of RF technology and the fast development of wireless communications, wideband signals have find their way into many more applications. One of them is the digital television, commonly known as the high-definition HDTV or the European DVB (*Digital Video Broadcasting*). Such applications generally demand the transmission of a large amount of information with minimum interference. So, one of the present interests is a better knowledge on the behavior of the RF signal propagation, which requires a complete characterization of the transmitting medium.

Techniques based on field integral equations have been suggested and used as they allow a full-wave characterization of the radio wave propagation phenomenon [1],[4]. However, they demand an extraordinary computational effort, limiting their applicability to simple practical scenarios [2]. For instance, in [2] a vertical polarization is assumed together with a near-grazing incidence, validating the approximation of the ground by a perfect magnetic conductor. So, after the application of the equivalence principle, only magnetic currents are left radiating in free-space, which can be solved by means of well-established integral equations. Besides, the terrain is assumed electrically smooth, such that the ground back-scattering can be neglected without largely compromising the accuracy of the analysis. That allows a recursive solution of the magnetic currents, without the need of the traditional full-matrix analysis of the moment method (MoM) technique [2].

In [2], a formulation based on the electric field integral equation (EFIE) is adopted, which for the present scenario is an integral equation of the second kind (i.e., magnetic currents radiating in free-space). That arises some questions, as such kind of integral equation should not be used for open surfaces, as in [2]. So, a magnetic field integral equation (MFIE) was proposed in [3] and [5] to solve that very same problem. The results obtained indicate that the MFIE is very accurate and has a better convergence (i.e., uses a small number of basis functions to represent the equivalent magnetic currents).

The objective of the present work is to investigate why the MFIE has a better convergence than the EFIE to characterize vertically-polarized radio wave propagation over terrains with smooth irregularities. For that, we compute and compare the magnitudes of the impedance-matrix elements for both EFIE and MFIE in a typical triangular terrain profile. It will be shown that the impedance-matrix yield by the EFIE presents regions where coupling between the magnetic currents do not exist, which may justify the adequacy of the use of the MFIE to numerically evaluate the equivalent magnetic currents.

II. The Integral Equations

For a vertically-polarized wave and near-grazing incidence, the terrain can be treated as a perfect magnetic conductor and only equivalent magnetic currents ($\vec{M}_S$) are considered [2]. The radiation integrals are then written as [2]–[5]

$$\vec{E}(\vec{r}) = T\left[\vec{E}_{in}(\vec{r}) + \vec{L}_2(\vec{M}_S)\right] \tag{1}$$

$$\vec{H}(\vec{r}) = T\left[\vec{H}_{in}(\vec{r}) + \frac{1}{\eta}\vec{L}_1(\vec{M}_S)\right] \tag{2}$$

where $\vec{E}_{in}$ and $\vec{H}_{in}$ represent the electric and magnetic incident fields at the observation point $\vec{r}$, η is the intrinsic impedance of the medium and $T = 1$ or 2 for an observer outside or at the surface, respectively. The operators L_1 and L_2 are given by:

$$\vec{L}_1(\vec{M}_S) = -jk\oint_{S'}\left[\vec{M}_S(\vec{r}\,')G - \frac{1}{k^2}\nabla'\cdot\vec{M}_S(\vec{r}\,')\nabla'G\right]ds' \tag{3}$$

$$\vec{L}_2(\vec{M}_S) = -\oint_{S'}\vec{M}_S(\vec{r}\,')\times\nabla'G\,ds' \tag{4}$$

where the position vectors $\vec{r}$ and $\vec{r}\,'$ are defined as in Fig. 1 and G is the free-space Green's function:

$$G = \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{4\pi|\vec{r}-\vec{r}\,'|}. \tag{5}$$

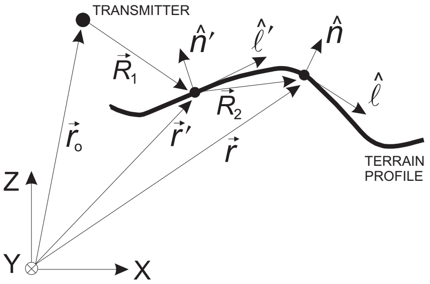


Fig. 1. Terrain profile

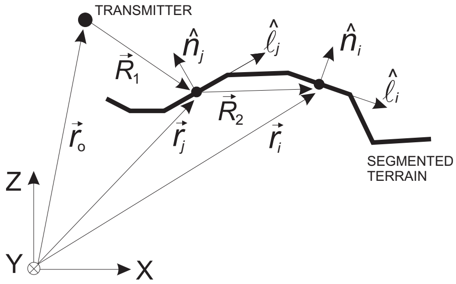


Fig. 2. Segmented terrain used for the MoM analysis

Applying the boundary conditions for a perfect magnetic conductor:

$$\hat{n} \times \vec{E}(\vec{r}) = -\vec{M}_S(\vec{r}) \tag{6}$$

$$\hat{n} \times \vec{H}(\vec{r}) = 0 \tag{7}$$

and from (1) and (2), with $T = 2$, we obtain the EFIE and MFIE:

$$\hat{n} \times \vec{E}_{in}(\vec{r}) = -\frac{\vec{M}_S(\vec{r}\,')}{2} - \hat{n} \times \vec{L}_2(\vec{M}_S) \tag{8}$$

$$\hat{n} \times \vec{H}_{in}(\vec{r}) = -\frac{1}{\eta}\, \hat{n} \times \vec{L}_1(\vec{M}_S) \tag{9}$$

Based on the considerations made, assuming the ground invariant perpendicular to the plane of incidence and that the transmitting antenna is sufficiently away from the terrain, we can treat the source as being a punctual source and the stationary phase method can be applied on the equivalent magnetic current, as done in [2]:

$$\vec{M}_S(\vec{r}\,') = \vec{M}_A(\vec{r}\,')\, e^{-jk\left|\vec{r}\,' - \vec{r}_0\right|} \tag{10}$$

Substituting (10) into (8) and (9) and after applying the stationary-phase method to solve one of the integrals, we can derive expressions where the only unknown elements are the current amplitudes $\vec{M}_A$ [2],[3]:

$$\hat{n} \times \vec{E}_{in} = -\frac{\vec{M}_A}{2}\, e^{-jkR_1} + k \int_{\ell'} (\hat{n} \cdot \hat{R}_2) \vec{M}_A\, G_2\, d\ell' \tag{11}$$

$$\hat{n} \times \vec{H}_{in} = \frac{k}{\eta} \int_{\ell'} \hat{n} \times \vec{M}_A\, G_1\, d\ell' \tag{12}$$

where ℓ' represents the terrain profile over the plane of incidence (see Fig. 1) and

$$G_1 = \frac{e^{j\pi/4}\ e^{-jk(R_1+R_2)}}{4\pi\sqrt{R_2(1 + R_2/R_1)/\lambda}} \tag{13}$$

$$G_2 = [1 - j/(kR_2)]\ G_1 \tag{14}$$

III. Moment Method Solution

The moment method (MoM) technique can be used to solve (11) and (12). The terrain is divided into several straight segments, as illustrated in Fig. 2. For the basis and weighting functions we choose unitary pulses and impulses, respectively [2]. To evaluate the ℓ' integrals in (11) and (12), the phase of M_A is assumed with a linear variation over each segment [5]. One thus obtain the following linear system:

$$[V_i] = [Z_{ij}]\ [M_j]\,, \tag{15}$$

where, for the EFIE,

$$\begin{aligned} V_i &= \vec{E}_{in} \cdot \hat{\ell}_i \\ Z_{ij} &= \frac{\sin\alpha}{\alpha}\, (\hat{n}_i \cdot \hat{R}_2)\, G_2\, k\Delta_j \\ Z_{ii} &= -\frac{e^{-jkR_1}}{2} \end{aligned} \tag{16}$$

and, for the MFIE,

$$\begin{aligned} V_i &= \eta\, (\vec{H}_{in} \cdot \hat{y}) \\ Z_{ij} &= \frac{\sin\alpha}{\alpha}\ G_1\ k\Delta_j \\ Z_{ii} &= \frac{e^{j\pi/4}\ e^{-jkR_1}}{2} \left[\frac{Fres\left(\sqrt{\frac{k\Delta j}{\pi}}(1 - \hat{R}_1 \cdot \hat{\ell}_j)\right)}{\sqrt{1 - \hat{R}_1 \cdot \hat{\ell}_j}} \right. \\ &\quad \left. + \frac{Fres\left(\sqrt{\frac{k\Delta j}{\pi}}(1 + \hat{R}_1 \cdot \hat{\ell}_j)\right)}{\sqrt{1 + \hat{R}_1 \cdot \hat{\ell}_j}} \right] \end{aligned} \tag{17}$$

where the vectors are those depicted in Fig. 2 and $Fres(x)$ is defined by the Fresnel integrals $S(x)$ and $C(x)$ as

$$Fres(x) = C(x) - jS(x) \tag{18}$$

If back-scattering is neglected (i.e., $Z_{ij} \approx 0$ for $j > i$), then the equivalent currents can be recursively calculated from the forward scheme [2]

$$M_i = \frac{1}{Z_{ii}} \left(V_i - \sum_{j=1}^{i-1} Z_{ij} M_j \right),\ i = 1, 2, \ldots \tag{19}$$

instead from the full-matrix linear system of (15).

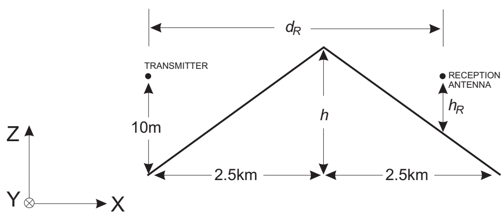


Fig. 3. Geometry of the wedge used in the simulations

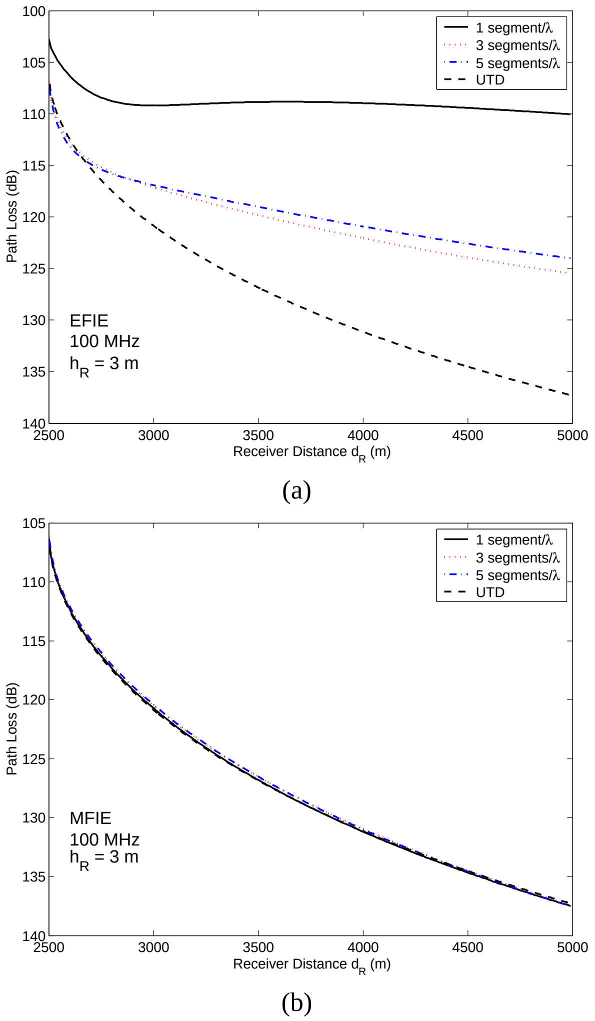


Fig. 4. Path-loss predictions at 100 MHz as a function of d_R for a) EFIE and b) MFIE

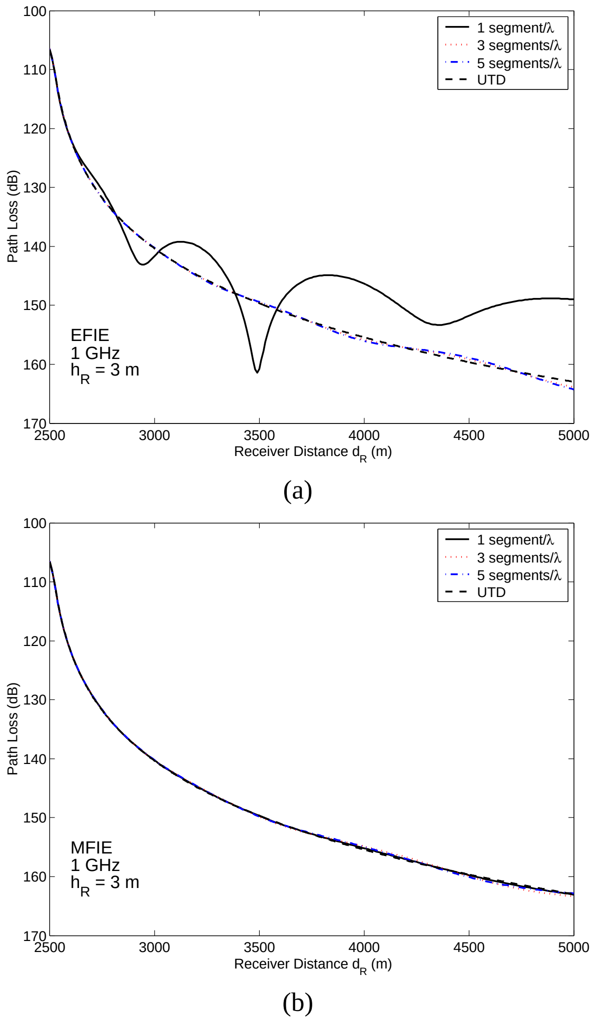


Fig. 5. Path-loss predictions at 1 GHz as a function of d_R for a) EFIE and b) MFIE

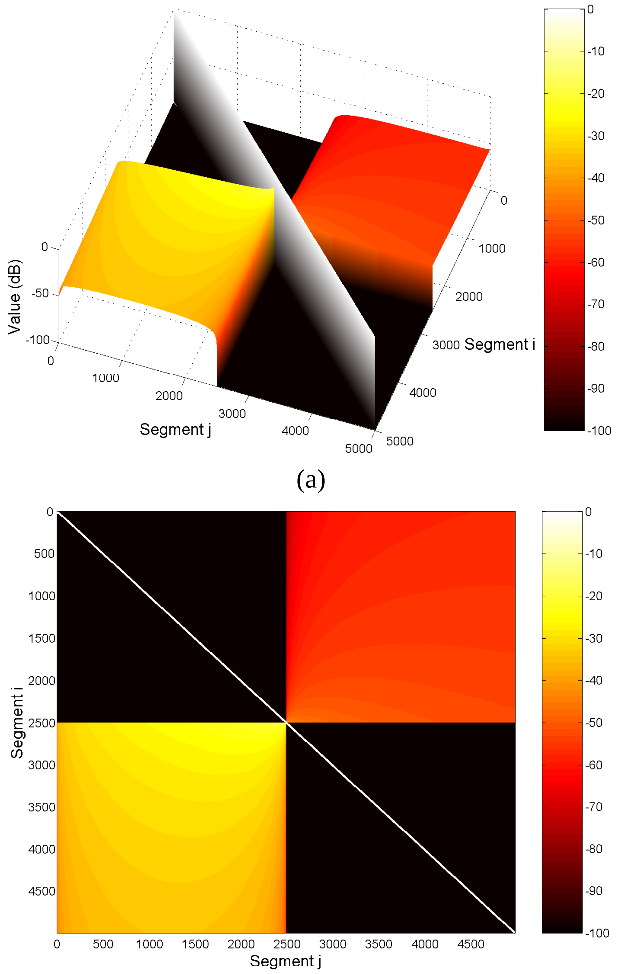
IV. RESULTS

In this section we will present the results for a theoretical irregular terrain profile, whose geometry is depicted in Fig. 3 together with the important geometrical parameters. The wedge's height was fixed at $h = 50$ m. The transmitting antenna's height was fixed at $h_T = 10$ m. The receiver's height was fixed at $h_R = 3$ m above the terrain profile and the distance between the antennas (d_R) was varied from 2500 to 5000 m (see Fig. 3). Numerical results for the path-loss prediction as a function of d_R were established with the help of (16)–(19) (i.e., the equivalent magnetic currents were recursively obtained) at 100 MHz and 1 GHz, using a vertical Hertz dipole as the transmitting antenna.

Figure 4 shows the path-loss predictions at 100 MHz for both the EFIE and MFIE, for different number of basis functions (i.e., segments) per wavelength (λ). Using the UTD (*Uniform Theory of Diffraction*) as reference, one immediately observes that the MFIE provides excellent path-loss predictions with just 1 basis function per λ, while 5 basis functions per λ are not sufficient to ensure the convergence of the EFIE-based procedure. Actually, one can show that the EFIE results in Fig. 4(a) never converge even if the number of basis functions per λ is increased.

In Fig. 5 we have the results at 1 GHz, still for the same triangular profile of Fig. 3. Again, we observe that the MFIE converges with just 1 basis functions per λ, while the EFIE provides reasonable results (with minor oscillations) with 3 basis functions per λ. To give a better picture of the practical importance of the faster convergence of the MFIE in the treatment of the radio wave propagation over irregular terrains, even for those cases where the recursive algorithm of (19) is applied, an increase in the number of segments (i.e., basis functions) by a factor N corresponds to an increase of N^2 in time. So, for the 1 GHz results depicted in Fig. 5, the MFIE provided an accurate prediction approximately 9 times faster than the EFIE.

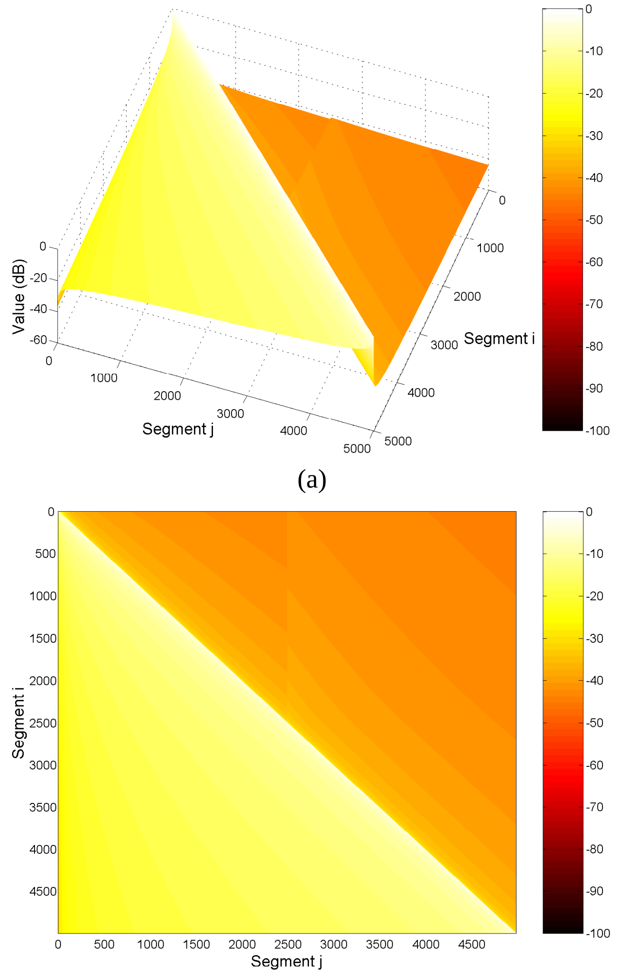
To try to explain the best convergence of the MFIE when compared to the EFIE, a study was made with the elements of the impedance matrix (Z_{ij}). In the forward scheme of (19), only the lower triangular portion of the Z-matrix (i.e., $i \leq j$) is considered [2]. This is equivalent to neglect the electromagnetic coupling between currents m_i and m_j for $j > i$ (i.e., no significant influence from the back-scattering, as the terrain profile is assumed electrically smooth).



(a)

(b)

Fig. 6. Normalized magnitude of the EFIE Z-matrix elements at 100 MHz: (a) 3D and (b) 2D representations



(a)

(b)

Fig. 7. Normalized magnitude of the MFIE Z-matrix elements at 100 MHz: (a) 3D and (b) 2D representations

To investigate the convergence problems of the EFIE, (16) and (17) were applied to obtain the full Z-matrix of the wedge profile of Fig. 3 at 100 MHz. The normalized absolute values of the Z_{ij} elements are illustrated in Figs. 6 and 7 for the EFIE and the MFIE, respectively. The black areas in Fig. 6 represent $Z_{ij} = 0$, characterizing a null coupling between the corresponding magnetic currents. This was expected because in such regions $\hat{n}_i \cdot \hat{R}_2 = 0$ in (16), as source and observer are on the same side of the wedge. Consequently, one observes from Fig. 6 that at a given row i, there are elements of the upper triangular portion of the Z-matrix (i.e., Z_{ij} with $j > i$) which are more significant than some of the lower triangular portion (i.e., Z_{ij} with $i > j$). That arouses concerns regarding the application of the forward scheme to evaluate the EFIE, as in such procedure the Z_{ij} elements with $j > i$ are neglected. In turn, from Fig. 7 one observes that such concerns do not exist when the same forward scheme is applied to evaluate the MFIE.

V. Conclusions

This work presented a study concerning the convergence of the EFIE and MFIE applied on the radio-wave pathloss prediction over irregular terrains for a vertical polarization and near-grazing incidence. It was verified that the convergence of the MFIE is archived with less basis functions when compared to the EFIE. The explanation for the poorer convergence of the EFIE-based procedure may be based on the small electric-field coupling between the surface equivalent magnetic currents, which probably makes the neglect of the terrain back-scattering a major source of error when a forward scheme is applied to evaluate the currents.

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