A PREDICTOR-CORRECTOR ALGORITHM FOR THE NUMERICAL EVALUATION OF REFLECTOR ANTENNA RADIATION INTEGRALS

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A. Introduction

One of the most widely used techniques for determining the radiation characteristics of reflector antennas is the Physical Optics method (PO) [1]. This technique requires the evaluation of complex integrals of the form

$$I = \iint_{S'} A(s')\, e^{-jkB(s')}\, ds', \tag{1}$$

where S' is the illuminated region of the reflector surface, $k = 2\pi/\lambda$, and $A(s')$ and $kB(s')$ are real functions describing the amplitude and phase of the integrand, respectively. $A(s')$ and $B(s')$ depend on the illuminating field, the observation point location, and the geometry of the reflector surface. To evaluate this integral a number of numerical methods have been used. Of particular relevance to this work is Simpson's algorithm (SA) [2], the Ludwig's algorithm (LA) [3], and the Crabtree's algorithm (CA) [4].

The SA is a nested scheme, representative of the numerical methods that do not take into account specific properties of the integrand (e.g., Newton-Cotes, Gaussian quadratures, etc.). The LA and CA use to their advantage the fact that the above integrand has in general a rapidly varying phase. They then divide the integration domain S' into a mosaic of cells, approximate the amplitude and phase of the integrand independently on each cell by functions that are integrable in closed form, and evaluate the integral by summing the results of each individual cell. The difference between the LA and the CA rests on the approximating functions used for $A(s')$ and $B(s')$, which are linear and biquadratic expressions, respectively.

A difficulty encountered when evaluating the above integral numerically is the fact that the phase $kB(s')$ can only be determined within a multiple of 2π radians, and this is not sufficient for the LA and CA. This ambiguity can be overcome by considering the path lengths involved in a particular geometry. However, whenever this is done, the algorithms lose their generality. Below a predictor-corrector scheme that eliminates this problem is presented, enabling both algorithms to maintain their inherent generality. The effectiveness of the CA, combined with the predictor-corrector scheme, is then investigated by applying it to a reflector antenna PO scattering computation example.

B. The Crabtree's Algorithm

The SA and LA are well known and the reader is referred to Refs. [2] and [3] for detailed information on them. However, since the CA is relatively recent, we briefly describe its operation below.

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To evaluate the radiation integral using CA, the reflector antenna surface is divided into a mosaic of rectangular cells by constructing a rectangular grid on suitable integration variables u and v. The radiation integral can then be expanded as

$$I = \sum_{c=1}^{N_{cell}} \iint_{cell} A_c(u,v)\, e^{-jkB_c(u,v)}\, du\, dv, \tag{2}$$

where N_{cell} is the total number of cells. Each cell contains 9 points where the integrand is evaluated, as depicted in Fig. 1. Over each cell the integrand amplitude is then approximated by the biquadratic expansion

$$A_c(u,v) = C_1u^2v^2 + C_2u^2v + C_3uv^2 + C_4u^2 + C_5v^2 + C_6uv + C_7u + C_8v + C_9, \tag{3}$$

and the phase by the incomplete biquadratic expansion

$$B_c(u,v) = D_1u^2 + D_2v^2 + D_3u + D_4v + D_5, \tag{4}$$

where C_i and D_l ($i = 1,2,...,9$ and $l = 1,2,...,5$) are the expansion coefficients. These two expansions lead to integrals that can be reduced to Fresnel's integrals, for which efficient numerical algorithms are available.

Note that the above integration technique can be used with any system of orthogonal coordinates, provided that the appropriate Jacobian determinant is incorporated into the amplitude term $A_c(u,v)$.

C. The Predictor-Corrector Algorithm

The basic idea of the predictor-corrector is to estimate (predict) the amplitude and phase of the integrand at each cell node and then to use these estimated values to eliminate (correct) any $n2\pi$ radians ($n = 0, \pm1, \pm2, ...$) ambiguity present. The predictor step is done using a biquadratic expansion (similar to Eq. 3) to extrapolate both A_c and B_c from one cell to another. The corrector step is then accomplished by comparing the predicted and actual values and adding $n2\pi$ radians to the actual phase value to bring it as close as possible to the predicted value. Care must be exercised with the zero crossings of A_c, by allowing it to become negative and treating the phase accordingly (this is the reason for having a predictor-corrector procedure on both A_c and B_c). Note that, for the predictor-corrector to work, it is crucial to start with a sufficiently small cell to guarantee that, inside the cell, the A_c amplitudes are not zero and kB_c changes by less than π radians from any node to its adjacent. Using this initially small cell the integrand values can be predicted and corrected in progressive larger adjacent cells, until any desired cell size is attained (see Fig. 2). Using geometrical optics it can be shown that the phase of the integrand at any two adjacent nodes does not differ by more than $4\pi\ell/\lambda$ radians, where ℓ is the physical distance between the nodes. An initial cell of sides of $0.1\,\lambda$ constitutes then a sufficiently small cell to start the predictor-corrector process.

D. Numerical Results and Conclusion

Since the present work is primarily concerned with evaluating the performance of the combination CA plus predictor-corrector algorithm, numerical simulations were performed on radiation integrals having a rectangular integration domain. The case of an axially-symmetric hyperboloidal reflector with eccentricity $e = 2.5$ and interfocal distance of $20\,\lambda$, illuminated by a feed located in its farthest focus, was selected as a test geometry. The projection of the reflector rim on a plane perpendicular to its axis is a square with sides of 10λ. The feed is a perfectly linearly polarized spherical wave source radiating an amplitude pattern described by $\cos^E(\theta_F)$, where θ_F is the angle measured from the feed boresight direction and $E = 38$ (this feed produces -10 dB of illumination taper at the reflector rim center). Typical electric field scattering results (feed direct radiation not included), evaluated using the three algorithms considered in this paper, are shown in Figs. 3-6.

Figs. 3, 4, and 5 depict the behavior of the SA, LA, and CA as the number of nodes is increased. N_S, N_L, and N_C are the number of nodes along the reflector side, for each algorithm, respectively—the total number of nodes, which is identical to the number of integrand evaluations, is then equal to either N_S^2, N_L^2, or N_C^2. In these figures the actual phase $kB(s')$ of the integrand was obtained using geometrical optics, and no predictor-corrector algorithm was then necessary. For good accuracy these figures show that $N_S^2 = 961$, $N_L^2 = 169$, and $N_C^2 = 81$ suffices, demonstrating the fact that the CA requires half the number of integrand evaluations of LA.

Fig. 6 shows the performance of CA combined with the predictor-corrector algorithm. In this case the integrand was evaluated as real and imaginary parts, and the task of determining the amplitude phase was left to the predictor-corrector algorithm. As expected, accurate results are obtained with $N_C^2 = 81$. Note that, with ambiguous amplitude and phase information, both LA and CA are incapable of performing the integral, a limitation that is not shared by SA.

This work demonstrates that CA yields a significant saving on the number of integrand evaluations as compared to the efficient LA. It also presents a predictor-corrector technique for using the CA without having to handle the integrand phase separately. An important advantage of using a predictor corrector algorithm is the fact that, by comparing the predicted and corrected integrand values, it is possible to perform a self check of the integration algorithm accuracy and hence determine when its results are reliable. The combination CA plus predictor-corrector then yields a very general, accurate, and numerically efficient technique for evaluating reflector antenna radiation integrals. As a final comment it should be mentioned that CA requires the computation of Fresnel's integrals, and hence is inherently more time consuming than LA. However, whenever the integrand evaluation requires considerable computational effort (e.g., array feed, computation of the main-reflector scattering in a dual-reflector system, etc.), the increase in computational burden of CA is generally not significant.

References

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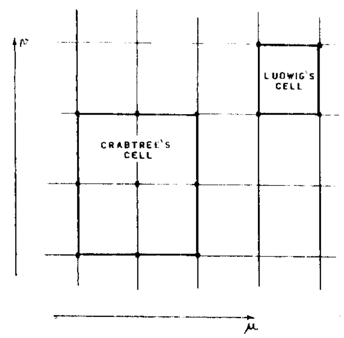

Fig. 1 – Ludwig's and Crabtree's integration cells.

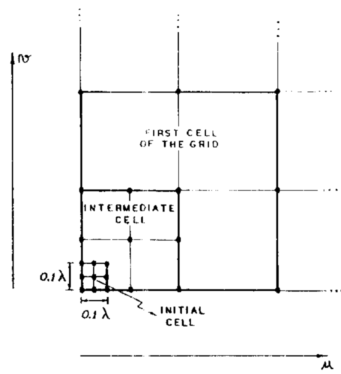

Fig. 2 – Predictor-corrector start-up cells.

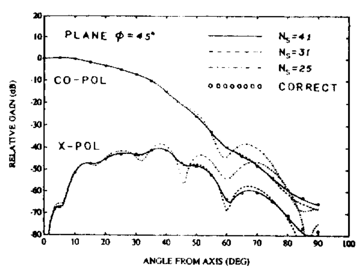

Fig. 3 – Reflector scattering, SA.

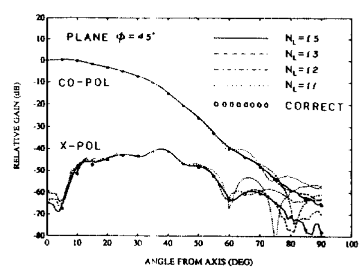

Fig. 4 – Reflector scattering, LA.

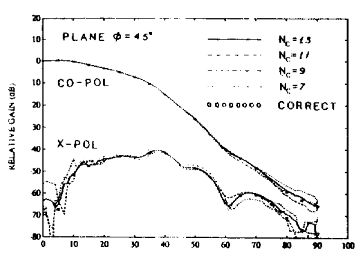

Fig. 5 – Reflector scattering, CA.

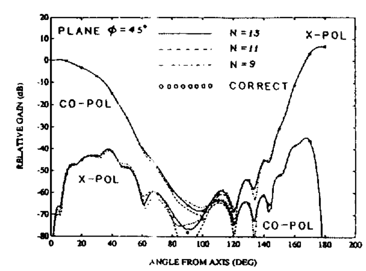

Fig. 6 – Reflector scattering, CA plus predictor-corector.