

# Equivalent edge currents for the time-domain analysis of reflector antennas

\*Cássio G. Rego<sup>1</sup>, Flávio J. V. Hasselmann<sup>2</sup> and Fernando J. S. Moreira<sup>1</sup>

<sup>1</sup>Universidade Federal de Minas Gerais, Depto. Engenharia Eletrônica, Av. Pres. Antônio Carlos, 6627, CEP 30161-970, Belo Horizonte – MG, Brazil, cassio@cpdee.ufmg.br

<sup>2</sup>Pontifícia Universidade Católica, CETUC, Rua Marquês de São Vicente, 225, CEP 22453-900, Rio de Janeiro – RJ, Brazil, flavio@cetuc.puc-rio.br

## 1 Introduction

Many methods for the determination of the scattered fields radiated by pulsed sources have been developed to provide solutions to the design of reflector antennas suited to short pulse radar applications. To be more practical, such methods must provide solutions directly in the time domain. There exist various asymptotic methods which are suitable for analytical studies of time-domain electromagnetic scattering, such as the time-domain uniform geometrical theory of diffraction (TD-UTD) introduced by Pathak and Rosseau [1, 2] and the time-domain physical optics (TDPO) introduced by Rusch and Sun [3]. The implementation of these techniques, and the development of a time-domain version of the uniform asymptotic theory of diffraction (TD-UAT), alongside with fringe wave currents (TD-FWCs) to correct the imperfections of the TDPO near surface edges, have been used in practical applications within the framework of reflector antennas illuminated by pulsed sources [4, 5].

It is a well-known fact that ray tracing methods fail in the determination of diffracted fields at caustic regions. This difficulty is a commonplace in the analysis of axially symmetric reflectors, where the observation points may be located over the antenna axis. This shortcoming can be overcome using equivalent edge currents (TD-EECs) obtained from a Fourier inversion of the formulation developed by Michaeli in the frequency domain [6]–[9]. These currents can be integrated over a reflector rim to provide the diffracted fields at any observation point, regarding that all possible singularities on the equivalent edge currents are removed [8, 9].

Recently, a work published by Altintas and Russer has introduced equivalent edge currents in time domain [10]. However, this formulation intended for the analysis of the electromagnetic scattering from conducting plates and does not take into account phase effects upon the equivalent edge currents associated with the scatterer curvature. Furthermore, the formulation introduced by Michaeli in the frequency domain [9] considers the scatterers' curvatures near edges and, consequently, appears to be more appropriate for the reflector antenna analysis.

In the present work, the frequency domain formulation developed by Michaeli [6]–[9] is extended to the time domain through an analytical transformation. The associated time-domain equivalent edge currents are henceforth named TD-EECs. These currents are implemented to provide the solution of the field diffracted by the rim of a paraboloidal reflector illuminated by a double-Gaussian pulse departing from the antenna focus. In order to verify its accuracy and applicability, the TD-EEC formulation is applied for observers located at the antenna symmetry axis and in the sidelobes' region as well. All numerical results obtained from the TD-EEC are compared against those obtained from a method-of-moments (MoM) frequency analysis transformed into time domain by a discrete Fourier inversion.

## 2 Formulation of the electric field radiated by the equivalent edge currents

The electric field radiated by a TD-EEC corresponding to the field diffracted by the edge of a perfectly conducting surface, illuminated by an incident electromagnetic field represented by  $\vec{e}^i(\vec{r}'_E, t)$  and  $\vec{h}^i(\vec{r}'_E, t)$ , is given by

$$\vec{e}^{EEC}(\vec{r}, t) = \vec{e}^{FWC}(\vec{r}, t) + \vec{e}^{POC}(\vec{r}, t), \quad (1)$$

where the first term in the righthand side represents the field radiated by the fringe wave component of the equivalent currents [8, 9], expressed as

$$\begin{aligned} \vec{e}^{FWC}(\vec{r}, t) = & -\frac{1}{4\pi r} \int_{c'} \left\{ \left[ \vec{e}^i\left(\vec{r}'_E, \tau - \frac{|\vec{r}'_E|}{c}\right) \cdot \hat{i} \right] D_e^{I,J}(\beta'_0, \beta'_0, \phi'_0, \phi_0) + \right. \\ & \left. \left[ \vec{h}^i\left(\vec{r}'_E, \tau - \frac{|\vec{r}'_E|}{c}\right) \cdot \hat{i} \right] \eta_0 D_h^{I,J}(\beta'_0, \beta_0, \phi'_0, \phi_0) \right\} [\hat{i} - (\hat{i} \cdot \hat{a}_r) \hat{a}_r] dl' + \\ & \frac{1}{4\pi r} \int_{c'} (\hat{a}_r \times \hat{i}) \left[ \vec{h}^i\left(\vec{r}'_E, \tau - \frac{|\vec{r}'_E|}{c}\right) \cdot \hat{i} \right] \eta_0 D_h^{M,J}(\beta'_0, \beta_0, \phi'_0, \phi_0) dl'. \end{aligned} \quad (2)$$

The second term in (1) represents the field radiated by the physical optics (PO) component of the equivalent currents [8, 9] and is obtained from the real part of the analytical representation [1, 4, 5]:

$$\begin{aligned} \vec{e}^{POC}(\vec{r}, t) = & -\frac{1}{4\pi r c} \int_{c'} \frac{\partial}{\partial t} \left\{ f(t) * \left[ \vec{e}^i\left(\vec{r}'_E, \tau - \frac{|\vec{r}'_E|}{c}\right) \cdot \hat{i} \right] \right\} D_e^{I,PO}(\beta'_0, \phi'_0) dl' - \\ & \frac{\eta_0}{4\pi r c} \int_{c'} \frac{\partial}{\partial t} \left\{ f(t) * \left[ \vec{h}^i\left(\vec{r}'_E, \tau - \frac{|\vec{r}'_E|}{c}\right) \cdot \hat{i} \right] \right\} D_h^{I,PO}(\beta'_0, \beta_0, \phi'_0, \phi_0) [\hat{i} - (\hat{i} \cdot \hat{a}_r) \hat{a}_r] dl' \\ & - \frac{\eta_0}{4\pi r c} \int_{c'} (\hat{a}_r \times \hat{i}) \frac{\partial}{\partial t} \left\{ f(t) * \left[ \vec{h}^i\left(\vec{r}'_E, \tau - \frac{|\vec{r}'_E|}{c}\right) \cdot \hat{i} \right] \right\} D_h^{M,PO}(\beta_0, \phi_0) dl'. \end{aligned} \quad (3)$$

In (2) and (3),  $\eta_0$  is the free space intrinsic impedance,  $c$  is the velocity of light,  $\vec{r}$  and  $\vec{r}'_E$  locate the observation point and the integration point over the reflector rim ( $c'$ ), respectively, and  $\tau = t - \frac{|\vec{r} - \vec{r}'_E|}{c}$  is the time delay due to the distance between the observation point and the integration points. The symbol  $*$  in (3) denotes a time convolution and  $f(t)$  is a time-domain transition function that keeps the regularity of the PO component in the vicinity of shadow boundaries. Actually, it corresponds to the analytical inverse transform of the transition function expressed by Equation (19) of Reference [9]. The angles  $\beta_0$ ,  $\beta'_0$ ,  $\phi_0$  and  $\phi'_0$  are calculated as in [6] and the angular coefficients  $D_e^{I,I}$ ,  $D_h^{I,I}$ ,  $D_h^{I,M}$ ,  $D_e^{PO,I}$ ,  $D_h^{PO,I}$  and  $D_h^{PO,M}$  can be obtained by comparing the integrands in (2) and (3) with their frequency-domain counterparts throughout References [6]–[9].

## 3 Numerical results

Figure 1a shows the geometry of a paraboloidal reflector antenna, with  $D = 7.5$  m and  $F/D = 0.4$ , illuminated by a spherical wave from the antenna focus with temporal behavior defined by a double-Gaussian pulse as shown in Fig. 1b. The applicability of the TD-EEC in the analysis of the reflector-antenna scattering is demonstrated by Fig. 2, where the scattered field was obtained at observation points located well into the sidelobes' region. In the results showed in figure 2a the two components of diffracted field are separated by the larger time delay between them. Figure 2b shows the antenna response with the observer at the reflector

axis ( $\theta = 180^\circ$ ), which is a focal region for the diffracted rays. The results are compared with those provided by a reference solution based on a frequency domain MoM transformed into the time domain using an inverse Fourier transform algorithm (MoM+IFFT). All the figures show that the results provided by both methods are indistinguishable.

#### 4 Acknowledgements

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#### References

- [1] P. R. Rosseau and P. H. Pathak, "Time-domain uniform geometrical theory of diffraction for a curved wedge," *IEEE Transactions on Antennas and Propagation*, vol. 43, no. 12, pp. 1375–1382, December 1995.
- [2] P. R. Rosseau and P. H. Pathak, "TD-UTD slope diffraction for a perfectly conducting curved wedge," *Proc. IEEE Antennas and Propagation Society International Symposium*, Newport Beach, California, USA, June 1995, pp. 856–859.
- [3] E.-Y. Sun and W. V. T. Rusch, "Time-domain physical-optics," *IEEE Transactions on Antennas and Propagation*, vol. 42, no. 1, pp. 9–15, January 1994.
- [4] C. G. Rego and F. J. V. Hasselmann, "Time-domain analysis of pulse-excited reflector antennas," in *Proceedings of IEEE Antennas and Propagation Society International Symposium*, Salt Lake City, Utah, USA, July 2000, pp. 2046–2049.
- [5] C. G. Rego and F. J. V. Hasselmann, "Time-domain analysis of pulse-excited reflector antennas—UAT approach," in *Proceedings of IEEE Antennas and Propagation Society International Symposium*, Boston, Massachusetts, USA, July 2001, pp. 376–379.
- [6] A. Michaeli, "Equivalent edge currents for arbitrary aspects of observation," *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 3, pp. 252–258, March 1984.
- [7] A. Michaeli, "Correction to "Equivalent edge currents for arbitrary aspects of observation", *IEEE Transactions on Antennas and Propagation*, vol. 33, no. 2, pp. 227, February 1985.
- [8] A. Michaeli, "Elimination of infinities in equivalent edge currents, part I: fringe current components," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 7, pp. 912–918, July 1986.
- [9] A. Michaeli, "Elimination of infinities in equivalent edge currents, part II: physical optics components," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 8, pp. 1034–1037, August 1986.
- [10] S. Altintas and P. Russer, "Time-domain equivalent edge currents for transient scattering," *IEEE Transactions on Antennas and Propagation*, vol. 49, no. 4, pp. 602–606, April 2001.

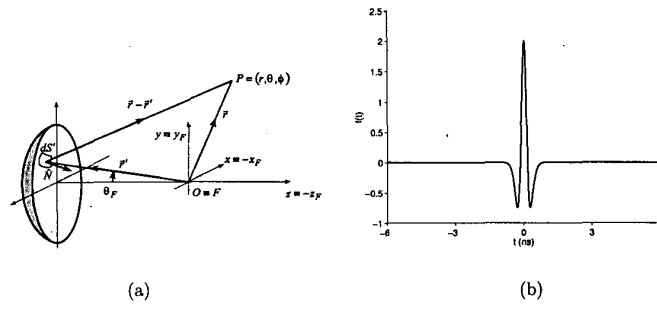


Figure 1: Geometry of the paraboloidal reflector and temporal behavior of the source.

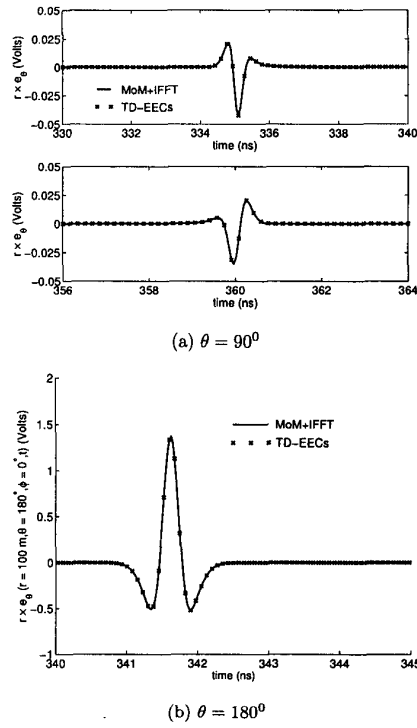


Figure 2: Reflector response at observer ( $r = 100 \text{ m}, \phi = 0^\circ$ ).