

A Plug-and-Play DGTD Implementation for General Dispersive Media

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Abstract—A discontinuous Galerkin implementation is presented for accurately calculating the electromagnetic scattering by general dielectric dispersive media in the time domain. To validate the formulation, the two-dimensional scattering of a TMz plane wave by a single pole Lorentz dispersive cylinder has been analyzed. The Fourier transform of the numerical solution in the time domain was compared with the analytical solution in the frequency domain at a given frequency. The results obtained are in agreement with the analytical solution.

Keywords—Discontinuous Galerkin, electromagnetic scattering, dispersive media.

I. INTRODUCTION

This study extends the Discontinuous Galerkin Time-Domain (DGTD) framework applied to Maxwell's equations [1] to accommodate general dispersive dielectric media using the Complex Conjugated Pole-Residue (CCPR) model [2] and solve electromagnetic scattering problems.

Differently from the standard approach, this implementation is modular, allowing for the selective update of regions containing dispersive media and absorbing layers with additional Auxiliary Differential Equations (ADEs). The domain truncation is performed using a uniaxial Perfectly Matched Layer (PML) [3], while incident fields are introduced using the Total Field Scattered Field (TFSF) technique [4].

The formulation is validated in two dimensions by analyzing the electromagnetic scattering of a TMz plane wave by a dispersive dielectric cylinder [5], ensuring the proposed approach's accuracy and reliability.

II. THEORETICAL FOUNDATION

Maxwell's curl equations for lossless isotropic linear media without sources [5] are defined as

$$\mu(\mathbf{r}) \frac{\partial \mathcal{H}(\mathbf{r}, t)}{\partial t} + \nabla \times \mathcal{E}(\mathbf{r}, t) = 0, \quad (1)$$

$$\epsilon(\mathbf{r}) \frac{\partial \mathcal{E}(\mathbf{r}, t)}{\partial t} - \nabla \times \mathcal{H}(\mathbf{r}, t) = 0, \quad (2)$$

where \mathcal{E} and \mathcal{H} are the electric and magnetic fields in the time-domain, respectively. Additionally, ϵ and μ are the electric permittivity and the magnetic permeability, respectively. Furthermore, \mathbf{r} and t indicate the spatial and temporal dependencies, respectively.

In conservation form, as required by the DGTD method, the aforementioned equations become [6]

$$Q(\mathbf{r}) \frac{\partial \vec{q}(\mathbf{r}, t)}{\partial t} + \nabla \cdot \vec{\mathcal{F}}(\vec{q}) = 0, \quad (3)$$

where

$$\vec{q}(\mathbf{r}, t) = \begin{bmatrix} \mathcal{H}(\mathbf{r}, t) \\ \mathcal{E}(\mathbf{r}, t) \end{bmatrix}, \quad Q(\mathbf{r}) = \begin{bmatrix} \mu(\mathbf{r}) & 0 \\ 0 & \epsilon(\mathbf{r}) \end{bmatrix},$$

$$\vec{\mathcal{F}}(\vec{q}) = [\vec{\mathcal{F}}_1, \vec{\mathcal{F}}_2, \vec{\mathcal{F}}_3]^\top, \quad \vec{\mathcal{F}}_i(\vec{q}) = \begin{bmatrix} \mathbf{e}_i \times \mathcal{E}(\mathbf{r}, t) \\ -\mathbf{e}_i \times \mathcal{H}(\mathbf{r}, t) \end{bmatrix},$$

with $\vec{\mathcal{F}}$ being the flux and \mathbf{e}_i representing the three Cartesian unit vectors.

A. Dielectric Dispersive Media Modeling

A general dispersive dielectric material can be modeled by considering its permittivity $\epsilon(\omega)$ as $\epsilon_0 \epsilon_\infty + \epsilon_0 \chi_e(\omega)$, where ϵ_∞ is the high frequency permittivity and $\chi_e(\omega)$ is the frequency dependent electric susceptibility. Specifically, using the CCPR model, $\chi_e(\omega)$ can be expanded as

$$\chi_e(\omega) = \sum_{r=1}^R \left(\frac{c_r}{j\omega - \alpha_r} + \frac{c_r^*}{j\omega - \alpha_r^*} \right), \quad (4)$$

where R is the number of poles, c_r and α_r are the model's parameters that can be obtained from data fitting.

This dispersion model can be incorporated via the electric constitutive equation using the ADE approach [2] resulting in

$$\nabla \times \mathcal{H}(\mathbf{r}, t) = \epsilon_0 \epsilon_\infty \frac{\partial \mathcal{E}(\mathbf{r}, t)}{\partial t} - 2\epsilon_0 \sum_{r=1}^R \Re \left\{ \frac{\partial \mathcal{P}_r(\mathbf{r}, t)}{\partial t} \right\}, \quad (5)$$

$$\frac{\partial \mathcal{P}_r(\mathbf{r}, t)}{\partial t} = a_r \mathcal{P}_r(\mathbf{r}, t) + c_r \mathcal{E}(\mathbf{r}, t), \quad (6)$$

where (2) is updated with an extra term and an additional ADE of a polarization field \mathcal{P}_r for every pole r is solved within the dispersive media.

B. Plug-and-Play Implementation

The approach adopted in this study extends the standard DGTD solution of (3) to address scattering problems. This extension involves applying techniques that enable the correction of field equations within specific regions using additional terms and ADEs, as illustrated in (5) and (6).

In a manner akin to the treatment of dispersive media, a uniaxial PML implemented through ADEs [3] was employed and the TFSF technique was used to introduce incident fields [4].

III. NUMERICAL RESULTS

In order to validate the presented formulation, the problem of a TMz plane wave scattered by a dispersive cylinder was analyzed. The schematic representation of the problem is illustrated in Figure 1.

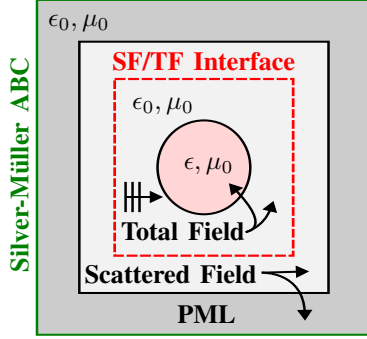


Fig. 1. Diagram of the analyzed study case.

In the two-dimensional case with TMz propagation, the set of differential equations to be solved simplify to the following system of equations

$$\frac{\partial \tilde{\mathcal{H}}_x}{\partial t} = -\frac{\partial \tilde{\mathcal{E}}_z}{\partial y} + (\tilde{\sigma}_x - \tilde{\sigma}_y) \tilde{\mathcal{B}}_x, \quad (7)$$

$$\frac{\partial \tilde{\mathcal{H}}_y}{\partial t} = \frac{\partial \tilde{\mathcal{E}}_z}{\partial y} - (\tilde{\sigma}_x - \tilde{\sigma}_y) \tilde{\mathcal{B}}_y, \quad (8)$$

$$\epsilon_\infty \frac{\partial \tilde{\mathcal{E}}_z}{\partial t} = \tilde{\nabla} \times \tilde{\mathcal{H}} \cdot \hat{z} - 2\Re \left\{ \frac{\partial \tilde{\mathcal{P}}_1}{\partial t} \right\} - \tilde{\sigma}_x \tilde{\mathcal{D}}_z - \tilde{\sigma}_y \tilde{\mathcal{E}}_z, \quad (9)$$

$$\frac{\partial \tilde{\mathcal{P}}_1}{\partial t} = \alpha_1 \tilde{\mathcal{P}}_1 + c_1 \tilde{\mathcal{E}}_z, \quad (10)$$

$$\frac{\partial \tilde{\mathcal{B}}_x}{\partial t} = -\frac{\partial \tilde{\mathcal{E}}_z}{\partial y} - \tilde{\sigma}_y \tilde{\mathcal{B}}_x, \quad (11)$$

$$\frac{\partial \tilde{\mathcal{B}}_y}{\partial t} = \frac{\partial \tilde{\mathcal{E}}_z}{\partial x}, \quad (12)$$

$$\frac{\partial \tilde{\mathcal{D}}_z}{\partial t} = \tilde{\nabla} \times \tilde{\mathcal{H}} \cdot \hat{z} - \tilde{\sigma}_y \tilde{\mathcal{D}}_z, \quad (13)$$

where \mathcal{B} , \mathcal{D} and σ come from the uniaxial PML [3]. It is pertinent to highlight that the fields, as described, are normalized [1] and may be categorized as either total or scattered, depending on the region in which they are defined.

In the simulation, an incident field was imposed using a cosine-modulated Gaussian waveform [7] traveling in the \hat{x} direction with center frequency $f_c = 300\text{MHz}$ (free-space wavelength $\lambda_0 \approx 1\text{m}$) and time constant $\tau = 1/600 \cdot 10^{-6}\text{s}$. The cylinder's radius corresponds to $0.5\lambda_0$, the total field region has dimensions $2.4\lambda_0 \times 2.4\lambda_0$, the scattered field region's width is $0.75\lambda_0$ and the PML's width is $0.50\lambda_0$.

A dispersive material with α_1 and c_1 corresponding to a single pole Lorentz model [7] with parameters $\epsilon_s = 2.0$, $\epsilon_\infty = 1.0$, $\omega_1 = 2\pi \times 375 \times 10^6$ rads and $\delta_1 = 0.3\omega_1$ was used and the PML parameters [3] σ_{\max} and p were respectively set to 150 and 2.

The simulation was carried out for $0.05\mu\text{s}$ using 4th order elements in a mesh with 308 triangular elements for 1252 time steps. Figure 2 shows the absolute value of the Fourier transform of the electric field time-domain solution at 300MHz. At this frequency, within the total field region, the relative L2 error was 1.85% comparing to the analytical solution.

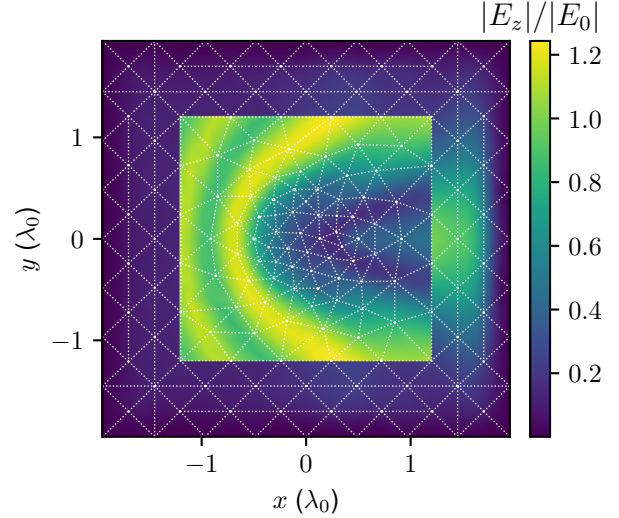


Fig. 2. Absolute value of the electric field normalized by the incident field.

IV. CONCLUSION

The standard DGTD method was extended to modularly accommodate general dispersive dielectric media using the CCPR model. The formulation was validated by analyzing the scattering of a TMz plane wave by a dispersive cylinder. The results obtained are in agreement with the analytical solution.

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