3. Wave Propagation, 9. Numerical Techniques

2D Scattering Integral Field Equation Solution through a IMLS Meshless-Based Approach

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Abstract – **In this work, we apply a meshless-based method to a set of integral equations arising in electromagnetic wave propagation and scattering. The objective is not only to solve these equations through a meshless-based method, but also to find a way to build shape functions that could work for any cross-sectional geometry. We have found that the Moving Least Squares (MLS) approximation is not able to provide useful shape functions in every situation. This technique relies on matrix inversions and, according to the geometry, singular matrices can occur. In order to avoid this problem, we have taken the Improved Moving Least Squares (IMLS) approximation, that doesn't depend upon matrix inversions and works well in a number of cross-sectional geometries.**

I. INTRODUCTION

Meshless methods have successfully been applied in Mechanics as an alternative to the traditional finite element method (FEM). Now they are being brought to the study of electrodynamics and presented as an additional aid in the solution of high-frequency [1] problems. In this work we take a different approach by applying a meshless discretization directly into the classical integral field equations instead of applying it to the weak form. We have already solved the problem for a circular cylinder using the MLS approximation [2], but the solution is not as general as to be applied to every cross-section form. For other geometries (e.g. rectangular) the MLS approximation fails in providing accurate results, because singular local matrices are obtained, leading to an inconsistent outcome. The IMLS approximation thus seemed to be the right one; despite having to build an orthogonal basis first, the associated matrices are diagonal, there is no need for inversions and the shape functions could then be built consistently for any cross-sectional geometry.

II. PROBLEM DESCRIPTION

In the investigated problems, a monochromatic incident plane wave is scattered by perfect electric conductor (PEC) cylinder infinite in the z-direction. For a TM^z polarized incident wave, the electric surface current density $\overrightarrow{J_s}$ is directed along the z-direction. One of the equations governing the phenomenon is the electric field integral equation (EFIE):

$$E_z^i(\vec{\rho}) = \frac{\omega\mu}{4}\oint J_s(\vec{\rho}')H_0^{(2)}(kR)dl' \qquad (1)$$

$E_z^i(\vec{\rho})$ is the incident electric field at $\vec{\rho}$, $R = \|\vec{R}\| = \|\vec{\rho} - \vec{\rho}'\|$, $\vec{\rho}$ and $\vec{\rho}'$ locate the observation and source points at the perimeter, respectively, $\omega = 2\pi f$, where f is the wave frequency, and $H_0^{(2)}$ is the zero-order Hankel function of the second type. The other is the magnetic field integral equation (MFIE):

$$[\hat{n} \times \vec{H}_i(\vec{\rho})].\hat{z} = \frac{1}{2}J_s(\vec{\rho}') + \frac{jk}{4}\oint J_s(\vec{\rho}')H_1^{(2)}(kR)[\hat{n}.\hat{R}]dl' \qquad (2)$$

where $\vec{H}_i(\vec{\rho})$ is the incident magnetic Field at $\vec{\rho}$, $H_1^{(2)}$ is the first-order Hankel function of the second type and $\hat{n}$ is the unit surface normal. The EFIE and MFIE formulation can be seen in [3]. To avoid spurious resonant solutions one can form the combined field integral equation (CFIE) through a linear combination from the EFIE and MFIE:

$$CFIE = \alpha\, EFIE + (1-\alpha)\,\eta\, MFIE \qquad (3)$$

where α is a parameter ranging from zero to one and η is the intrinsic impedance of the exterior medium [3].

III. THE MESHLESS APPROACH

The meshless approach begins by spreading nodes over the domain of the problem to be solved. In the present study, the domain of interest is the perimeter of the cylinder cross section. To each node a shape function with compact support is associated. The vicinal region in which the shape function is different from zero is called the *influence domain* of the corresponding node [4]. The main difference between meshless methods and mesh-based methods (like the FEM) is that the *element* concept is not present. The influence domains are arbitrary (the only restriction is that the set of influence domains must cover the entire domain) and can overlap. So, the nodes can be distributed arbitrarily without generating an element mesh.

Once the shape functions have been determined, the approximated value of a function u in a point $\vec{x}$ is given by u^h and is expressed as

$$u^h(\vec{x}) = \textstyle\sum_{I=1}^{N} \phi_I(\vec{x})\,\hat{u}_I \qquad (4)$$

where $I = 1 \ldots N$ are the nodes whose influence domains include the point $\vec{x}$ and the numbers $\hat{u}_I$ associated to each node are called the *nodal parameters*.

There are several ways to build the shape functions [4]; among them we chose the moving least squares approximation (MLS). The MLS approximation begins by expressing u^h as

$$u^h(\vec{x}) = \sum_{i=1}^{m} p_i(\vec{x}) a_i(\vec{x}) = p^T(\vec{x}) a(\vec{x}) \quad (5)$$

where $p_i, i = 1 \dots m$ are monomial functions, m is the number of terms in the basis p and a_i are coefficients (e.g., in two dimensions, one could have $p^T = [1, x, y]$). The next step is to force the difference between the approximation u^h and the exact value at the nodal points to reach a minimum through the minimization of a weighted functional; this takes several matrix manipulations whose detailed descriptions can be found in [4]. The results are:

$$[\phi_1(\vec{x}), \phi_2(\vec{x}), \dots, \phi_N(x)] = p^T(\vec{x}) A^{-1}(\vec{x}) B(\vec{x}), \quad (6)$$

$$A(\vec{x}) = P^T W(\vec{x}) P \text{ and } B(\vec{x}) = P^T W(\vec{x}) \quad (7)$$

$$P = \begin{pmatrix} p_1(\vec{x}_1) & \cdots & p_m(\vec{x}_1) \\ \vdots & \ddots & \vdots \\ p_1(\vec{x}_N) & \cdots & p_m(\vec{x}_N) \end{pmatrix} \quad (8)$$

and W is a diagonal matrix whose elements are

$$[W(\vec{x})]_{II} = w(\vec{x} - \vec{x}_I) \quad (9)$$

$w(\vec{x} - \vec{x}_I)$ is a weight function with compact support (i.e. a cubic spline [4]).

The MLS approximation doesn't always provide useful shape functions, because sometimes the A-matrices become singular, preventing inversion. This phenomenon occurs for some geometries, like the rectangular one. The reason is that for points lying in regions along the sides, *away from the corners*, the parameter that describes the contour line experiments variation only in one variable (x only or y only). For a given point $\vec{x}$ in the rectangle upper side, for example, the y-variable is a constant $y = c$. So the basis becomes $p^T = [1, x, c]$. One sees that all N nodes whose influence domains act upon $\vec{x}$ have the y-coordinate equal to c. Consequently, P has two constant columns. So, the product $A = P^T W P$ will have two linearly dependent columns. Hence, A is singular.

One way to solve that is to make the nodal influence domains bigger than the side of the rectangle, in order to assure that inside this domain there will be points distributed along two adjacent sides. By doing this, both x and y will vary, P will no longer have two linearly dependent columns and A shall not be singular. But it revealed to be a bad approach: one sees that as the influence domains become larger, the local perspective of the method is destroyed. Besides that, the results are not so much accurate.

To remedy this problem, a different approximation was developed: the improved moving least squares (IMLS) [5]. In the IMLS, it is required that the terms of the basis p be orthogonal to each other. In order to do so, they are viewed as belonging to a Hilbert space in which the following inner product between functions f and g is defined:

$$\langle f, g \rangle = \sum_{I=1}^{N} w(\vec{x} - \vec{x}_I) f(\vec{x}_I) g(\vec{x}_I) \quad (10)$$

The orthogonality condition is assured through the property ($k, j = 1, 2, \dots, m$):

$$\langle p_k, p_j \rangle = \sum_{I=1}^{N} w(\vec{x} - \vec{x}_I)\, p_k(\vec{x}_I) p_j(\vec{x}_I) = \begin{cases} A_k, & k = j \\ 0, & k \neq j \end{cases} \quad (11)$$

After some matrix manipulations [5], one concludes that inversions are no longer necessary. The shape functions are

$$[\phi_1(\vec{x}), \phi_2(\vec{x}), \dots, \phi_N(x)] = p^T(\vec{x}) \bar{A}(\vec{x}) B(\vec{x}) \quad (12)$$

where $\bar{A}$ is a diagonal matrix whose elements are given by

$$[\bar{A}(\vec{x})]_{ii} = \frac{1}{\langle p_i, p_i \rangle} \quad (13)$$

IV. PRELIMINARY RESULTS

For a rectangular cross-section cylinder, we have begun by spreading nodes over the contour and applying the IMLS approximation to build the shape functions. We can express the unknown surface current density at a nodal point $\vec{\rho}_i$ along the contour as in (4), with u^h replaced by J and $\vec{x}$ by $\vec{\rho}_i$. The next step is to insert that expression for J in (3) and get a linear system that can be solved in order to find the nodal parameters. The following results were obtained (scattering by a square cylinder with sides $2a$, where $a = 1/k$ and $k = 2\pi/\lambda$ is the wave number, λ being the wavelength):

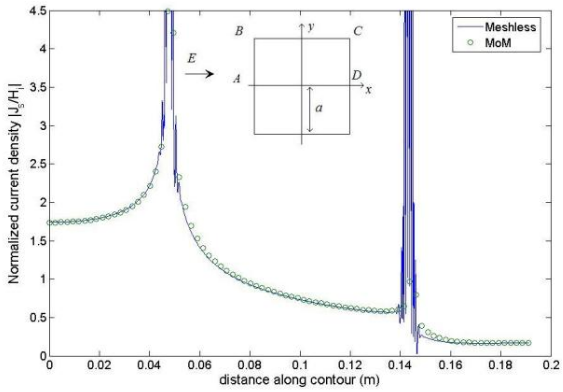

Fig. 1. Normalized current density along cylinder contour

The figure above shows the absolute value of the normalized current density (with respect to the incident magnetic field H_i) along the path ABCD. One sees that the result is accurate when compared to the method of moments (MoM). The peaks in the current density at the vertices B and C are theoretically predicted for a TM incident wave.

Further studies on the precision and the convergence of the method will be presented in the final paper. We have found out that the MLS approximation is not general enough to deal with any cross-sectional geometry. It should be noted that the IMLS approximation runs faster than the MLS, as there is no need for matrix inversion.

VI. REFERENCES

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