Optimal Number of Basis Functions in MoM Analysis of Bodies of Revolution

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Abstract — This work investigates the minimum number of basis functions for convergence of the method of moments (MoM) analysis of electromagnetic scattering by bodies of revolution. The analysis is based on the CFIE and Müller formulations for conductor and dielectrics surfaces, respectively. Empirical formulas are derived for the optimal number of basis functions, based on the analysis of electrically small and large spherical bodies.

I. INTRODUCTION

One of the most efficient numerical techniques to analyze the scattering by homogeneous bodies is the surface integral equation numerically evaluated by the MoM [1],[2]. For the special case of BOR's (bodies of revolution) the problem is formulated in terms of integrals over generatrices, which significantly reduces the computational effort [2]-[5]. For simple homogeneous media, proper combinations of surface electric and magnetic field integral equations (EFIE and MFIE, respectively) have proved to be highly appropriate. Combined field integral equation (CFIE) and Müller formulation are recognized to be the most efficient ones for closed perfect electric conductor (PEC) [4] and dielectric [5],[6] BOR's, respectively. For composite bodies the most used and efficient formulations are EFIE or CFIE for conducting surfaces, and Müller or PMCHWT (solution proposed by Poggio, Miller, Chang, Harrington, Wu and Tsai) for dielectric surfaces [7],[8].

Although electromagnetic scattering from conducting, dielectric, and composite bodies have been successfully studied using MoM [1]-[8], they are often limited to small geometries and small relative permittivities (ε_r), and the optimal number of basis functions has not been addressed. In this work, adopting the versatile triangular basis functions (TBF) to represent the current along the BOR generatrix, closed conducting, dielectric, and composite spheres are analyzed by CFIE and Müller formulation, respectively, to obtain the minimum number of segments (i.e., basis functions) necessary to reach the desired accuracy for the current representation. The study leads to empirical formulas that provide the optimal (minimum) number of segments (ONS) in terms of the sphere relative permittivity (for dielectrics) and electrical radius. Such formulas may guide the selection of the number of basis functions used in the analysis complex BOR geometries.

II. INTEGRAL EQUATION FORMULATIONS

The EFIE and MFIE formulations

$$\hat{n} \times \mathbf{E}_i^s(\mathbf{J},\mathbf{M}) + \hat{n} \times \mathbf{E}_i^{inc} = -\mathbf{M}\,, \tag{1}$$

$$\hat{n} \times \mathbf{H}_i^s(\mathbf{J},\mathbf{M}) + \hat{n} \times \mathbf{H}_i^{inc} = \mathbf{J}\,, \tag{2}$$

where i = 1, 2 represent the external and internal media, are properly combined to yield integral equations suited for the analysis of the BOR scattering. For PEC bodies, the EFIE (1), with $\mathbf{M} = 0$, is chosen for open shells [4]. For closed-surface PEC bodies, the CFIE avoids spurious resonances and, consequently, provides stable numerical solutions [4]. The CFIE is a linear combination of (1) and (2) with $\mathbf{M} = 0$:

$$\alpha' \,\mathrm{EFIE}_2 + \beta' \,\mathrm{HFIE}_2\,. \tag{3}$$

For dielectric surfaces, (1) and (2) can be combined as:

$$\mathrm{EFIE}_1 + \alpha \,\mathrm{EFIE}_2\,, \tag{4}$$

$$\mathrm{HFIE}_1 + \beta \,\mathrm{HFIE}_2\,, \tag{5}$$

where the use of $\alpha = \varepsilon_1/\varepsilon_2$ and $\beta = \mu_1/\mu_2$ is known as the Müller formulation, while $\alpha = \beta = -1$ defines the PMCHWT formulation [5],[6].

In the present work the CFIE and Müller integral equations are numerically evaluated by the MoM technique. Triangular basis functions (TBF) are employed for the equivalent current representation and the Galerkin's method adopted for the numerical evaluation of the current coefficients. All integrals appearing in the Z-matrix terms are evaluated by Gaussian quadrature with appropriate singularity treatment [2],[3].

III. MEAN RELATIVE ERROR

In order to demonstrate the formulation accuracy the surface currents for several different dielectric, conducting, and composite spheres were computed. For all performed tests the spheres were illuminated by a plane wave polarized in $\hat{x}$ and propagating in $\hat{z}$ directions, respectively. The numerical results were compared with analytical data provided by Mie-series, using the following mean relative error:

$$E_{MR}(\%) = \frac{E_{RJt} + E_{RJ\phi} + E_{RMt} + E_{RM\phi}}{4}\,, \tag{6}$$

where E_{MR} takes into account the mean relative errors of the tangential $\hat{t}$ and $\hat{\phi}$ components of $\mathbf{J}$ and $\mathbf{M}$ (E_{RX}), according to:

$$E_{RX}(\%) = 100\left[\left|\left|X^{MoM}\right| - \left|X^{Mie}\right|\right| / \left|X^{Mie}\right|_{max}\right]_{max}\,, \tag{7}$$

where X^{MoM} and X^{Mie} denote numerical MoM and the analytical Mie-series solutions, respectively.

Figures 1, 2, and 3 show the computed surface electric current in $\hat{t}$ direction, J_t, as functions of the distance $S(\lambda_0)$ measured along the BOR generatrix for PEC, dielectric, and PEC coated spheres.

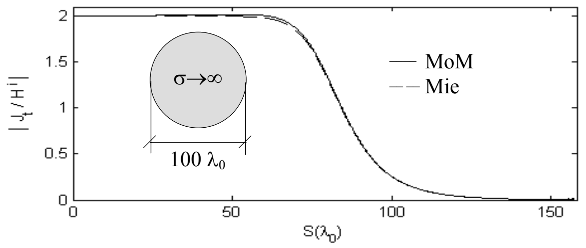

Fig. 1. Electric current, J_t, for a PEC sphere with radius $R_C = 50\lambda_o$. The current was represented by 2352 TBF and $E_{MR} = 0.977$ %.

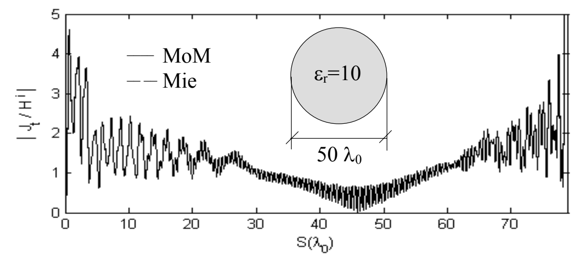

Fig. 2. Electric current, J_t, for a dielectric sphere with radius $R_D = 25\lambda_o$ and $\varepsilon_r = 10$. The current was represented by 1957 TBF and $E_{MR} = 1.1$ %

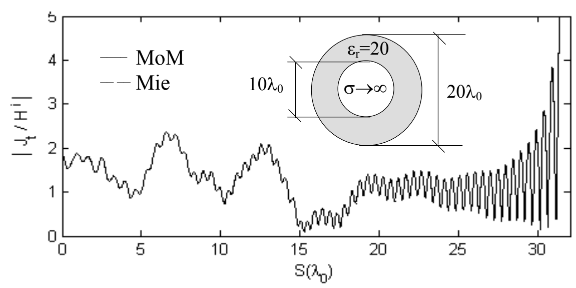

Fig. 3. Electric current, J_t, (in the dielectric surface) for a PEC sphere with radius $R_C = 5\lambda_o$ coated by a dielectric layer with $\varepsilon_r = 20$ and radius $R_D = 10\lambda_o$. The current in each surface was represented by 1099 TBF and $E_{MR} = 0.949$ %.

IV. OPTIMUM NUMBER OF BASIS FUNCTIONS

To obtain the optimum number of segments (i.e., basis functions) per λ_0, the plane-wave scattering by PEC, dielectric, and PEC coated spheres were analyzed for several different ε_r and electrical radii. For each computation, the optimum number (ONS) was determined for $E_{MR} < 1\%$, with respect to the Mie series solutions. Fig. 4(a) shows the ONS per λ_0 for PEC spheres as a function of $R_C(\lambda_0)$ and Fig. 4(b) shows the ONS per λ_0 for dielectric spheres, with $0{,}5 \leq R_D(\lambda_0) \leq 75$, as function of ε_r. For PEC spheres, it can be observed that as $R_C(\lambda_0)$ increases (i.e., as the sphere curvature diminishes), less segments per λ_0 are needed. For dielectric spheres, with $0{,}5 \leq R_D(\lambda_0) \leq 75$, it can be observed that the ONS per λ_0 does not vary with the increase of $R_D(\lambda_0)$, but as ε_r increases more segments are needed. From the numerical data of Figs. 4(a) and (b) empirical formulas for the ONS per λ_0 were obtained:

$$\text{ONS}\Big|_{\text{PEC}} = 15 + 26\,e^{-0.48 R_C/\lambda_0}, \tag{8}$$

$$\text{ONS}\Big|_{\text{dielectric}} = 0.00024\,\varepsilon_r^3 - 0.036\,\varepsilon_r^2 + 1.87\,\varepsilon_r + 17.71\,. \tag{9}$$

Figure 5 shows the ONS per λ_0 for PEC coated spheres. Two cases are considered. In the first, the thickness of the dielectric layer, t_D, is equal to the radius of the conducting sphere, R_C. In the second case $t_D = 0.1R$. The ONS per λ_0 is presented as function of ε_r of the dielectric layer for different R_C and t_D values. In this case it can be observed that the ONS increases with t_D and ε_r and decreases with R_C, as expected. The smallest value of t_D tested was equal to $0.05\ \lambda_0$. It was verified that to reach the desired accuracy it is necessary to use the same number of segments in the PEC and in dielectric surfaces. From the numerical data (ND) shown in Fig. 5 an empirical formula (EF) for the ONS per λ_0 was obtained:

$$\text{ONS}\Big|_{\text{PEC coated}} = \begin{cases} \text{ONS}\Big|_{\text{PEC}}\left[1.3679 - e^{-\varepsilon_r^{0.3}}\right]\varepsilon_r^{t_D} & \varepsilon_r < 2 \\ \left[28.5 + 13.8\,e^{-0.22 R_C}\right]\left[1.28 - 0.55\,e^{-0.2\varepsilon_r}\right]1.08^{t_D} & \varepsilon_r \geq 2 \end{cases} \tag{10}$$

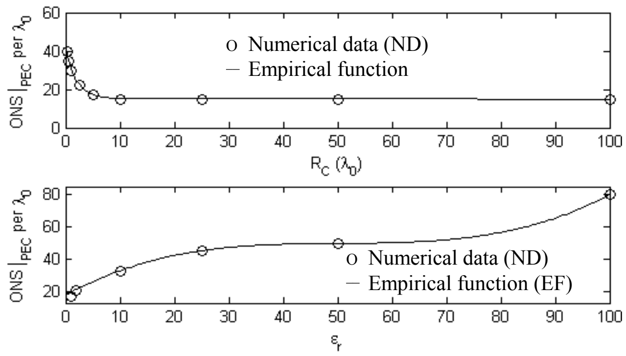

Fig. 4. ONS per λ_o for: a) PEC and b) dielectric spheres.

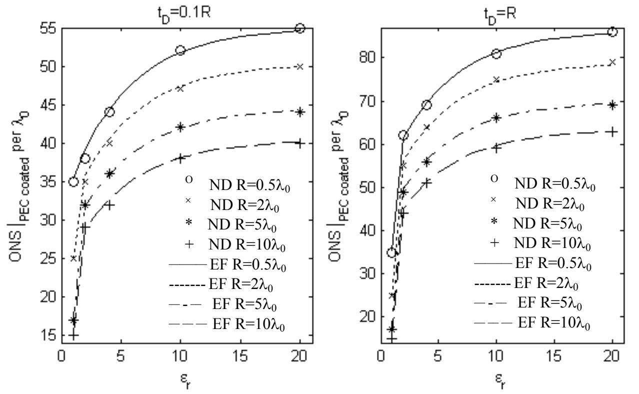

Fig. 5. ONS per λ_o for PEC coated spheres.

V. REFERENCES

[1] L. Medgyesi-Mitschang, J. Putnam, and M. Gedera, "Generalized method of moments for three-dimensional penetrable scatters," J. Opt. Soc. Am. A, Opt. Image Sci., vol. 11, pp. 1383-1398, Apr. 1994.

[2] J. R. Mautz and R. F. Harrington, "An improved E-Field solution for a conducting body of revolution," Tech. Report TR-80-1, Dept. Elect. Comp. Engineering, Syracuse University, 1980.

[3] A. W. Glisson, "On the development of numerical techniques for treating arbitrarily-shaped surfaces," Ph.D. Dissertation, University of Mississippi, June 1975.

[4] J. R. Mautz and R. F. Harrington, "H-Field, E-Field and combined field solutions for bodies of revolution," Tech. Report TR-77-2, Dept. Elect. Comp. Engineering, Syracuse University, 1977.

[5] J. R. Mautz and R. F. Harrington, "Electromagnetic scattering from a homogeneous body of revolution," Tech. Report TR-77-10, Dept. Elect. Comp. Engineering, Syracuse University, 1977.

[6] P. Ylä-Oijala and M. Taskinen, "Well-conditioned Müller formulation for electromagnetic scattering by dielectric objects," IEEE Trans. Antennas and Propagation, vol. 53, no. 10, pp. 3316–3323, Oct. 2005.

[7] A. A. Kishk and L. Shafai, "Improvement of the numerical solution of dielectric bodies with high permittivity," IEEE Antennas and Propagation, vol. 37, no. 11, pp. 1486–1490, Nov. 1989.

[8] B. M. Kolundzija, "Electromagnetic modeling of composite metallic and dielectric structures," IEEE Trans. Microwave Theory Tech., vol. 47, pp. 1021–1032, July 1999.