GENERALIZED CLASSICAL AXIALLY-SYMMETRIC
DUAL-REFLECTOR ANTENNAS

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A. Introduction

Classical axially-symmetric Cassegrain and Gregorian reflectors are widely used in high-gain antenna applications [1]. The main disadvantage of these configurations is the subreflector blockage, which causes a number of deleterious effects. However, this problem can be reduced by decreasing the main-reflector radiation toward the subreflector. This may be accomplished either by shaping both reflectors [2] or by using alternative classical configurations [3]. This work considers the second option by presenting, in an unified way, generalized classical axially-symmetric configurations that prevent, from a Geometrical Optics (GO) stand point, the main-reflector scattered energy from striking the subreflector surface. Starting from initial design variables, closed-form expressions are derived for the relevant surface parameters, as well as for the corresponding aperture field distributions. These expressions can be used as effective design tools to determine the final antenna geometry, or even to establish an initial configuration for a shaping procedure.

B. Generalized Classical Axially-Symmetric Configurations

There are four different generalized classical axially-symmetric configurations that avoid the main-reflector scattering toward the subreflector. Their generating curves and relevant parameters are depicted in Figs. 1–4. They are obtained from GO concepts by imposing an uniform-phase field distribution over the antenna aperture (assumed at the plane z = 0), starting from a spherical-wave feed source at the antenna focus (located at the coordinate-system origin). The reflector surfaces are yield by spinning the generating curves about the z-axis (symmetry axis). From the figures, \(D_M\) and \(D_S = 2|X_S|\) are the main- and subreflector diameters, respectively, where \(X_S\) is the z-coordinate of the subreflector rim. \(D_R\) is the blockage diameter, which sets the z-coordinate of the main-reflector lower point. \(V_M\) and \(V_S\) are, respectively, the z-coordinates of the main- and subreflector points corresponding to the principal ray. \(F\) is the focal length of the parabola generating the main reflector and \(2c\) and \(e\) are, respectively, the inter-focal distance and eccentricity of the hyperbola or ellipse generating the subreflector. \(\theta_E\) is the subreflector edge angle and \(\beta\) is the tilt angle between the z-axis and the axis of the subreflector generating conic. The angle \(\theta\) shown in the figures defines an arbitrary feed-ray direction in the plane \(y = 0\). In this work, positive (negative) angular values correspond to counterclockwise (clockwise) angles in the plane shown in Figs. 1–4.

Geometries I (Fig. 1) and II (Fig. 2) have feed-rays with \(\theta > 0\) mapped at the aperture region with \(z > 0\). However, the subreflector edge ray (\(\theta_E > 0\)) is mapped at \(z = D_M/2\) for Geometry I and \(z = D_R/2\) for Geometry II. These two geometries are discussed in Ref. [3]. Geometries III (Fig. 3) and IV (Fig. 4) have feed-rays with \(\theta < 0\) mapped at the aperture region with \(z > 0\). However, the subreflector edge ray (\(\theta_E < 0\)) is mapped at \(z = D_M/2\) for Geometry III and \(z = D_R/2\) for Geometry IV.

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In all these configurations the main-reflector generating curve is a parabola, while the subreflector generating curve is a hyperbola for Geometries I and IV and an ellipse for Geometries II and III. The feed is located at one of the hyperbola/ellipse foci and the parabola focus coincides with the other hyperbola/ellipse focus. The basic parameters of the four antenna configurations are summarized in Table 1.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Parabola</td>
<td>Parabola</td>
<td>Parabola</td>
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<tr>
<td>Subreflector</td>
<td>Hyperbola</td>
<td>Ellipse</td>
<td>Ellipse</td>
<td>Hyperbola</td>
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<tr>
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<tr>
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<td>$\theta_L$</td>
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</table>

Table 1: Parameters of the generalized geometries.

In Geometries I and IV the hyperbola can be convex ($e > 1$) or concave ($e < -1$), and $|e| \to \infty$ yields a straight line. In all geometries $V_S$ is a positive quantity and $V_M$ can be either positive or negative. Note that great care should be taken in designing Geometries III and IV to prevent the subreflector reflected rays from intersecting the feed and/or the subreflector surface.

C. Design Equations

For design purposes, the reflectors’ generating curves of Figs. 1–4 can be conveniently established from the starting parameters $D_M$, $D_S = 2|X_S|$, $D_B$, $\theta_E$, and $\ell_o$, where $\ell_o$ is the total path length from the feed to the antenna aperture (note that $\ell_o/2$ is approximately equal to the distance between the main- and subreflector surfaces). From Figs. 1–4 and the conic-section equations one obtains (for all geometries)

$$\tan \left( \frac{\theta_1}{2} \right) = \frac{D_1}{2\ell_o} , \quad \tan \left( \frac{\theta_2}{2} \right) = \frac{D_2 - 2X_S}{2\ell_o - 2X_S \tan(\theta_E/2)} ,$$

$$\tan \beta = \frac{\sin \theta_E - \sin \theta_2 + \sin(\theta_E + \theta_2)}{\cos \theta_E + \cos \theta_2 - \sin(\theta_E + \theta_2)/\tan(\theta_E/2)} , \quad \begin{cases} \beta < 0, & \text{Geom. I and IV} \\ \beta > 0, & \text{Geom. II and III} \end{cases}$$

$$V_S = \frac{X_S \sin(\theta_E + \theta_2) \sin(\beta + \theta_1)}{\sin \theta_E \sin \theta_1 \sin(\beta + \theta_2)} , \quad V_M = V_S - \frac{D_1}{2 \tan \theta_1} ,$$

$$2e = \frac{V_S \sin \theta_1}{\sin(\beta + \theta_1)} , \quad e = \frac{\sin \theta_1}{\sin \beta + \sin(\beta + \theta_2)} , \quad F = \frac{D_1 - 4e \sin \beta}{4 \tan(\theta_E/2)} ,$$

where $D_1$, $D_2$, $\theta_1$, and $\theta_2$ are defined in Table 1. The commonly encountered classical Cassegrain and Gregorian configurations can be derived from Geometries I and III, respectively, by taking the limit $D_B \to 0$ in Eqs. 1–4 to obtain
\[
\begin{align*}
\theta_L = \beta = 0, \quad \tan \left( \frac{\theta_U}{2} \right) &= \frac{D_M - 2X_S}{2\varepsilon - 2X_S \tan(\theta_B/2)}, \\
V_S &= \frac{X_S}{2} \left( \cot(\theta_E/2) - \tan(\theta_U/2) \right), \quad V_M = V_S - \frac{\varepsilon}{2}, \\
2c &= \frac{2V_S \sin(\theta_E + \theta_U)}{\sin \theta_U - \sin \theta_E \sin(\theta_E + \theta_U)}, \quad e = \frac{2c}{2V_S - 2c}, \quad F = 2c - V_M.
\end{align*}
\]

D. GO Aperture Field Distribution

For design purposes, most of the relevant radiation characteristics of the above configurations can be obtained from the GO aperture distribution. The GO fields are derived by spinning the generating curves of Figs. 1–4 about the z-axis and noting that the astigmatic tube of rays leaving the subreflector has two principal centers of curvature: one located at the ring caustic defined by the rotated parabola focal point and the other located at the intersection of the reflected ray with the z-axis. Assuming the feed radiation as

\[
\vec{E}_f(R_f, \theta_f, \phi_f) = [E_\phi(\theta_f, \phi_f) \hat{\phi}_f + E_\theta(\theta_f, \phi_f) \hat{\theta}_f] \frac{\exp(-jkR_f)}{R_f},
\]

where \(\vec{E}_f\) is the radiated electric field and \(R_f, \theta_f,\) and \(\phi_f\) are the usual observation point spherical coordinates associated with the feed system, the GO electric-field Cartesian components \(E_x\) and \(E_y\) at the plane \(z = 0\) (aperture) can be shown to be

\[
\begin{bmatrix}
E_x(\rho_A, \phi_A) \\
E_y(\rho_A, \phi_A)
\end{bmatrix} = \begin{bmatrix}
\cos \phi_A & -\sin \phi_A \\
\sin \phi_A & \cos \phi_A
\end{bmatrix} \begin{bmatrix}
\cos(\phi_f - \phi_A) E_\theta(\theta_f, \phi_f) \\
\cos(\phi_f - \phi_A) E_\phi(\theta_f, \phi_f)
\end{bmatrix} \times \exp(-jk\varepsilon + j\Psi) \frac{\tan(\theta/2)}{4F(e^2 - 1)} \left[ A_1 (1 + \cos \theta) - A_2 \sin \theta \right]^{1/2},
\]

where

\[
A_1 = 1 - e \cos \beta, \quad A_2 = (e A_1 + e F) \sin \beta, \quad A_3 = F (1 + e \cos \beta) + e A_2 \sin \beta,
\]

\(\rho_A\) and \(\phi_A\) are the cylindrical coordinates of the aperture point and \(\Psi\) is the Gouy phase shift (equal to 0, \(\pi/2, \pi,\) and \(3\pi/2\) for Geometries I, II, III, and IV, respectively). The values of \(\theta, \theta_f,\) and \(\phi_f\) are given by

\[
\tan \left( \frac{\theta}{2} \right) = \frac{\sin \beta + \Delta \cos \beta - \Delta}{e (\cos \beta - \Delta \sin \beta) + 1}, \quad \Delta = \frac{\rho_A - 2c \sin \beta}{2F},
\]

\[
\theta_f = [\theta, \phi_f = \begin{cases}
\phi_A & \text{when } \theta > 0 \text{ (Geometries I and II)}, \\
\phi_A + \pi & \text{when } \theta < 0 \text{ (Geometries III and IV)}.
\end{cases}
\]

E. Conclusions

This work presented closed-form expressions for the surface parameters and aperture field distributions of all possible classical axially-symmetric dual-reflector antenna configurations, including the ones where a blockage diameter is incorporated to reduce the main-reflector scattered energy that strikes the subreflector surface. These expressions are useful design tools and permit considering classical axially-symmetric configurations in an unified and simple way.
References


Fig. 1 - Geometry I.

Fig. 2 - Geometry II.

Fig. 3 - Geometry III.

Fig. 4 - Geometry IV.