First-Order Aberrations on Generalized Classical Axially-Symmetric Dual-Reflector Antennas

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Abstract— **This work presents a first-order treatment of aberrations caused by small feed displacements in classical axially-symmetric dual-reflector antennas. A general and closed-form analysis (based on geometrical optics principles) of the aberrated aperture field is derived in terms of the reflector geometry and feed illumination. The consequences to the aperture efficiency and main-beam tilt are illustrated for axially-displaced-ellipse configurations.**

I. Introduction

Recent developments on personal communication services demand the use of high-gain antennas with very compact geometries (e.g., wireless internet access). A suitable configuration for such applications is the axially-displaced-ellipse (ADE) reflector antenna, which yields high gain levels with smaller lengths between sub- and main-reflectors. The ADE is an axially-symmetric dual-reflector configuration and its classical (conic) generating curves are depicted in Fig. 1. One of the first authors to investigate the ADE was Yerukhimovich [1]. Since then, the ADE has been used in many applications (e.g., [2]). Recently, a closed-form and general design procedure was derived for the four different families of classical axially-displaced configurations, including the ADE [3]–[6]. The formulation, based on geometrical optics (GO), also provides closed-form expressions for the field representation at the antenna aperture, enabling the parameterization of the output aperture ray in terms of the associated input feed ray.

The present work is concerned with the aperture aberration provoked by small feed displacements from the primary focus of classical axially-symmetric dual-reflector antennas. Such study is of great interest in the design of single-fed reflectors operating over a wide frequency range or even in applications involving feed-arrays.

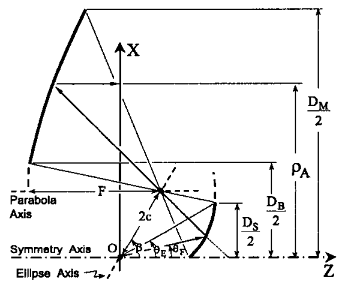

Fig. 1. ADE generating-curve configuration.

A first-order treatment is derived, partially based on previous works conducted by Dragone [7]. The present investigation is incited by the fact that Dragone's formulation can not be directly applied to the present antennas due to the curvature discontinuity at the subreflector vertex.

II. The Aberrated Aperture Field

There are four different kinds of classical axially-symmetric dual-reflector antennas: the axially-displaced Cassegrain (ADC), Gregorian (ADG), ellipse (ADE) and hyperbola (ADH) [3]–[6]. Their geometries are uniquely determined from five input parameters: the main-reflector diameter D_M, the subreflector diameter D_S, the blockage diameter D_B, the subreflector edge-angle θ_E, and the constant path length ℓ_o from the primary antenna focus (at the origin) to the aperture plane (at the plane $z = 0$). Some relevant parameters are depicted in Fig. 1 for the ADE. The classical Cassegrain and Gregorian configurations are obtained from the ADC and ADG, respectively, by setting $D_B = 0$.

The phase of the aperture field is pro-

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portional to the path length (V) from the feed phase center to the aperture (along the corresponding ray path), which can be described by means of a first-order treatment based on [7]:

$$V \approx \ell_o - [(x_o \cos\phi_F + y_o \sin\phi_F)\sin\theta_F + z_o \cos\theta_F], \quad (1)$$

where x_o, y_o, and z_o are the Cartesian coordinates locating the feed phase center and θ_F and ϕ_F are the usual spherical coordinates representing the feed-ray direction. Assuming small feed displacements, the amplitude and polarization of the aperture field can be assumed unaltered (i.e., as if $x_o = y_o = z_o = 0$) and are directly given by the formulation in [4].

In order to define the aberration terms, it is convenient to describe V in terms of the aperture polar coordinates ρ_A and ϕ_A (see Fig. 1). This is accomplished with the help of the relation between θ_F,ϕ_F and ρ_A,ϕ_A [4]:

$$\tan\left(\frac{\theta_F}{2}\right) = \frac{2\,A_3 - A_1\,\rho_A}{2\,A_4 - A_2\,\rho_A}, \quad (2)$$

$$|\phi_F - \phi_A| = \begin{cases} 0, & \text{ADC and ADE}, \\ \pi, & \text{ADG and ADH}, \end{cases} \quad (3)$$

where

$$A_1 = 1 - e\cos\beta, \quad (4)$$
$$A_2 = e\sin\beta, \quad (5)$$
$$A_3 = [c\,(1 - e\cos\beta) + e\,F]\sin\beta, \quad (6)$$
$$A_4 = F\,(1 + e\cos\beta) + c\,e\sin^2\beta, \quad (7)$$

F is the focal length of the main-reflector generating parabola, e and $2c$ are the eccentricity and the inter-focal distance of the subreflector generating ellipse/hyperbola, respectively, and β is the angle between the axes of the generating conics. All geometrical parameters are uniquely determined from the five input parameters [4]. Substituting (2) and (3) into (1) one obtains

$$V = \ell_o - z_o \sum_{n=0}^{\infty} B_n\,\rho_A^n - \epsilon\,(x_o\cos\phi_A + y_o\sin\phi_A)\sum_{n=0}^{\infty} C_n\,\rho_A^n, \quad (8)$$

where $\epsilon = 1$ for the ADC and ADE and $\epsilon = -1$ for the ADG and ADH. The coefficients B_n and C_n can be readily calculated with the help of (3.6.22) in [8].

In order to minimize the aperture-efficiency degradation caused by a certain feed displacement one should attempt to minimize the magnitude of the coefficients B_n and C_n in (8). It can be shown that

$$B_n, C_n \propto \frac{A_1\,A_4 - A_2\,A_3}{(A_3^2 + A_4^2)^{n+1}}, \quad \text{for } n \geq 1, \quad (9)$$

indicating [with the help of (4)–(7)] that the aberration diminishes as $|e| \to 1$. The condition $|e| = 1$ is never met in practice, as the subreflector generating conic tends to a parabola and, consequently, $2c \to \infty$. However, $|e| \to 1$ as $|\theta_E| \to 0$ [4]. Nevertheless, it is interesting to note that the present formulation is also applicable to the classical Cassegrain and Gregorian antennas, obtained from the ADC and ADG, respectively, with $D_B = 0$. For such antennas, $\beta = 0$ and, consequently, $A_2 = A_3 = 0$. From (4)–(7) it can be shown that, under these circumstances, $B_n = 0$ only for odd values of n, whereas $C_n = 0$ only for even n-values. In these cases, tilt (C_1), defocus (B_2), coma (C_3), and spherical (B_4) are the only possible primary aberrations, as expected [7].

It is important to note from (8) that the classical axially-symmetric dual-reflector antennas present aberration terms that are not observed in the usual axially-symmetric optical systems [9]. This is due to the discontinuity of the surface curvature at the subreflector vertex (see Fig. 1), which transforms the feed principal ray ($\theta_F = 0$) into a cylindrical locus of output rays. So, the one-to-one correspondence between input feed rays and output aperture rays fails for the principal ray. As a consequence, the bilinear transformation derived in [7] can not be applied to the present antennas.

III. Axial Feed Displacements

In the present section only axial feed displacements (i.e., $x_o = y_o = 0$) are considered. The theory of Sect. II was employed on a parametric investigation of the aperture efficiency (η_{ap}) decay with z_o [6]. This was accomplished by integrating the aberrated GO aperture field [with V directly given by (1)] to obtain the antenna gain (G_o) and the corresponding efficiency:

$$\eta_{ap} = G_o\,\lambda_o^2/(\pi\,D_M)^2, \quad (10)$$

where λ_o is the free-space wavelength. Here, only the results for the ADE are presented. A detailed explanation of the analysis and the results for the other antenna configurations are presented in [6].

Due to the adopted GO principles, similar antennas (antennas with identical angular dimensions) will have the same efficiency η_{ap} provided that both the feed illumination and z_o/λ_o remain unchanged. For this reason the parametric investigation was conducted with the antenna linear dimensions normalized to D_M and with $|z_o|$ expressed in terms of λ_o (the results are symmetric about $z_o = 0$). The values of $D_S = D_B = 0.1\,D_M$ were specified, aiming highly efficient antennas [5]. A linearly-polarized raised-cosine feed model (RCF) was adopted as the excitation and its exponent was adjusted for every single configuration in order to provide the maximum possible efficiency with a focused feed ($z_o = 0$) [5]. The resulting η_{ap} values are shown in Fig. 2 in terms of $|z_o|/\lambda_o$, θ_E, and ℓ_o/D_M. From the results one observes that η_{ap} decreases as $|z_o|$ increases, as expected. Also, the efficiency decay is less pronounced for smaller values of θ_E, as discussed in Sect. II. Furthermore, the variation of ℓ_o/D_M has a small impact on η_{ap}.

As GO does not account for diffraction mechanisms, the absolute values of η_{ap} depicted in Fig. 2 are not expected to be met but by reflector arrangements with very large electrical dimensions. However, the relative decay of η_{ap} can be reasonably predicted. To demonstrate this, some representative antenna configurations were analyzed by the moment-method (MoM) technique [6]. Only the results for two particular ADE antennas are presented here: ADE 1 (with $\theta_E = 20°$ and $\ell_o = 50\lambda_o$) and ADE 2 ($\theta_E = 30°$ and $\ell_o = 100\lambda_o$). Both have $D_M = 100\lambda_o$ and $D_B = D_S = 10\lambda_o$. In order to attain a more realistic comparison, the main-reflector generating parabola was extended toward the symmetry axis to obtain a main-reflector without an undesirable opening at its surface (see Fig. 1). The same RCF model was adopted, with exponents equal to 36.136 (ADE 1) and 15.796 (ADE 2). For $z_o \approx 0$, the present formulation (GO plus aperture method) estimates $\eta_{ap} \approx 91\%$ for both geometries (see Fig. 2), while the MoM gives $\eta_{ap} \approx 76\%$

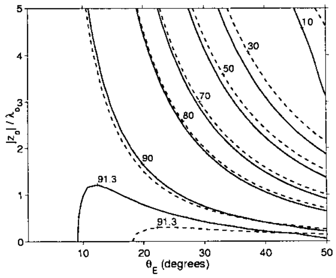

Fig. 2. ADE η_{ap} variation (values in percent) with $|z_o|/\lambda_o$ and θ_E: $\ell_o/D_M = 0.5$ (solid lines) and 1 (dashed lines).

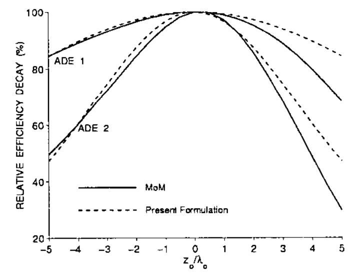

Fig. 3. Relative aperture efficiency decay for the ADE 1 and ADE 2. Results obtained from the MoM (solid lines) and the present formulation (dashed lines).

and 80% for ADE 1 and ADE 2, respectively. However, as demonstrated by the results of Fig. 3, the relative efficiency decay calculated from the present theory agrees reasonably well with the one obtained by the MoM. The agreement is better for the ADE 1, since the aberrations are less pronounced for smaller θ_E angles. The agreement deteriorates as z_o increases, as the feed approaches the subreflector and the adopted first-order treatment of aberrations is rapidly violated. Results for the other antenna configurations are presented in [6].

IV. Off-Axis Feed Displacements

A similar analysis can be conducted for off-axis feed displacements ($z_o = 0$), which can benefit applications where the present antennas are fed by arrays. The formulation not only permits parametric investigations of the aperture efficiency but also the approximate determination of the direction of maximum radiation, which is given by [7]

$$\sin\theta_\infty \, e^{j\,\phi_\infty} = -\,\frac{x_o + j\,y_o}{f_o}\,, \qquad (11)$$

where θ_∞ and ϕ_∞ are the spherical coordinates of the principal ray (maximum radiation direction) leaving the aperture and f_o is the equivalent focal distance for the classical axially-symmetric dual-reflector antennas, given by:

$$f_o \approx \frac{D_M}{8\epsilon}\left[\sum_{n=0}^{\infty}\frac{C_n}{3+n}\left(\frac{D_M}{2}\right)^n\right]^{-1}. \quad (12)$$

The previous equation was derived following the procedure in [7], but assuming an aperture field with a uniform amplitude and that $D_M \gg D_B \geq D_S$. For the classical Cassegrain and Gregorian antennas, it can be shown that (12) reduces to $f_o \approx F\,|(e+1)/(e-1)|$, as expected.

The values of f_o/D_M for several ADE geometries are depicted in Fig. 4, which shows that f_o does not heavily depend on D_S (assuming $D_S = D_B$) nor on ℓ_o (specially when $\ell_o > D_M$). Similar results where observed for the other three axially-displaced families. The present theory (GO plus aperture method) was further employed to investigate the main-beam tilt caused by the variation of x_o on the previously used ADE 2 geometry. The pattern cuts ($\phi = 0°$) are shown in Fig. 5. From it, the values of θ_∞ are $-0.26°$ and $-0.44°$ for $x_o = 1\lambda_o$ and $2\lambda_o$, respectively. From (12), with $n \leq 4$, the θ_∞ values are $-0.24°$ and $-0.48°$, respectively. One will find a better agreement for the ADE 1 geometry, as it presents a smaller θ_E angle.

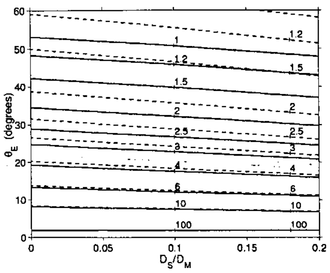

Fig. 4. ADE f_o/D_M variation with D_S/D_M ($D_B = D_S$) and θ_E: $\ell_o/D_M = 0.5$ (solid lines) and 1 (dashed lines).

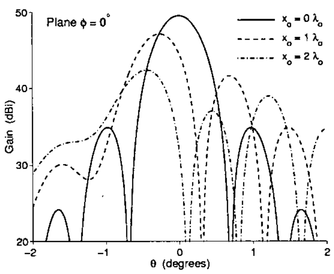

Fig. 5. Far-zone radiation patterns of the ADE 2 for different values of x_o.

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