EMFIE and MEFIE Formulations for Numerical Solution of Single and Composite Bodies of Revolution

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Abstract — In this paper, we present and investigate two formulations, EMFIE (electric-magnetic field integral equation) and MEFIE (magnetic-electric field integral equation), for solving electromagnetic scattering by dielectric and composite bodies of revolution. The formulations are generated by the standard electric and magnetic field surface integral equations. Numerical results are compared to analytical ones demonstrating the accuracy of the proposed formulations.

I. INTRODUCTION

Electromagnetic scattering by conducting, dielectric, and composite bodies is an important and challenging problem in the field of computational electromagnetics. Analytical solutions are available only for very limited geometries. For bodies having arbitrary shapes, one has to resort to numerical techniques. A variety of approaches have been developed to study the problem. For homogeneous bodies, method of moments (MoM) is preferred as the problem can be formulated in terms of surface integrals over the conducting and dielectric surfaces [1],[2]. For bodies of revolution (BOR) the problem is formulated in terms of integrals over generatrices [1],[2]. For perfectly conducting (PEC) BOR the problem has been exhaustively studied and the most accurate formulations are the EFIE (electric field integral equation) and CFIE (combined field integral equation, which is a linear combination of the EFIE and MFIE—magnetic field integral equation) for open and closed conducting bodies, respectively [1]. For dielectric BOR, many combinations of the EFIE and MFIE have been investigated [2],[3]. Some of these are the PMCHWT (solution proposed by Poggio, Miller, Chang, Harrington, Wu and Tsai) and Müller integral equations. Such solutions (or combinations of them) have also been used in the scattering analysis of composite BOR's [4]-[6]. For bodies with regions of different materials the most used formulation is the PMCHWT and for layered bodies many formulations have been applied [4]-[6].

The reported investigations are generally conducted for electrically small BOR (i.e., with dimensions of the order of the wavelength) [1]-[6]. In this work we present and investigate two formulations for solution of the electromagnetic scattering by large and small BOR. Plane-wave scattering from dielectric and composite spheres of different electrical sizes and relative permittivities (ε_r) are evaluated and results are compared to analytical data provided by Mie series. The study shows that the proposed formulations accuracy is approximately equal to classical Müller and PMCHWT formulation acuracy in scattering analysis by different types of BOR.

II. PROBLEM FORMULATION

For a BOR with permittivity ε_1 and permeability μ_1, immersed in an infinite and homogeneous medium with permittivity ε_0 and permeability μ_0, the equivalence principle can be applied to establish a set of four integral equations to solve for the electric (**E**) and magnetic (**H**) fields in terms of equivalent electric (**J**) and magnetic (**M**) surface currents [2]. Assuming that the sources of the incident field ($\mathbf{E}^{inc}$, $\mathbf{H}^{inc}$) are outside the body, the integral equations are [2]:

$$[\eta_0 \mathbf{L}_0(\mathbf{J}) + \mathbf{K}_0(\mathbf{M})]_{\tan} = \mathbf{E}^{inc}_{\tan}, \tag{1}$$

$$[\mathbf{L}_0(\mathbf{M}) - \eta_0 \mathbf{K}_0(\mathbf{J})]_{\tan} = \mathbf{H}^{inc}_{\tan}, \tag{2}$$

$$[\eta_1 \mathbf{L}_1(\mathbf{J}) + \mathbf{K}_1(\mathbf{M})]_{\tan} = 0, \tag{3}$$

$$[\mathbf{L}_1(\mathbf{M}) - \eta_1 \mathbf{K}_1(\mathbf{J})]_{\tan} = 0, \tag{4}$$

where the operators L_i and K_i are

$$\mathbf{L}_i(\mathbf{X}) = \frac{j}{k_i}\int_{s'}\left[k_i^2\, \mathbf{X}(\mathbf{r}')\, G_i(\mathbf{r},\mathbf{r}') - \nabla' \mathbf{X}(\mathbf{r}')\nabla' G_i(\mathbf{r},\mathbf{r}')\right] ds' \tag{5}$$

$$\mathbf{K}_i(\mathbf{X}) = \upsilon_i\, \hat{\boldsymbol{n}} \times \frac{\mathbf{X}(\mathbf{r})}{2} + \int_{s'} \mathbf{X}(\mathbf{r}') \times \nabla' G_i(\mathbf{r},\mathbf{r}')\, ds' \tag{6}$$

the index i represents the interior (i = 1) and exterior (i = 0) regions, **X** is either **J** or **M** over the surface s, υ_i is a constant that values 1 for a field point outside the surface and –1 inside the surface, and G_i is the free space Green's function.

In (1)-(4) there are four equations and two unknowns (**J** and **M**), allowing a number of different combinations to solve for **J** and **M**.

A. PEC bodies

For PEC bodies, the EFIE (1) is the choice for open shells [1]. For closed-surfaces, the CFIE avoids spurious resonances [1] and is a linear combination of (1) and (2):

$$\alpha' \,\mathrm{EFIE}_0 + \beta' \,\mathrm{HFIE}_0, \tag{7}$$

where EFIE_0 and MFIE_0 represent (1) and (2), respectively, and α' and β' are linear-combination weights [4].

B. Homogeneous dielectric bodies

For dielectric bodies, (1)-(4) can be combined in many different ways to yield two integral equations suited to determine **J** and **M** [2]. The commonly adopted combinations are:

i) EFIE: using the EFIE's (1) and (3).

ii) MFIE: using the MFIE's (2) and (4).

iii) CFIE: combining (1) with (2) and (3) with (4) to obtain two new integral equations, (7) and [2]:

$$\alpha' \,\mathrm{EFIE}_1 + \beta' \,\mathrm{HFIE}_1, \tag{8}$$

where EFIE_1 and MFIE_1 represent (3) and (4), respectively. Another way to combine the (1)-(4) is [2]-[3]:

$$\mathrm{EFIE}_0 + \alpha\, \mathrm{EFIE}_1, \tag{9}$$

$$\mathrm{HFIE}_0 + \beta\, \mathrm{HFIE}_1, \tag{10}$$

where linear-combination weights α and β are

iv) PMCHWT: $\alpha = \beta = -1$.

v) Müller: $\alpha = \varepsilon_1/\varepsilon_2$ and $\beta = \mu_1/\mu_2$.

The proposed formulations are:

vi) EMFIE (electric-magnetic field integral equation): using $EFIE_0$, (1), and $MFIE_1$, (4).

vii) MEFIE (magnetic-electric field integral equation): using $MFIE_0$, (2), and $EFIE_1$, (3).

The proposed formulations only use one integral equation for each region. So, they require a computational effort 30% lesser than classical Müller and PMCHWT formulation to fill the Z-matrix.

C. Composite bodies

For composite bodies, PEC surfaces are analyzed by EFIE or CFIE, while the interfaces between dielectric regions can be treated by any combination cited in Sect. II-B.

In the present work the integral equations are numerically evaluated by the MoM technique. Triangular basis functions (TBF) are employed to describe equivalent currents and Galerkin's method is adopted for numerical evaluation of the TBF's coefficients. All Z-matrix integrals are evaluated by Gaussian quadrature with appropriate singularity treatment.

III. NUMERICAL RESULTS

To demonstrate the accuracy of the EMFIE and MEFIE, the surface equivalent currents of several different dielectric, conducting, and composites spheres were computed. For all performed tests the spheres were illuminated by plane waves polarized in $\hat{x}$ and propagating in $\hat{z}$ directions. The numerical results were compared to analytical data obtained from the Mie series. The following mean relative error was adopted:

$$E_{MR}(\%) = \left[E_{RJt} + E_{RJ\phi} + E_{RMt} + E_{RM\phi}\right]/4, \qquad (6)$$

where E_{MR} takes into account the mean relative errors of the tangential components of $\mathbf{J}$ and $\mathbf{M}$ (E_{RX}), according to:

$$E_{RX}(\%) = 100\left[\left|\left|X^{MoM}\right| - \left|X^{Mie}\right|\right| / \left|X^{Mie}\right|_{max}\right]_{max}, \qquad (7)$$

where X^{MoM} and X^{Mie} denotes the MoM and Mie series solutions, respectively.

Figures 1, 2, and 3 show the equivalent electric current in $\hat{t}$ direction, J_t, as functions of the distance $S(\lambda_0)$ measured along the BOR generatrix for dielectric, PEC and dielectric coated spheres. Figure 4 shows the computed external electric current current in $\hat{t}$ direction for a bisected sphere.

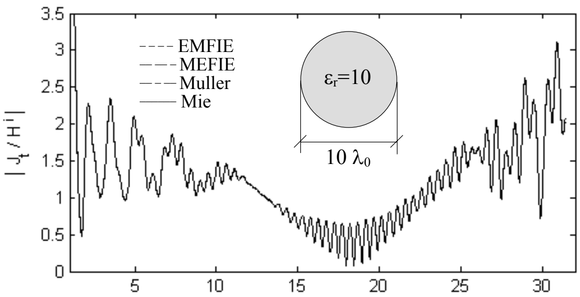

Fig. 1. Electric currente ,J_t. Current representation by 785 TBF. E_{MR} = 0.592 %, E_{MR} = 1.05 % and E_{MR} = 0.80% for EMFIE, MEFIE and Müller formulation, respectively.

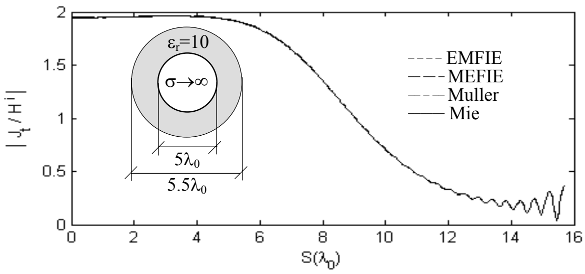

Fig. 2. J_t in external surface. Current representation in each surface by 471 TBF. E_{MR} = 0.44 %, E_{MR} = 1.25 % and E_{MR} = 0.478 % for EMFIE, MEFIE and Müller formulation, respectively.

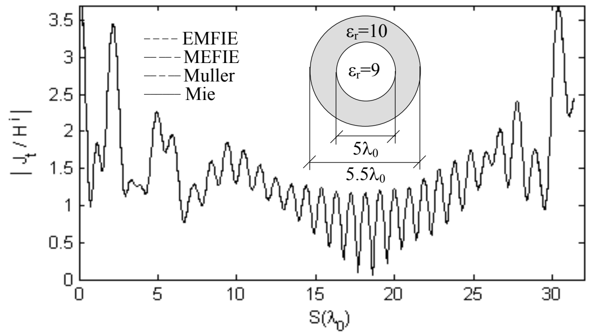

Fig. 3. J_t in external surface. Current representation in each surface by 550 TBF and E_{MR} = 0.98 %, E_{MR} = 0.81 % and E_{MR} = 0.256 % for EMFIE and MEFIE and Müller formulation, , respectively

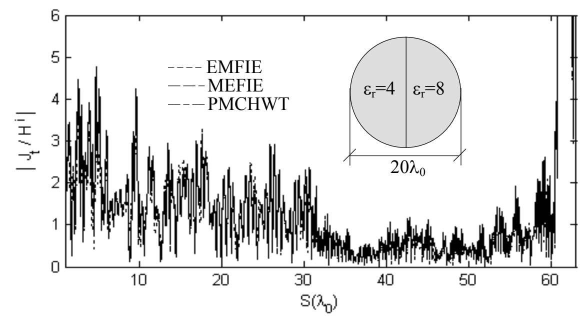

Fig. 4. J_t in external surface. Current representation in external by 942 TBF. E_{MR} in relation to PMCHWT formulation is equal to 6.21 % and 3.12 % for EMFIE and MEFIE, respectively.

For dielectric, PEC and dielectric coated spheres the proposed and Müller formulations accuracy are similar. For bisected spheres PMCWHT is the best formulation. It was observed that E_{MR} erro lesser than 5% does not influence in the fields results accuracy.

IV. REFERENCES

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