

A Ray-Tracing Algorithm Based on a Double-Tree Image Theory

Daniela N. Schettino and Fernando J. S. Moreira

Abstract— A novel ray-tracing algorithm based on the image theory is presented. The main idea is the use of the images of both transmitter and receiver to establish the corresponding images' trees and the pertinent ray paths. Although it may seem a paradox, the implementation of both trees, in some cases, actually reduces the total amount of stored data, as the last level of the transmitter's tree adopted in conventional algorithms is not necessary. Some case studies are present to demonstrate this feature.

Keywords— Electromagnetic propagation, geometrical optics, ray tracing.

I. INTRODUCTION

The growing demand of mobile communications, specially at urban areas, leads toward the adoption of the micro-cellular concept. It is then desirable to perform accurate coverage predictions upon such micro-cells in order to minimize on-site measurements. Statistical prediction models can show considerable error, as they do not account for effects of radio wave propagation around buildings and similar obstacles. However, deterministic techniques can be used to overcome such shortcomings. Some techniques have been proposed in the last decades, but those based on ray tracing and the uniform theory of diffraction (UTD) seem to be the most efficient for the problems at hand [1]–[6].

Basically, there are two approaches for tracing rays throughout the radio propagation channel: the shooting-and-bouncing ray (SBR) technique and the image theory (IT). The IT is more rigorous than the SBR as it is capable of determining all ray-path components without redundancy [1]. However, the IT technique has two major drawbacks. One is the amount of computer memory necessary to store the tree corresponding to the successive transmitter's images (also known as the virtual sources), specially when several multi-path components (including reflections, transmissions, and diffractions) are necessary to fully characterize the radio channel and also when a large number of segments (in 2-D problems) or flat patches (3-D) are on demand to represent the surrounding environment. The second problem is the time spent to determine the pertinent ray paths from the information contained in the tree and from the geometry describing the environment. This is basically due to the recursive com-

putation of the associated specular points, which slows down the evaluation of the ray path.

However, to our best knowledge, no attention has been paid to the fact that, based on Fermat's principle and for reciprocal media (which is an excellent assumption for practical purposes), a certain ray path from the transmitter to the receiver is exactly the same as one would obtain if the transmitter and receiver were interchanged. So, the main objective of this work is to present a ray-tracing algorithm based on the IT where the images' trees of both transmitter and receiver are used to establish the necessary ray-path components that characterize the associated multi-path phenomenon.

Although it may seem a paradox, when both trees are used, the necessary number of virtual sources to be calculated and stored, in some cases, is reduced. This is due to the fact that the last tree level (as adopted by the conventional algorithms which only use the tree generated by the transmitter) is not necessary to determine the ray paths, which will become clearer once the algorithm is presented in the next section. As the data of different receivers' trees can be stored in the same workspace (there is no need to retain such data once the coverage prediction for a certain location is evaluated), the total allocated memory is tremendously reduced, specially for environments with a large number of obstacles and, consequently, multi-path components.

II. THE ALGORITHM

The objective of this section is to describe the double-tree IT algorithm and how to use it to establish the ray paths. For the sake of simplicity, a 2-D model is assumed, where the geometry of the environment is described by straight segments. However, the main concepts of the present algorithm can be readily extended to 3-D problems or quasi ones [2]. Only reflections are considered in here, but the effects from wedge diffraction can be straightforwardly regarded by assuming the corresponding diffraction points as virtual receivers or transmitters for those rays reaching or leaving such points [2]. Radio transmission through walls can also be accounted for with minor modifications in the algorithm [3]. The present algorithm can also be combined with any kind of accelerating technique, including those used to obtain the successive images [4] and those used to reduce the number of profile segments to be checked in the shadowing analysis of a ray path [5]. Finally, the double-tree concept may also benefit those techniques based on ray tubes [6].

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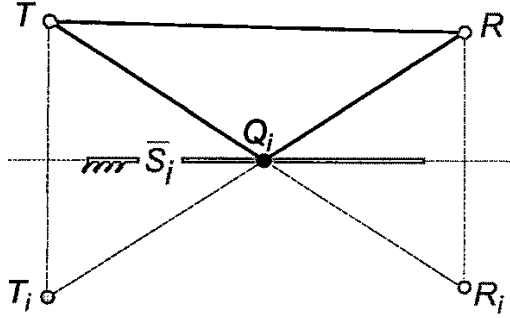


Fig. 1. Direct and one-reflection ray paths.

The algorithm will be presented on a step-by-step basis in order to properly introduce the notation.

A. Images' Trees

The first step is to determine the trees corresponding to the successive images (virtual sources) of both the transmitter (T) and the receiver (R), which can be accomplished by simple vector algebra and an appropriate coordinate system. Any kind of accelerating technique can be adopted when necessary, like those presented in [4] and [5]. For the proper ordering of such virtual sources, the segments representing the environment profile are numbered from 1 to N (the total number of segments). A certain i -th segment is described as \bar{S}_i . The image of T through \bar{S}_i is described as T_i . Furthermore, the image of T_i through \bar{S}_j is described as T_{ij} (such that $i \neq j$) and so forth. A similar notation is applied to those images corresponding to the receiver by just substituting T by R .

B. Direct and One-Reflection Ray Paths

To determine if the line-of-sight (LOS) is present or not, one just need to check if the segment \overline{TR} is obstructed by any one of the profile segments (see Fig. 1).

If both T_i and R_i are present in their corresponding trees, then it is possible to exist a ray path that is reflected just once and exactly by \bar{S}_i , as depicted in Fig. 1. The existence of such path is confirmed if $\overline{T_i R}$ (or $\overline{TR_i}$) intersects \bar{S}_i (at the specular point Q_i , which can be easily determined from simple vector algebra) and if neither $\overline{TQ_i}$ nor $\overline{Q_i R}$ are intersected by the remaining profile segments \bar{S}_j (where $j \neq i$).

C. Two-Reflection Ray Path

If both T_i and R_j (with $i \neq j$) are present, then it is possible to exist a ray path that reaches R after being reflected by \bar{S}_i (first reflection) and \bar{S}_j (second reflection), as illustrated in Fig. 2. The ray path really exists if the next requirements are completely fulfilled: the segment $\overline{T_i R_j}$ must intersect both \bar{S}_i (at the specular point Q_i) and \bar{S}_j (at Q_j); then one must check if $\overline{T_i R_j} \cdot \overline{Q_i Q_j} > 0$. This condition is necessary to avoid false-reflection scenarios

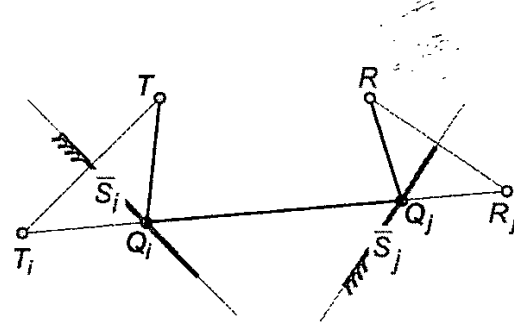


Fig. 2. Two-reflection ray path.

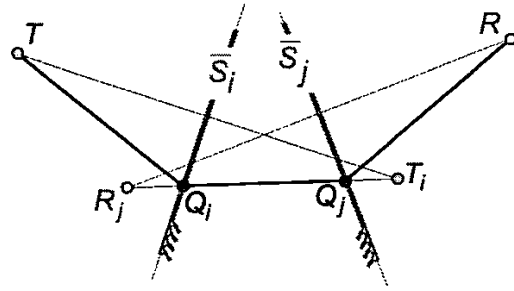


Fig. 3. False-reflection scenario: $\overline{T_i R_j} \cdot \overline{Q_i Q_j} < 0$

like the one presented in Fig. 3. Finally, $\overline{TQ_i}$, $\overline{Q_i Q_j}$, and $\overline{Q_j R}$ can not be obstructed by any one of the remaining profile segments.

Note that the two-reflection component is fully determined without the need of T_{ij} (or R_{ji}), what would be the case if one adopts an IT technique based solely on a tree generated by T (or R). Furthermore, Q_i and Q_j can be independently determined, enabling the parallel computation of both. If just the transmitter's tree was used, then one would need to obtain Q_j first and use it to finally obtain Q_i .

D. Three-Reflection Ray Path

If both T_{ik} and R_{jk} (with $i \neq k$ and $j \neq k$, but i can be equal to j) are present, then it is possible to exist a ray path that reaches R after being reflected by \bar{S}_i (first reflection), \bar{S}_k (second), and \bar{S}_j (third), as depicted in Fig. 4. To ensure the existence of such ray component the following requirements must be fulfilled. The segment $\overline{T_{ik} R_{jk}}$ must intersect both \bar{S}_i (at Q_i) and \bar{S}_k (at Q_k), while $\overline{T_{ik} R_j}$ must intersect both \bar{S}_k (at the very same Q_k) and \bar{S}_j (at Q_j). Then one must check if $\overline{T_i R_{jk}} \cdot \overline{Q_i Q_k} > 0$ and $\overline{T_{ik} R_j} \cdot \overline{Q_k Q_j} > 0$, in order to avoid false-reflection scenarios similar the one depicted in Fig. 3. Finally, $\overline{TQ_i}$, $\overline{Q_i Q_k}$, $\overline{Q_k Q_j}$, and $\overline{Q_j R}$ can not be obstructed by any one of the remaining profile segments.

A remarkable feature is that the three-reflection component is fully determined without the need of T_{ikj} or R_{jki} . Also, Q_i , Q_k , and Q_j can be determined via a parallel

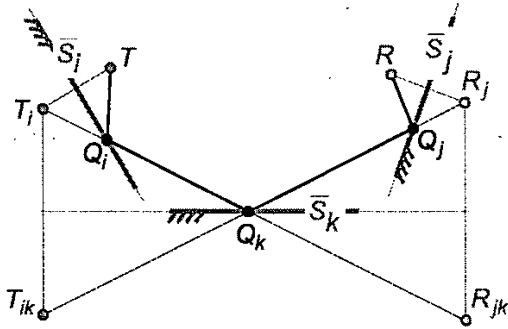


Fig. 4. Three-reflection ray path.

computation (with a smart coding, the redundant calculation of Q_k can be easily avoided). If the single tree of the transmitter was adopted, besides the need of T_{ikj} , the specular points would need to be recursively calculated.

E. Several Reflections

The present algorithm can be readily extended to account for ray-path components going through several reflections by simple induction from the previous cases. For instance, a four-reflection ray path may occur if both T_{ikl} and R_{jlk} ($i \neq k$, $k \neq l$, $l \neq j$, and no restrictions otherwise) are present, corresponding to a ray that may reach R after being reflected through \bar{S}_i , \bar{S}_k , \bar{S}_l , and \bar{S}_j , successively. If such ray really exists, Q_i corresponds to the intersection between \bar{S}_i and $\overline{T_i R_{jlk}}$, Q_k corresponds to the intersection between \bar{S}_k and $\overline{T_i R_{jlk}}$ (or $\overline{T_{ik} R_{jl}}$), Q_l corresponds to the intersection between \bar{S}_l and $\overline{T_{ik} R_{jl}}$ (or $\overline{T_{ikl} R_j}$), and Q_j corresponds to the intersection between \bar{S}_j and $\overline{T_{ikl} R_j}$. Note that all these specular points can be independently calculated. Tests similar to those presented for the two- and three-reflection cases must be conducted to avoid false-reflection scenarios and to check if the total path is obstructed by any one of the remaining profile segments.

III. IMPORTANT FEATURES

The use of the double-tree algorithm still sustains an easy determination of the ray paths, as in a conventional single-tree technique. The search starts from a simple comparison between the same levels in both trees. For example, the presence of T_{ik} and R_{jk} (one already has $i \neq k$ and $j \neq k$ if the trees were constructed rightly) in the third levels (regarding T and R as the first ones) of their corresponding trees indicates that a three-reflection component reflected through \bar{S}_i , \bar{S}_k , \bar{S}_j , successively, is possible.

A feature, that may be very attractive for those cases dealing with a large number of specular reflections, is that the specular points can be found independently from each other, by the (possible) intersection between the corresponding profile segment and the one generated by points

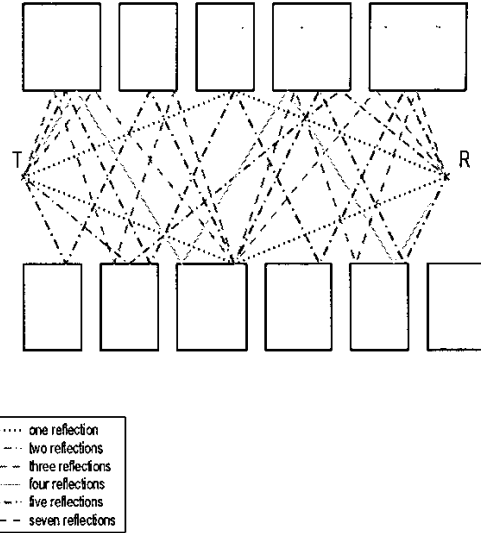


Fig. 5. Seven-reflection in a simplified 2-D environment.

contained in both trees at different levels, following a reasonable ordering. This fact enables the parallel computation of such points, which may be an important factor in a complex multi-path environment. For instance, regarding the three-reflection case depicted in Fig. 4, Q_j corresponds to the intersection between \bar{S}_j and $\overline{T_{ik} R_j}$, such that R_j is present in the receiver's tree as R_{jk} is. On the other hand, Q_i , given by the intersection between \bar{S}_i and $\overline{T_i R_{jk}}$, can be determined independently from Q_j . Finally, the vector algebra necessary to obtain the specular points and to verify if the total path is obstructed by the profile is basically the same as presented in a conventional single-tree algorithm.

However, the main advantage is that, for some specific cases, the use of both trees tremendously reduces the total amount of virtual sources, as the last tree level of the single-tree technique is not necessary, assuming that different receiver's trees can be temporarily stored in the same workspace. Two examples of such cases will be presented in the next section.

IV. NUMERICAL RESULTS

In order to demonstrate the capability of the double-tree algorithm in reducing the amount of stored data, two case studies are investigated in this section. Their geometries are depicted in Figs. 5 and 6, which may correspond to 2-D simplified models of outdoor environments. Both the single-tree IT and the double-tree IT were applied for each case study, always seeking for ray-path components up to seven reflections.

In Fig. 5, all the valid ray paths are being indicated (without considering possible diffractions), except for the

V. CONCLUSIONS

This work presented a new ray-tracing technique suited for wave propagation prediction. The method is based on the image theory where the images generated by both transmitter and receiver (double-tree algorithm) are employed to obtain the ray-path components. A 2-D version of the algorithm was presented, but the concepts are readily extendable to 3-D problems. Also, only multiple reflections were considered, but wedge diffraction can be easily incorporated as in the conventional single-tree technique. The major advantages of the double-tree IT are the tremendous reduction of the total amount of virtual sources, for some specific profiles, necessary for the deterministic prediction (specially for highly complex multipath environments) and the fact that the specular points associated to a certain ray path can be independently determined, enabling a parallel processing of such points.

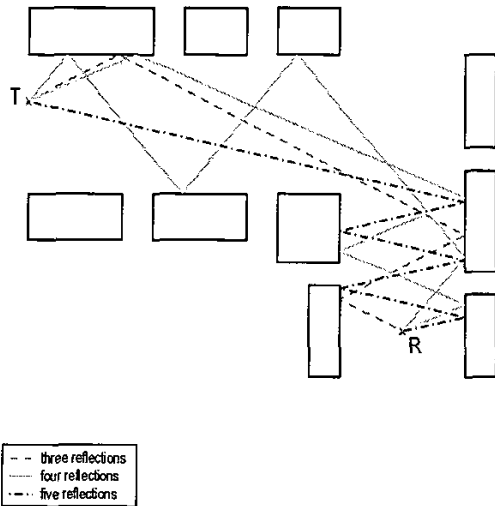


Fig. 6. Seven-reflection in a simplified 2-D environment.

direct-ray component. One also observes that there is no valid six-reflection component. For the second case study depicted in Fig. 6, although the algorithms were supposed to account for up to seven reflections, only three-, four- and five-reflection components are present, while the direct ray component is obstructed.

The ray paths provided by both techniques were, obviously, the same (as illustrated in Figs. 5 and 6); but the number of images used by the double-tree algorithm to trace such rays was quite smaller. For instance, in order to account for up to seven-reflection components, the single-tree IT technique required a total of 363,101 and 387,843 virtual sources for the configurations proposed in Figs. 5 and 6, respectively. However, for the double-tree algorithm, these numbers decreased to 132,202 and 134,690, respectively. The decrease in the amount of virtual sources would be more dramatic in a more complex environment (say, a 3-D one), specially when wedge diffractions should be considered.

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