

ASYMPTOTIC APPROXIMATIONS FOR THE MUTUAL COUPLING IN FINITE ARRAYS OF RECTANGULAR WAVEGUIDES

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The antenna for BRASILSAT has to operate over C-Band and has to provide beams for national and regional coverages. Antennas consisting of an offset parabolic reflector that is illuminated by a reconfigurable feed array pose as a natural solution for this problem. For the array, pyramidal horns are usually employed as feed elements. In the synthesis of the excitations of the elements, the mutual coupling effects can be considered by the analysis presented by Bird (1) that uses the integral equation and Green's function approach. The analysis is centered in the calculation of self- and mutual-admittances, that involves the numerical evaluation of double integrals. Thus, design relies on an efficient computation, specially when a large number of elements is employed.

In order to simplify computation, we introduce an asymptotic approximation in the expression for the mutual-admittance similar to the one suggest by Bird (2) in the analysis of mode coupling for a planar circular waveguide array. Here, the use of that approximate expression for the mutual-admittances is explored in the design of such antennas in conjunction with more stringent specifications for BRASILSAT.

MUTUAL COUPLING ASYMPTOTIC EXPRESSIONS

The mutual coupling analysis presented by Bird (1) assumes the rectangular apertures of the feeds embedded in an infinite ground plane and the aperture fields represented by a finite series of rectangular waveguide modes (TE and TM). For two apertures D_i and D_j placed in a plane, as depicted by Figure 1, the mutual admittance y_{ij} ($m,n;p,q$) of mode (m,n) in aperture i and mode (p,q) in aperture j is given by

$$y_{ij}(m,n,p,q) = [jk\pi \frac{y_0}{4}] [\alpha_{mn} \alpha_{pq}] [C_x I_x + C_y I_y + C_z I_z] \quad (1)$$

where the expressions for the parameters α_{uv} and $C_{x,y,z}$ can be found in Reference (1). The indexes in the integrals $I_{x,y,z}$ represent the following possible combinations of mode coupling:

$$I_{y,z} = \int_{D_1} \int_{D_2} G(x-x', y-y', z) \frac{\sin \left[\frac{m\pi x}{a_i} \right]}{\cos \left[\frac{m\pi x}{a_i} \right]} \frac{\sin \left[\frac{p\pi x'}{a_i} \right]}{\cos \left[\frac{p\pi x'}{a_i} \right]} dx' dy' dz'$$

$$\times \frac{\cos \left[\frac{m\pi y}{a_i} \right]}{\cos \left[\frac{m\pi y}{a_i} \right]} \frac{\cos \left[\frac{q\pi y'}{a_j} \right]}{\cos \left[\frac{q\pi y'}{a_j} \right]} dS dS' \quad (2)$$

that involve two double integrals. Reduction of the order of these integrals is obtained by variable conversion resulting in expressions with double and single integrals for mutual- and self-admittances, respectively. For apertures that are separated by more than one wavelength, the Green's function can be expanded to terms of R where R is the distance between the centers of the apertures, as follows:

$$G(|r-r'|) \cong G(R_{ik}) \exp \left(-jk_0 f_1(\Delta x, \Delta y) \right) \times \left[1 - \frac{1}{R_{ik}} [f_1(\Delta x, \Delta y) + f_2(\Delta x, \Delta y)]^2 \left(\frac{jk_0}{2} + \frac{1}{2R_{ik}} - \frac{jk_0}{R_{ik}} f_1(\Delta x, \Delta y) \right) + \frac{k_0^2}{8R_{ik}} (f_2(\Delta x, \Delta y))^2 \right] + \frac{1}{R_{ik}^2} (f_1(\Delta x, \Delta y))^2 \quad (3)$$

$$f_1(\Delta x, \Delta y) = \Delta x \cos(\phi_{ik}) + \Delta y \sin(\phi_{ik}) \quad (4.a)$$

$$f_2(\Delta x, \Delta y) = \Delta x \sin(\phi_{ik}) + \Delta y \cos(\phi_{ik}) \quad (4.b)$$

$$\text{being } \Delta x = x - x', \quad \Delta y = y - y', \quad R_{ik}^2 = X_{ik}^2 + Y_{ik}^2,$$

$\phi_{ik} = \tan^{-1}(Y_{ik}/X_{ik})$, for X e Y described in Figure 1. Substitution of this asymptotic approximation into (2), it yields the following expressions for the integrals

$$I_{y,z} = G(R_{ik}) \left\{ E_0 F_0 - \frac{1}{R_{ik}} [E_1 F_0 \cot(\phi_{ik}) + E_0 F_1 \tan(\phi_{ik})] - E_1 F_1 \left[jk_0 + \frac{3}{R_{ik}} \right] + \frac{E_2 F_0}{2} \left[jk_0 + \frac{s_1(\phi_{ik})}{R_{ik}} \right] + \frac{E_2 F_2}{2} \left[jk_0 + \frac{s_2(\phi_{ik})}{R_{ik}} \right] \right\} + \frac{jk_0}{R_{ik}^2} \{ \cot(\phi_{ik}) (E_3 F_0 + E_1 F_2 s_2 \phi_{ik}) + \tan(\phi_{ik}) (E_0 F_3 + E_2 F_1 s_1 \phi_{ik}) \} - \frac{k_0^2}{8R_{ik}^2} [E_4 F_0 + E_0 F_4 - 4(E_3 F_1 + E_1 F_3) + 6E_2 F_2] \quad (5)$$

where

$$s_1(\phi_{ik}) = 1 - 2 \cot^2(\phi_{ik}) ; \quad s_2(\phi_{ik}) = 1 - 2 \tan^2(\phi_{ik}) \quad (6)$$

$$E_L = \sin^L(\phi_{ik}) \sum_{l=0}^L (-1)^l \frac{L!}{l!(L-l)!} IX \frac{s}{c} (l, L-1) \quad (7)$$

$$F_L = \cos^L(\phi_{ik}) \sum_{l=0}^L (-1)^l \frac{L!}{l!(L-l)!} IY \frac{c}{s} (l, L-1) \quad (8)$$

The index c, s in the functions IX and IY refers to the combinations mentioned before (2). These functions are given by

$$IX \frac{s}{c} (t, u) = \int_0^{a_k} \int_0^{a_l} \gamma^t \beta^u \frac{\sin \left[\frac{p_n \pi \alpha}{a_k} \right]}{\cos \left[\frac{p_n \pi \alpha}{a_k} \right]} \frac{\sin \left[\frac{p_m \pi \beta}{a_l} \right]}{\cos \left[\frac{p_m \pi \beta}{a_l} \right]} \times e^{jk_x(\alpha-\beta)} d\beta d\alpha \quad (9)$$

$$IY \frac{c}{s} (t, u) = \int_0^{b_k} \int_0^{b_l} \gamma^t \beta^u \frac{\cos \left[\frac{q_n \pi \alpha}{b_k} \right]}{\sin \left[\frac{q_n \pi \alpha}{b_k} \right]} \frac{\cos \left[\frac{q_m \pi \beta}{a_l} \right]}{\sin \left[\frac{q_m \pi \beta}{a_l} \right]} \times e^{jk_y(\alpha-\beta)} d\beta d\alpha \quad (10)$$

$$k_x = k_0 \cos(\phi_{ik}) ; \quad k_y = k_0 \sin(\phi_{ik}) \quad (11)$$

The above integrals can be solved analytically, leading to simple closed formulas for the mutual-admittances. Figures 2.a-d show a comparison between the values obtained for the coupling coefficients of the modes TE_{10} - TE_{10} and TE_{10} - TE_{01} in identical square waveguides, for two different relative positions. The full line curves refer to the values obtained by using the exact expressions and the dashed and dotted lines refer to the values obtained by using the terms of R_n to the second and third order, respectively.

DESIGN EXAMPLE

The antenna for BRASILSAT has to generate a national beam with a minimum illumination of 24 dBi within the national boundary and 27 dBi within the polygon (A) shown in Fig 4.a. It also has to provide a regional beam covering the frequency band of 3.625-3.7 GHz that has to illuminate the east coast with a minimum of 29 and 31 dBi within the polygons (B) and (C), respectively. Designs for this specification are reported in Reference 3. To illustrate the performance of the asymptotic approximations presented above, they were employed in the design of a shaped beam antenna that has to comply with a more stringent set of specifications for BRASILSAT. In order to explore the design possibilities for the national beam and to provide higher level of illumination over the areas of more intense traffic, it was required, besides the 24 and 27 dBi specifications, a minimum illumination of 29 and 31 dBi over the areas defined by the polygons (B) and (C), respectively.

The configuration considered in the study is composed of a feed array with 17 identical elements that illuminate an offset parabolic reflector with an aperture diameter of 1.8 meters as described in Fig.3. The 12 modes model (2) was used to expand the feed aperture fields and the feed excitations are synthesized by using a procedure described in Reference (3). To calculate the self-admittance and the mutual-admittance for modes in adjacent apertures the exact expression given by Bird (2) was used. The asymptotic approximation presented here was used to calculate the coupling between modes in non-adjacent apertures. It corresponds to apertures that are separated by more than 0.6λ (wavelength) in the down-link (3.74-4.2GHz) and more than 0.9λ for the up-link. By using the synthesized excitations for the feed array, the radiation patterns obtained by Physical Optics (PO) analysis are shown in Figures 4a-c and 5a-c. For the design at mid frequency of the up-link (6.15 GHz), the copolar radiation shown in Fig. 5.a meets the specifications and presents a copolar peak of 31.53 dBi. The crosspolar radiation shown in Fig. 5.b is below -34 db. For the down-link design, although the copolar radiation has concentrated energy on the east coast, the optimized pattern does not meet the goals as shown in the contour plots in Fig. 4.a for the mid-frequency (3.95 GHz). This limitation can be overcome by the use of a slightly larger reflector in the design ($D=2.2$ m).

To evaluate the approximation given by the expressions presented here, the radiation patterns were calculated by using the exact expressions for the admittances and the same feed excitations given by the synthesis. These patterns were compared to the ones in Figures 4.a-b and 5.a-b and the discrepancies between them for the crosspolar component are depicted in Figures 4.c and 5.c in dB referred to the copolar peak. It is observed that the differences are well below -38 dB. Although differences of -20 dB were found in the copolar pattern for the region considered in the figures, its shape is not affected, since the differences occur over the main coverage area.

Besides the accuracy furnished by the asymptotic approximations, computation time is reduced to one third of that one required to calculate the coupling by using the exact expressions.

REFERÊNCIAS

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2. Bird, T.S., 1979, IEE J. Microw. Opt. Acoust. (MOA), 5, 172-180
3. Moreira, F.J.S. and Bergmann, J.R., 1992, Report CETUC-PAA-04/92

This work was supported by TELEBRAS under contract PUC-TELEBRAS 415-91-JDpqD.

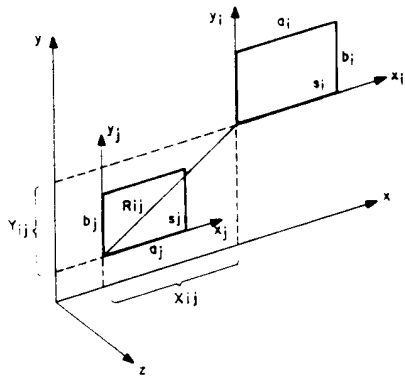


Fig. 1. Rectangular waveguides in ground plane

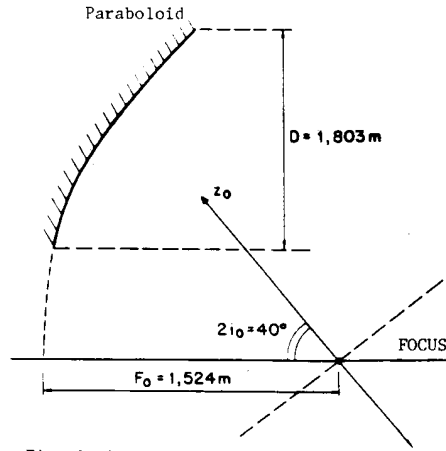


Fig. 3. Antenna geometry

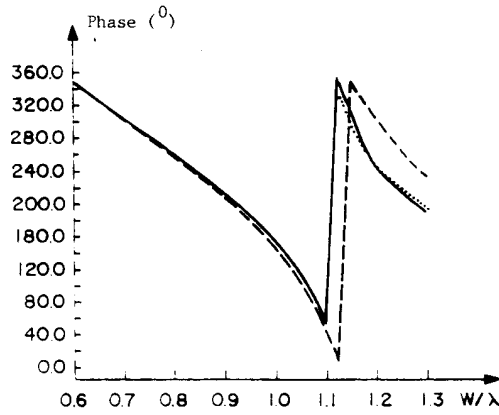
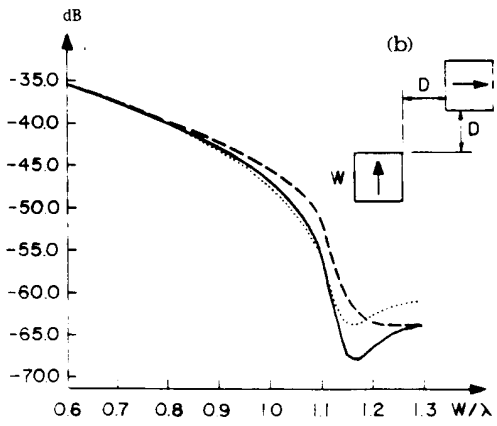
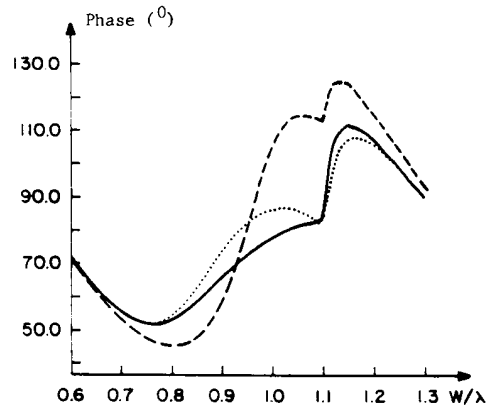
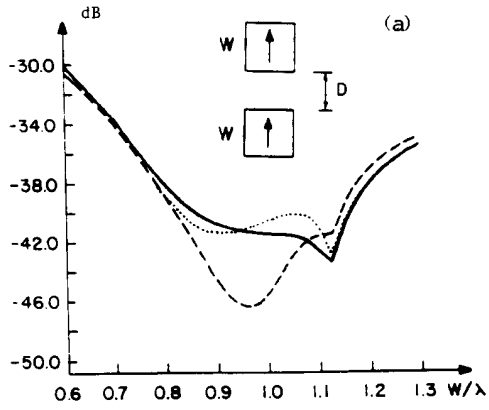
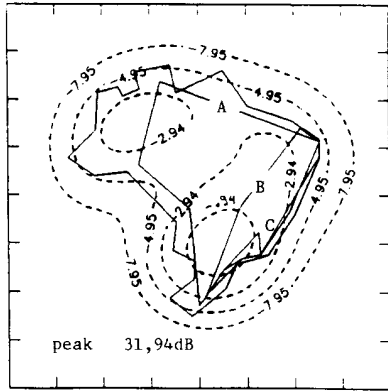
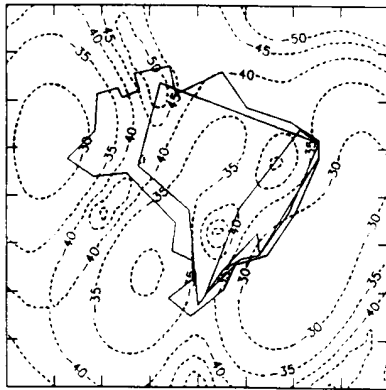


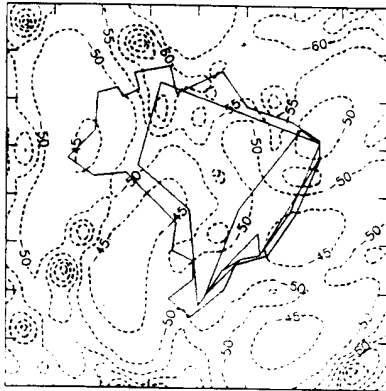
Fig. 2. Coupling coefficients of the TE_{10} - TE_{01} (a) and TE_{10} - TE_{10} (b)



(a)

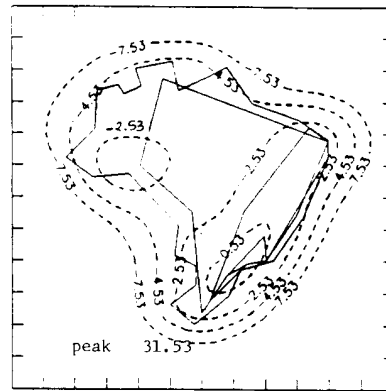


(b)

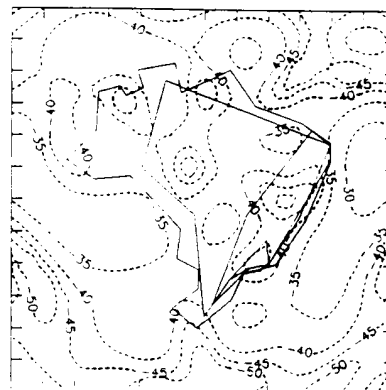


(c)

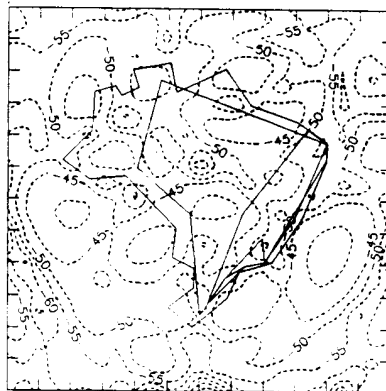
Fig. 4. Copolar (a) and Crosspolar (b) Radiation patterns, and crosspolar discrepancies at 3.95 GHz



(a)



(b)



(c)

Fig. 5. Copolar (a) and Crosspolar (b), Radiation Patterns, and Crosspolar discrepancies at 6.15 GHz