Numerical Convergence of Method of Moments in the Analysis of Bodies of Revolution

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Abstract — **The scattering analysis by method of moments may lead to integral equations with severe singularities in their kernels. In this work we demonstrate that when these singularities are removed by the extraction technique and enough basis functions are used to describe surface current behavior, the numerical solution converges with Z-matrix integrals numerically evaluated using 2-point Gaussian quadrature. The numerical convergence is investigated in the analysis of the electromagnetic scattering by conducting and dielectric spheres.**

I. INTRODUCTION

In the analysis of electromagnetic scattering by homogeneous bodies, the numerical evaluation of surface integral equations by the method of moments (MoM) has proved its efficiency to treat surfaces with arbitrary shape. The choice of suitable basis functions to represent the surface currents, the numerical evaluation of singular integrals, and efficient algorithms for the matrix inversion are fundamental to obtain accuracy and convergence from the MoM analysis. However, the numerical evaluation of singularities arising in the integral kernels is not a simple task. In [1] a robust numerical technique (extraction technique) is presented to remove singularities arising in the scattering by conducting bodies of revolution (BOR's). In [2] the method is extended to handle triangular basis functions in a Galerkin scheme.

For homogeneous dielectric BOR's the accuracy of the MoM analysis deteriorates as the dielectric constant increases [3]. In [4] different integral equation formulations are used to overcome this problem, but still with relatively small accuracy. This drawback may be partially overcome by increasing the number of segments used to represent the BOR generatrix, which should be made proportional to the dielectric constant and, consequently, increases the Z-matrix size. In [5] triangular functions for both basis and testing functions were used, simply represented by a series of pulses. It was observed that, to attain certain accuracy, more pulses had to be used to represent the triangular functions as the dielectric constant of the BOR increased. In some sense this leads to the perception that accuracy increases with the number of quadrature points used to evaluate the integrals.

In this work we analyze the plane-wave scattering by conducting and dielectric spheres. We observe that when the extraction technique [2] is applied and a sufficient number of triangular basis functions (TBF's) is used to represent the surface geometry, numerical convergence is attained when 2-point Gaussian quadratures are applied to evaluate the integrals of the MoM Z-matrix. The accuracy increases if more TBF's are used but does not change with the number of quadrature points.

II. INTEGRAL EQUATION EVALUATION

The MoM solution involving surface electric (EFIE) and magnetic (MFIE) field integral equations leads to many different formulations [3], [6]. The EFIE formulation is the choice for open conducting shells. For closed conducting surfaces the combined field integral equation (CFIE) avoids spurious resonances [6]. The CFIE is a linear combination of EFIE and MFIE. For dielectric bodies, EFIE and MFIE can be linearly combined in several forms [3]. One of the most adopted combination is the Müller formulation [3]. All these formulation, when applied to the analysis of the scattering by BOR's, lead to integrals of the form [2]:

$$\mathrm{I} = \int_{\varphi=0}^{2\pi}\int_{\alpha=-1}^{1}\int_{\alpha'=-1}^{1}\left(\frac{ab}{c}\right)F(\varphi)\frac{e^{-jkR}}{R^{d}}\,d\alpha'\,d\alpha\,d\varphi\,, \tag{1}$$

where R=|**r** − **r**′| is the distance between source and observation points, a may be equal to 1, α' or α'^2, b may be equal to 1, α or α^2, c may be equal to 1, ρ, ρ' or $\rho\rho'$, and the exponent d = 1 or 3. The integral (1) has removable singularities whenever the observation point **r** is very close to the source point **r**′. The concept adopted in [2] to treat these singularities is to split the corresponding integrands into two: one that is regular and can be numerically evaluated by a Gaussian quadrature and another that contains a removable singularity and can be integrated analytically.

III. NUMERICAL RESULTS

To evaluate the numerical convergence we consider the CFIE and Müller formulation to the plane-wave scattering by perfect electric conductor (PEC) and dielectric spheres, respectively. The accuracy of the numerical results was verified against analytical solutions based on Mie series using the RMS error:

$$\mathrm{E_{RMS}}(\%) = \left(\mathrm{E}_{\mathrm{J}t} + \mathrm{E}_{\mathrm{J}\phi} + \mathrm{E}_{\mathrm{M}t} + \mathrm{E}_{\mathrm{M}\phi}\right)/4\,, \tag{2}$$

with $\mathrm{E_X}$ representing a RMS defined by

$$\mathrm{E_X}(\%) = \sqrt{\frac{1}{N}\sum_{m=1}^{N}\left(\left|\mathrm{X}_\mathrm{m}^{\mathrm{MoM}} - \mathrm{X}_\mathrm{m}^{\mathrm{Mie}}\right|/\left|\mathrm{X}_\mathrm{m}^{\mathrm{Mie}}\right|\right)^2}\,, \tag{3}$$

where X is any one of the electric (J_t or J_ϕ) or magnetic (M_t or M_ϕ) surface current components while X^{MoM} and X^{Mie} represent the corresponding numerical and analytical solutions, respectively.

The first case study is a PEC sphere, as illustrated in Fig. 1. The sphere radius is $1\lambda_0$ (the wavelength in vacuum). Different numbers of segments per λ_0 were used to represent the sphere generatrix. The TBF is defined over two consecutive segments [1]. Over each segment the regular integrals in α and α' of (1) were both evaluated using a NIP-point Gaussian quadrature, with NIP varied from 1 to 10. Singularities were removed by extraction technique [2]. Figure 2 shows the RMS error as function of NIP. Converge is attained using NIP = 2, independently from the number of segments per λ_0, which is determinant for the RMS error. When singularities are not removed the RMS error increases, as expected.

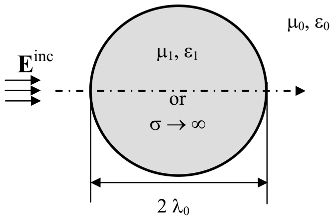

Fig. 1. Plane-wave scattering by homogeneous sphere

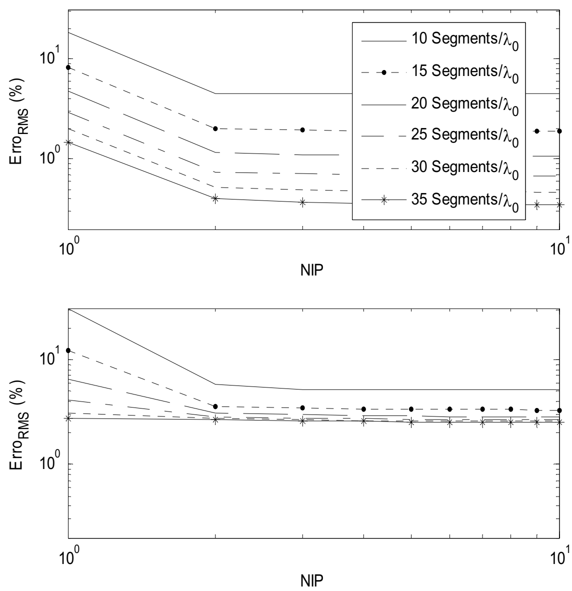

Fig. 2. Error for PEC sphere a) with singularities removal b) without singularities removal

The RMS error for dielectric spheres are illustrated in Fig. 4 for relative permittivity ε_r = 2 and 20. Once more, when singularities are removed numerical convergence is attained for NIP = 2. The number of segments/λ_0 influences the RMS error, which diminishes as more segments are used. When singularities are not removed, convergence is attained only for larger NIP values, especially for large values of ε_r.

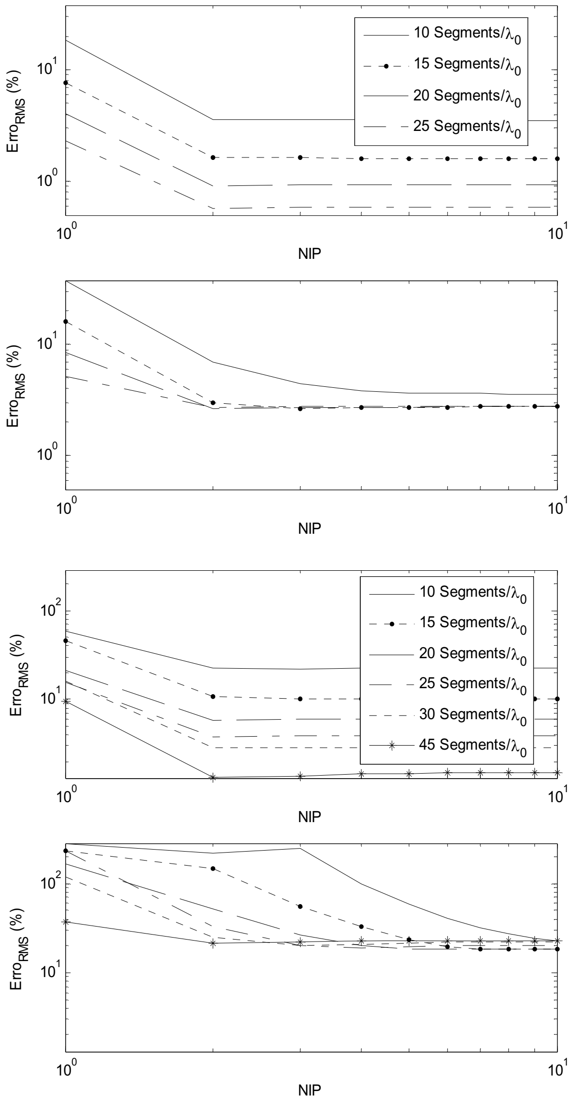

Fig. 3. Error for dielectric sphere a) ε_r =2 with singularities removal b) ε_r =2 without singularities removal c) ε_r =20 with singularities removal d) ε_r =20 without singularities removal

IV. REFERENCES

[1] J. Mautz and R. Harrington, "An improved E-Field solution for a conducting body of revolution," Tech. Report TR-80-1, Dept. Electrical and Computer Engineering, Syracuse University, 1980.

[2] U. Resende, F. Moreira, and O. Pereira Filho, "Efficient evaluation of singular integral equations in moment method analysis of bodies of revolution," Journal of Microwaves and Optoelectronics, vol. 6, no. 2, pp. 373-391, Dec. 2007.

[3] J. Mautz and R. Harrington, "Electromagnetic scattering from a homogeneous material body of revolution," AEÜ – Int. Journal of Electronics and Communication, vol. 33, pp. 71-80, 1979.

[4] A. Kishk and L. Shafai, "On the accuracy limits of different integral equation formulations for numerical solution of dielectric bodies of revolution," Can. J. Phys., vol. 63, pp. 1532-1539, 1985.

[5] A. Kishk, and L. Shafai, "Improvement of the numerical solution of dielectric bodies with high permittivity," IEEE Trans. Antennas Propagat., vol. 37, pp. 1486-1490, Nov. 1989.

[6] J. Mautz and R. Harrington, "H-field, E-field and combined field solutions for bodies of revolution," IEEE Trans. Antennas Propagat., vol. 27, no. 4, pp. 157-164, July 1979.