# Time-Domain Analysis of Classical Omnidirectional Axis-Displaced Dual-Reflector Antennas

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***Abstract*— This work presents an approach to obtain the time-domain electric far field radiated by classical omnidirectional dual-reflector antennas fed by a TEM coaxial horn. The time-domain formulation is developed by means of analytical inversion of the corresponding frequency domain solution obtained using geometrical optics (GO) principles and the aperture method.**

***Index Terms*—Reflector antennas, omnidirectional dual-reflector antennas, time-domain analysis..**

## I. Introduction

Communication services are constantly requiring higher data rates, which drives interest in the development of systems that operates at higher frequencies [1]. Omnidirectional dual-reflector antennas, which are capable of operating in microwave and millimeter wave bands with compact designs, have been extensively investigated and analyzed in frequency-domain (FD) [1]–[4]. For example, the electromagnetic performance of omnidirectional dual-reflector antennas in millimeter waves was studied in [1]. However, for large operation bandwidths, it may be impractical to perform the transient analysis of electromagnetic fields radiated by reflector antennas in the FD [5]. Several formulations intended to obtain the radiated fields in the time-domain (TD) have been presented, but only single-reflector antennas were considered in them [5]–[7].

In this paper, a formulation to obtain the electric far field radiated by classical omnidirectional dual-reflector antennas in TD is developed. These antennas are composed of two circularly symmetric reflectors generated by axis-displaced conic sections [2]. Initially, the radiated field in FD is derived using geometrical optics (GO) principles and the aperture method. Later, by applying an inverse Fourier transform, the TD electric field step response is obtained. The feed utilized is a transverse electromagnetic (TEM) coaxial horn that yields an azimuthally-uniform coverage with vertical polarization [3]. At the end, a brief case study and some conclusions are presented.

## II. Formulation

In this work, it is adopted the design procedure of classical omnidirectional dual-reflector antennas with an arbitrary main-beam direction in the elevation plane discussed in [2]. As presented in [2] and [3], such antennas are classified in four different configurations: ominidirectional axis-displaced Cassegrain (OADC), Gregorian (OADG), ellipse (OADE) and hyperbola (OADH). Fig. 1 illustrates the OADE configuration and Fig. 2 shows the main geometric parameters for an OADC configuration (the parameters for other configurations are defined similarly) [4].

### *A. Frequency-Domain Fields*

The TEM spherical radiation emitted from the principal focus ($O$) of the antenna can be described as [3]

$$\vec{E}_F\left(\vec{r}_F,\omega\right) = \left[F\left(\theta_F\right)\theta_F + P\left(\theta_F\right)\phi_F\right]\frac{e^{-j\frac{\omega}{c}r_F}}{r_F}, \quad (1)$$

where $c$ is the free-space speed of light and $\left(r_F,\theta_F,\phi_F\right)$ are the standard spherical coordinates locating the subreflector point $S$ (see Fig. 2).

Following GO principles, after the two reflections, the GO field at the conical aperture point $A$ can be represented as [3], [4]

$$\vec{E}_A\left(\vec{r}_A,\omega\right) = \xi\left[F\left(\theta_F\right)\rho_M\hat{\rho} + P\left(\theta_F\right)\phi_M\right]A_{GO}\left(\theta_F\right)e^{-j\frac{\omega}{c}\ell}, \quad (2)$$

$$\vec{H}_A\left(\vec{r}_A,\omega\right) = \xi\left[F\left(\theta_F\right)\phi_M - P\left(\theta_F\right)\hat{\rho}_M\right]\frac{A_{GO}\left(\theta_F\right)}{Z_0}e^{-j\frac{\omega}{c}\ell}, \quad (3)$$

where $\xi = 1(= -1)$ for the OADC and OADE (OADG and OADH), $Z_0$ is the free-space impedance, $\ell$ is the constant path length from $O$ to $A$, $A_{GO}$ is the GO attenuation factor, and

$$\begin{aligned}\hat{\rho}_M &= \cos\gamma\rho - \sin\gamma\hat{z},\\ \phi_M &= \phi.\end{aligned} \quad (4)$$

The electric far field radiated by the antenna in free-space can be approximated as [8]

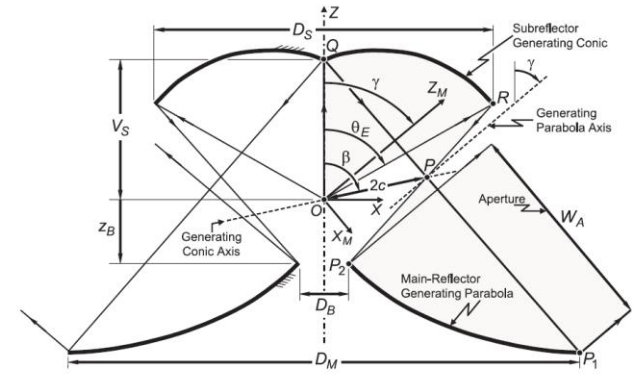


Fig. 1. OADE configuration [4].

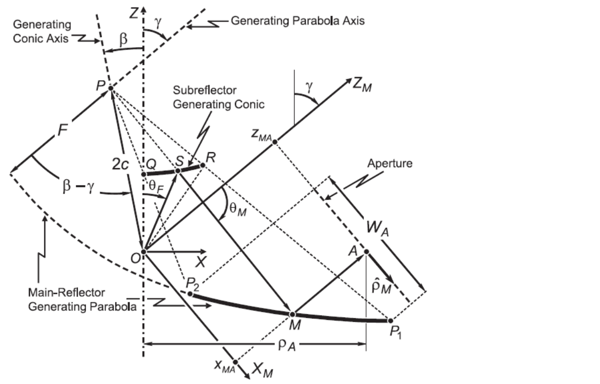


Fig. 2. Geometrical parameters [4].

$$\vec{E}(\vec{r},\omega) \approx -j\omega\frac{Z_0}{4\pi c}\frac{e^{-j\frac{\omega}{c}r}}{r}\int_{S_A}\left[\vec{J}_A-\left(\vec{J}_A\cdot\hat{r}\right)\hat{r} + \frac{1}{Z_0}\vec{M}_A\times\hat{r}\right]e^{j\frac{\omega}{c}\vec{r}'\cdot\hat{r}}dS_A, \qquad (5)$$

where

$$\vec{J}_A = \hat{n}_A\times\vec{H}_A = -\vec{E}_A/Z_0, \qquad (6)$$

$$\vec{M}_A = -\hat{n}_A\times\vec{E}_A = -Z_0\vec{H}_A, \qquad (7)$$

$$\vec{r}'\cdot\hat{r} = \rho_A(\theta_F)\sin\theta\cos(\phi-\phi') + z_A\cos\theta, \qquad (8)$$

$$dS_A = \rho_A(\theta_F)\,dx_{MA}\,d\phi', \qquad (9)$$

in which $\hat{n}_A$ is the normal vector to the aperture, $\rho_A$ is the distance from $A$ to the symmetry axis, $z_A$ and $x_{MA}$ are the $z$ and $x_{MA}$-coordinates of $A$, respectively.

Substituting (6) - (9) into (5) and rewriting it according to ($\theta_F$,$\phi_F$) parameters, we find

$$\begin{aligned}E_\theta(\vec{r},\omega) \approx\ & j\omega\frac{1}{4\pi c}\frac{e^{-j\frac{\omega}{c}r}}{r}\int_0^{2\pi}\int_0^{\theta_E}\Big\{E_T(\theta_F,\omega)\big[\sin\gamma\sin\theta \\ & +(1+\cos\gamma\cos\theta)\cos(\phi-\phi_F)\big] \\ & +E_P(\theta_F)(\cos\gamma+\cos\theta)\sin(\phi-\phi_F)\Big\} \\ & e^{j\frac{\omega}{c}\left[\rho_A(\theta_F)\sin\theta\cos(\phi-\phi')+z_A\cos\theta\right]} \\ & J(\theta_F)\rho_A(\theta_F)\,d\theta_F\,d\phi_F\end{aligned} \qquad ,(10)$$

$$\begin{aligned}E_\phi(\vec{r},\omega) \approx\ & j\omega\frac{1}{4\pi c}\frac{e^{-j\frac{\omega}{c}r}}{r}\int_0^{2\pi}\int_0^{\theta_E}\Big\{E_P(\theta_F,\omega)\big[\sin\gamma\sin\theta \\ & +(1+\cos\gamma\cos\theta)\cos(\phi-\phi_F)\big] \\ & -E_T(\theta_F)(\cos\gamma+\cos\theta)\sin(\phi-\phi_F)\Big\} \\ & e^{j\frac{\omega}{c}\left[\rho_A(\theta_F)\sin\theta\cos(\phi-\phi')+z_A\cos\theta\right]} \\ & J(\theta_F)\rho_A(\theta_F)\,d\theta_F\,d\phi_F\end{aligned} \qquad , (11)$$

where $J = \left|dx_{MA}/d\theta_F\right|$ and $E_T$ and $E_P$ are the components of $\vec{\mathrm{E}}_{\mathrm{A}}$ in the directions $\hat{\rho}_M$ and $\hat{\phi}_M$, respectively.

In the present work, the adopted feed is a TEM coaxial horn with external and internal radii $R_e$ and $R_i$, respectively, whose model is given by [3]:

$$\mathrm{F}(\theta_\mathrm{F}) = \frac{\left[J_0(\omega R_i\sin\theta_F/c) - J_0(\omega R_e\sin\theta_F/c)\right]}{\sin\theta_F}, \qquad (12)$$
$$P(\theta_\mathrm{F}) = 0.$$

Thus, (10) and (11) become

$$E_\theta(\vec{r},\omega) \approx j\omega\int_0^{2\pi}\int_0^{\theta_E} E_T(\theta_F,\omega)B(\theta_F,\phi_F)\,d\theta_F\,d\phi_F, \qquad (13)$$

$$E_\phi(\vec{r},\omega) \approx 0, \qquad (14)$$

where $E_T$ can be written as

$$E_T(\theta_\mathrm{F},\omega) = \xi\left[\frac{J_0\left(\frac{\omega R_i\sin\theta_\mathrm{F}}{c}\right) - J_0\left(\frac{\omega R_e\sin\theta_\mathrm{F}}{c}\right)}{\sin\theta_\mathrm{F}}\right]A_{GO}(\theta_\mathrm{F})e^{-j\frac{\omega}{c}\ell} \qquad (15)$$

and

$$B(\theta_F,\phi_F)=\frac{\rho_A(\theta_F)J(\theta_F)}{4\pi rc}\Big[(1+\cos\gamma\cos\theta)\cos(\phi-\phi_F) +\sin\gamma\sin\theta\Big]e^{-j\frac{\omega}{c}\left[r-\rho_A(\theta_F)\sin\theta\cos(\phi-\phi')-z_A\cos\theta\right]} \quad (16)$$

### B. Time-Domain Fields

In order to obtain the time domain radiated field, it can be applied to its frequency domain equation an inverse Fourier transform defined as [9]

$$\vec{\varepsilon}(\vec{r},t)=\mathcal{F}^{-1}\left[\vec{E}(\vec{r},\omega)\right]=\frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{E}(\vec{r},\omega)e^{j\omega t}d\omega. \quad (17)$$

Using the convolution theorem [9], the time-domain electric field step response is given by

$$\varepsilon_\theta^u(\vec{r},t)\approx\int_0^{2\pi}\int_0^{\theta_E}E_T(\theta_F,t)*B(\theta_F,\phi_F,t)d\theta_F d\phi_F, \quad (18)$$

where

$$E_T(\theta_F,t)=\mathcal{F}^{-1}\left[E_T(\theta_F,\omega)\right] =\frac{\xi c A_{GO}(\theta_F)}{2\pi\sin\theta_F}\left[\frac{1}{\sqrt{(R_i\sin\theta_F)^2-(l-ct)^2}} -\frac{1}{\sqrt{(R_e\sin\theta_F)^2-(l-ct)^2}}\right] ,(19)$$

$$B(\theta_F,\phi_F,t)=\mathcal{F}^{-1}\left[B(\theta_F,\phi_F,\omega)\right]=\frac{\rho_A(\theta_F)J(\theta_F)}{4\pi rc}\left[(1+\cos\gamma\cos\theta)\cos(\phi-\phi_F)+\sin\gamma\sin\theta\right]\delta\left(t-\frac{r-\rho_A(\theta_F)\sin\theta\cos(\phi-\phi')-z_A\cos\theta}{c}\right). \quad (20)$$

Performing the convolution of (19) and (20) and substituting the result into (18), we finally have

$$\varepsilon_\theta^u(\vec{r},t)\approx\int_0^{2\pi}\int_0^{\theta_E}C(\theta_F,\phi_F)\left[\frac{1}{\sqrt{(R_i\sin\theta_F)^2-\left[l-c(t-a(\theta_F,\phi_F))\right]^2}} -\frac{1}{\sqrt{(R_e\sin\theta_F)^2-\left[l-c(t-a(\theta_F,\phi_F))\right]^2}}\right]d\theta_F d\phi_F , \quad (21)$$

where

$$C(\theta_F,\phi_F)=\frac{\xi A_{GO}(\theta_F)\rho_A(\theta_F)J(\theta_F)}{8\pi^2 r\sin\theta_F}\left[(1+\cos\gamma\cos\theta)\cos(\phi-\phi_F)+\sin\gamma\sin\theta\right] , \quad (22)$$

$$a(\theta_F,\phi_F)=\frac{r-\rho_A(\theta_F)\sin\theta\cos(\phi-\phi')-z_A\cos\theta}{c}. \quad (23)$$

## III. Case Study

Using the formulation discussed in Section II, the TD electric far field radiated by an OADE antenna was calculated and preliminary numerical results are presented here. The antenna geometry is depicted in Fig. 3 and it was established with $\gamma = 102^o$, $W_A = 10\lambda$, $D_M = 24\lambda$, $V_S = 9.77\lambda$, $D_B = 2.4\lambda$ and $Z_B = 0$, where $\lambda = 0.1$ m. Also, the coaxial horn parameters are: $R_e = 1.17\lambda$ and $R_i = 0.3\lambda$.

Fig. 4 presents the magnitude of the step response for an observer located at $r = 5000$m, $\theta = 102^o$ and $\phi = 0^o$. In Fig. 3 and Fig. 4, the results were obtained considering the observer located at $\theta = 112^o$ and $\theta = 122^o$, respectively.

## IV. Conclusions

A formulation to obtain the TD electric field step response from classical omnidirectional dual-reflector antennas fed by a TEM coaxial horn was developed. The response due to an arbitrary excitation can be obtained by performing a temporal convolution with the step response. In addition, the procedure used here can be extended to establish the TD radiated field from omnidirectional dual-reflector antennas supplied by any feeders.

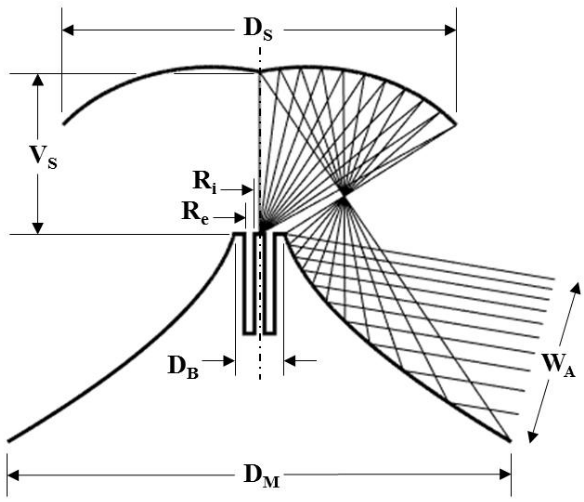


Fig. 3. OADE antenna draw of the case study.

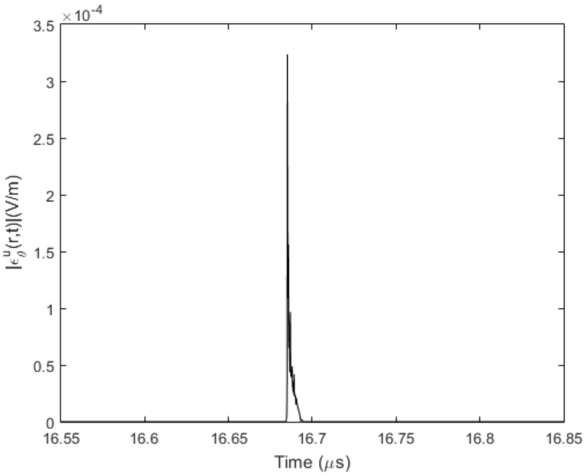


Fig. 4. Magnitude of the electric field step response at observation point located at *r* = 5000m, θ = 102º and ϕ = 0º.

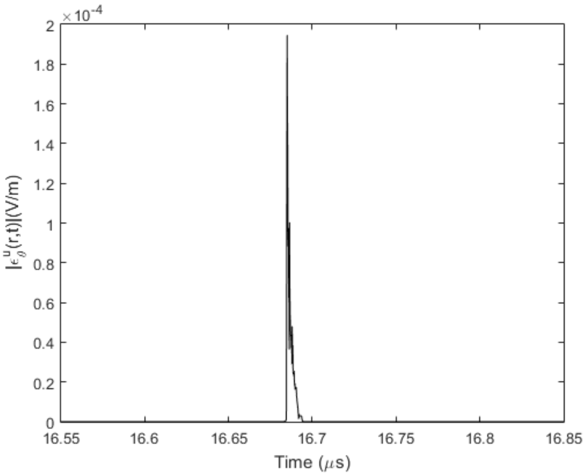


Fig. 5. Magnitude of the electric field step response at observation point located at *r* = 5000m, θ = 112º and ϕ = 0º. .

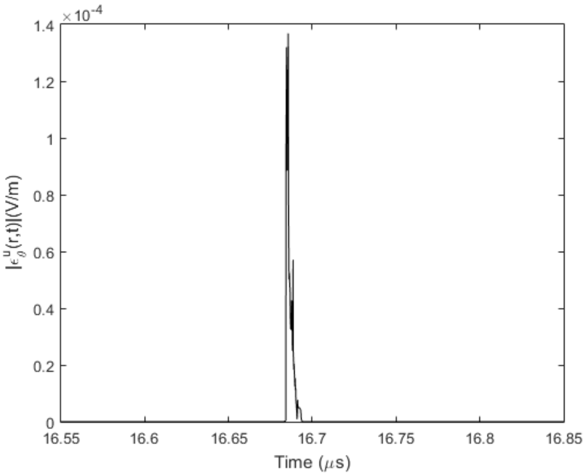


Fig. 6. Magnitude of the electric field step response at observation point located at *r* = 5000m, θ = 122º and ϕ = 0º.

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