Propagation Prediction Based on Time Domain Electric Field Integral Equation for Smoothly Irregular Terrains

Ruã Luz Barbosa[1] and Fernando José da Silva Moreira[2]

[1] Electrical Engineering Graduate Program, UFMG, Belo Horizonte, Brazil, rua.barbosa@gmail.com

[2] Dept. Electronics Engineering, UFMG, Belo Horizonte, Brazil, fernandomoreira@ufmg.br

***Abstract*—This paper proposes a formulation to predict the propagation of a vertically polarized electromagnetic wave in the time domain using the Electric Field Integral Equation over an irregular terrain for a near grazing incidence. The results show good agreement when compared with the ones obtained from the Magnetic Field Integral Equation and the Uniform Theory of Diffraction, both in the time domain.**

***Index Terms*—electric field integral equation, time domain integral equation**

I. INTRODUCTION

The performance of a radio communication link does not depend only of the antennas and devices that process the signals to be transmitted. It depends also on the radio channel, which affect the propagation of the electromagnetic waves [1]. The radio channel is less predictable than the wired communication channel and that is why special care need to be taken when planning wireless links to avoid problems after their implementation.

With the requirement of higher transmission rates, the use of Ultra Wide Band (UWB) signals has got attention. As the bandwidth of UWB signals are relatively large, it is more suitable to study its propagation in time domain to properly characterize the radio channel. The study in the time domain can be done using two approaches: (1) calculating the field amplitude in the frequency domain and then applying the Inverse Fourier Transform (IFT); (2) calculating the field strength directly in time domain [2].

The propagation prediction in the frequency domain using integral equations has been developed in several works. Here, particularly important are references [3] and [4]. In [3], equations based on the Electric Field Integral Equation (EFIE) were proposed to predict signals over a smoothly irregular terrain. A near grazing incidence was assumed, considering radio propagation over great distances. For a vertical polarization and grazing incidence, the ground was treated as a perfect magnetic conductor. Assuming no transversal variation in the terrain profile, the integral over the surface was reduced to a line integral by using the stationary phase method, which means that the contribution to the scattering comes from the segments along the plane of incidence. By using the Method of Moments (MoM), the ground equivalent magnetic current can be found and the scattered electric field can be calculated from it. With the backscattering being neglected, a forward scheme was used to recursively calculate the equivalent magnetic currents. The formulation was applied to a wedge profile and compared with the Uniform Theory of Diffraction (UTD). The method was also applied to a terrain profile in Denmark and compared with measured data, showing good agreement. Following a similar procedure, in [4] a prediction method was developed using the Magnetic Field Integral Equation (MFIE). The MFIE was found to have a better numerical convergence than the EFIE for the scenario investigated in [3].

By applying the Inverse Fourier Transform (IFT) to the frequency domain MFIE formulation developed in [4], a time domain MFIE (TD-MFIE) was proposed in [5]. With the backscattering neglected, a march-on-in-time scheme was used to evaluate the time-domain integral and, consequently, obtain the time-domain equivalent magnetic currents. Good agreement between the TD-MFIE and a time domain Uniform Theory of Diffraction (TD-UTD) [2] was obtained.

In this work, a formulation based on a time-domain Electric Field Integral Equation (TD-EFIE) is developed to predict the field strength over a smooth and irregular terrain for a vertically polarized signal. The TD-EFIE is applied to the propagation over a ground with a smooth wedge profile, as in [3]. Results are compared to TD-MFIE, TD-UTD and the Discrete Inverse Fourier Transform of the results obtained from the frequency-domain EFIE (EFIE+DIFT).

II. EFIE FORMULATION IN THE FREQUENCY DOMAIN

For typical radio links in rural scenarios, a near grazing incidence can be considered and ground can be assumed a perfect magnetic conductor. Using the equivalence principle with boundary conditions for a perfect magnetic conductor, the scattered field will be radiated by equivalent magnetic currents at the ground surface, which lead us to the frequency domain EFIE [6]:

$$\vec{E}(\vec{r}) = T[\vec{E}_{in}(\vec{r}) + \vec{L}_2(\vec{M}_s)] \tag{1}$$

$$\vec{L}_2(\vec{M}_s) = -\iint_{S'} [\vec{M}_s(\vec{r}\,') \times \nabla' G] \tag{2}$$

where T is 1 if the fields are evaluated out of the surface and 2 if evaluated on it, $\vec{M}_S$ is the equivalent magnetic surface current, $\vec{E}_{in}$ is the incident electric field radiated by the transmitting antenna, and G is the free-space Green's function.

Fig. 1 shows the terrain profile and the coordinate system adopted. The position of the transmitter antenna is given by $\vec{r}_o$, primed coordinates locate the source, and non-primed ones locate the observer. The normal and tangential unit vectors of the surface are given by $\hat{n}$ and $\hat{l}$, respectively.

Considering the transmitter far away from the terrain, it can be assumed that the current has a spherical phase distribution:

$$\vec{M}_S(\vec{r}) = \vec{M}_a(\vec{r})e^{-jk|\vec{r}\,'-\vec{r}_o|}, \qquad (3)$$

where $k = 2\pi/\lambda$ is the wave number and $|\vec{r}\,'-\vec{r}_o|$ is the distance between the antenna and the surface. Substituting (3) into (1) and applying the boundary condition for a perfectly conducting magnetic surface ($\hat{n}\times\vec{E} = -\vec{M}_S$), the surface integral can be reduced to a line one by applying the stationary phase method. The incident field at the surface is then given by:

$$\hat{n}\times\vec{E}_{in} = -\frac{\vec{M}_a}{2}e^{-jkR_1} + k\int_{l'}(\hat{n}\cdot\hat{R}_2)\vec{M}_a G_2 dl', \qquad (4)$$

$$G_2 = \left(1-\frac{j}{kR_2}\right)\frac{e^{-jk(R_1+R_2)+j\pi/4}}{4\pi\sqrt{(1+R_2/R_1)R_2/\lambda}}, \qquad (5)$$

where R_1 is the distance from the transmitter to the source point on the surface and R_2 is the distance from the source to the observer, as shown in Fig. 1.

Dividing the terrain profile in the incidence plane into N segments, the MoM can be used to calculate the magnetic currents in (4). By neglecting backscattering, the currents can be recursively calculated following the approach in [3], using:

$$M_i = \frac{1}{Z_{ii}}\left(V_i - \sum_{j=1}^{i-1} Z_{ij}M_j\right),\ i = 1,2,\ldots,N, \qquad (6)$$

$$V_i = \vec{E}_{in}\cdot\hat{l}_i, \qquad (7)$$

$$Z_{ij} = (\hat{n}\cdot\hat{R}_2)G_2 k\Delta_j\frac{sen(\alpha)}{\alpha},\ i \neq j, \qquad (8)$$

$$Z_{ii} = -\frac{1}{2}e^{-jkR_1},\ i = j, \qquad (9)$$

$$\alpha = \frac{k\Delta_j}{2}(\hat{R}_{1j} - \hat{R}_{2ij})\cdot\hat{l}\,', \qquad (10)$$

where i represents the observer segment, j represents the source segment, and Δ_j is the length of the source segment.

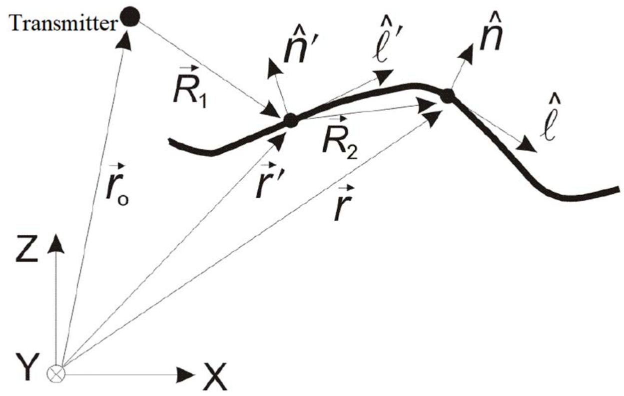

Fig. 1 - Terrain profile.

After numerically determining the magnetic current, the electric field at the receiver antenna can be calculated using:

$$\vec{E}(\vec{r}) = \vec{E}_{in}(\vec{r}) - \sum_{j=1}^{N} M_j\frac{sen(\alpha)}{\alpha}(\hat{y}\times\hat{R}_2)G_2 k\Delta_j \qquad (11)$$

III. EFIE FORMULATION IN THE TIME DOMAIN

Recalling that $\vec{M}_S(\vec{r}) = \vec{M}_a(\vec{r})e^{-jkR_1}$ and applying the inverse Fourier transform in (4), the magnetic current in time domain can be found using the MoM. The result of the extensive procedure leads to the following system:

$$[V_{i,p}] = [Z^1_{ij,p}][M_{i,p} - M_{i,p-1}] + [Z^2_{ij,p}][M_{i,p}], \qquad (12)$$

where,

$$M_{i,p} = \frac{1}{Z^2_{ii,p}}\left[V_{i,p} - \sum_{j=1}^{i-1}\sum_{p_i=2}^{p} Z^1_{ij,p_i}(M_{j,p_i} - M_{j,p_i-1}) - \sum_{j=1}^{i-1}\sum_{p_i=1}^{p} Z^2_{ij,p_i}M_{j,p_i}\right], \qquad (13)$$

$$V_{i,p} = \vec{E}_{in}(\vec{r}\,',t_p)\cdot\hat{l}_i, \qquad (14)$$

$$Z^1_{ij,p} = \frac{(\hat{n}_i\cdot\hat{R}_2)\Delta_j}{6\pi R_2\Delta t_i}\sqrt{\frac{2c}{R_2(1+R_2/R_1)}} \times\left[\sqrt{\tau - t_{p-1}}\left(2\tau + t_{p-1} - 3t_p + \frac{3R_2}{c}\right) - \sqrt{\tau - t_p}\left(2\tau - 2t_p + \frac{3R_2}{c}\right)\right],\ i \neq j, \qquad (15)$$

$$Z^1_{ii,p} = 0,\ i = j, \qquad (16)$$

$$Z^2_{ij,p} = \frac{(\hat{n}_i\cdot\hat{R}_2)\Delta_j}{2\pi R_2}\sqrt{\frac{2c}{R_2(1+R_2/R_1)}} \times\left[\sqrt{\tau - t_{p-1}} - \sqrt{\tau - t_p}\right],\ i \neq j, \qquad (17)$$

$$Z_{ii,p}^{2} = -\frac{1}{2},\ i = j, \tag{18}$$

i and j represent the observer and source segments, respectively, and p represents the time instants. $\vec{E}_{in}(\vec{r}\,',t_p)$ is the incident electric field at the observation segment at the time instant t_p. The retarded time τ is the instant time at the source segment ($\tau = t_p - R_2/c$), c is the speed of light, and Δt_i is the temporal increment at the observation segment.

The electric field at the receiving antenna can be calculated by applying the inverse Fourier transform in (10):

$$\vec{E}(\vec{r},t) = \vec{E}_{in}(\vec{r},t) - \sum_{i=1}^{N}\sum_{p_i=2}^{p}\left[(\hat{y}\times\hat{R}_2)\frac{\Delta_i}{4\pi} \times\sqrt{\frac{2c}{R_2\left(1+R_2/R_1\right)}}(I_1+I_2)\right], \tag{19}$$

where

$$I_1 = \frac{2}{c}\frac{M_{i,p}-M_{i,p-1}}{\Delta t_i}\left(\sqrt{\tau-t_{p-1}}-\sqrt{\tau-t_p}\right), \tag{20}$$

$$I_2 = \frac{2(M_{i,p}-M_{i,p-1})}{3R_2\Delta t_i}\Big[(2\tau+t_{p-1}-3t_p)\sqrt{\tau-t_{p-1}} -2(\tau-t_p)\sqrt{\tau-t_p}\Big]+\frac{2M_{i,p}\left(\sqrt{\tau-t_{p-1}}-\sqrt{\tau-t_p}\right)}{R_2}, \tag{21}$$

R_2 is the distance from the source segment to the receiving antenna and Δ_i is the length of the source segment.

IV. Numerical Results

To evaluate the results of the TD-EFIE formulation presented in the previous section, the propagation of a pulse defined in [7], over a wedge profile terrain with 200m of extension and 2m height [5], was analyzed. The transmitter was positioned at one end of the terrain at 5m height and the receiver at the other end at 5m height, as shown in Fig. 2. The pulse is illustrated in Fig. 3 in time and frequency domains.

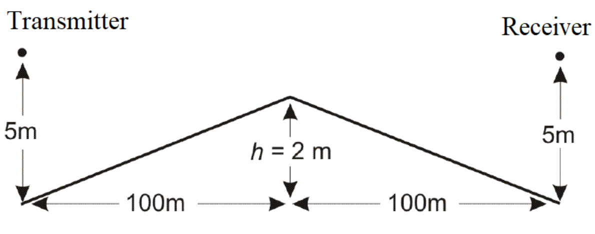

Fig. 2 - Perfect magnetic conductor wedge.

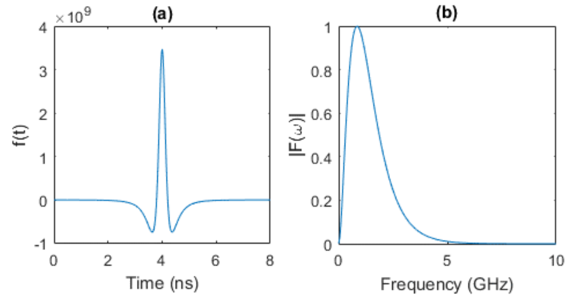

Fig. 3 - (a) Pulse in time domain; (b) pulse spectrum.

First, the received pulse was calculated in time domain after applying the discrete inverse Fourier transform to the numerical results obtained from the frequency domain EFIE (EFIE+DIFT) in (11). The field strength was calculated for a frequency range, up to 7GHz, of the pulse spectrum and, for each frequency analyzed, the terrain was segmented in 2 wavelengths per segment. The pulse was then obtained directly in time domain (TD-EFIE) using (19). In this analysis, the terrain was segmented in 2 wavelengths per segment based on the highest significant frequency (approximately 7 GHz) of the pulse spectrum [Fig. 3(b)]. A sampling frequency of 40 GHz was adopted for time discretization.

Figures 4 and 5 show the magnitude of the scattered electric field as a function of time. As shown in Fig. 4, the results from the TD-EFIE have good agreement with the results of the TD-MFIE and TD-UTD. Fig. 5 shows that the EFIE+DIFT approach has an accuracy comparable to the ones of the TD-EFIE and TD-UTD, with the first one having a very small time delay for the scattered pulse.

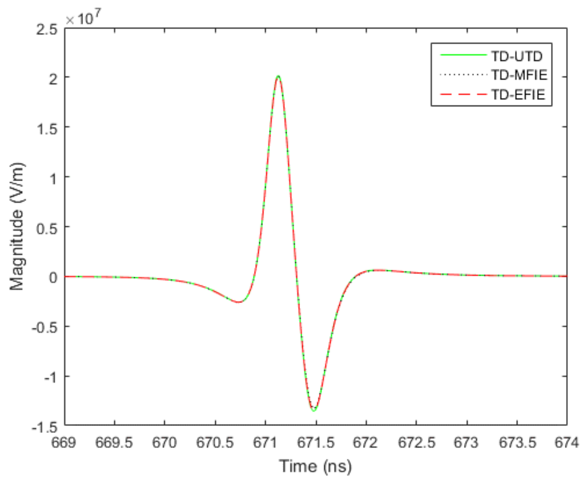

Fig. 4 - Magnitude of the time-domain scattered field, calculated by the TD-EFIE, TD-MFIE, and TD-UTD.

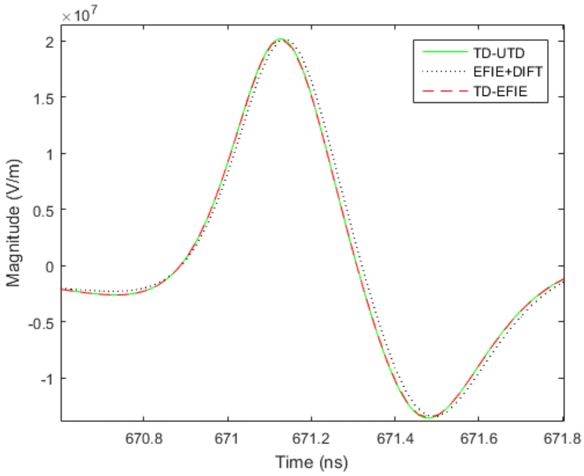

Fig. 5 - Enlarged view of the magnitude of the time-domain scattered field, calculated by the TD-EFIE, TD-UTD, and EFIE-DIFT.

V. Conclusions

This work presented a method to predict the propagation of a vertically polarized electromagnetic wave over smoothly irregular terrains in time domain. The method is based on the Electric Field Integral Equation (EFIE). Good results were obtained when compared with TD-UTD and TD-MFIE. The TD-EFIE also shows good agreement with the frequency-domain EFIE after applying a discrete inverse Fourier transform to the frequency results.

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