AXISYMMETRIC ELECTROMAGNETIC RESONANT CAVITIES SOLUTION BY A MESHLESS LOCAL PETROV-GALERKIN METHOD

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Abstract

This work describes a meshless approach to obtain the resonant frequencies of axisymmetric electromagnetic cavities. The Meshless Local Petrov-Galerkin is used with shape functions generated by Moving Least Squares. Boundary conditions are imposed by a collocation method that does not require integrations. The proposed analysis has a simple implementation and reduces computational effort. Results for TE and TM modes of a spherical cavity are in agreement with analytical solutions.

1 Introduction

The Element Free Galerkin Method (EFGM) has been successfully applied in the solution of wave scattering problems [1]. EFGM requires a background mesh to perform the numerical integration. Recently, Meshless Local Petrov-Galerkin (MLPG), which does not use a mesh even for integration, has been used to solve propagation problems [1] and 3D static problems [6].

The present work determines the resonant frequencies of axisymmetric cavities using MLPG. Similar problems have been solved in [3] using a meshless collocation method. However, Petrov-Galerkin methods present higher convergence rates than collocation methods [4], which is the main advantage of the present approach.

2 Problem Formulation

We investigate the use of MLPG to calculate the resonant wavenumbers of an axisymmetric cavity. To simplify the analysis, only TE and TM modes without ϕ dependence are investigated. This enables a 2D implementation of the problem based on the ϕ-component of the electric (TE) or magnetic (TM) field. With these considerations a pseudoscalar formulation can be employed [7]:

$$\iint_{\Omega} \frac{\nabla\psi \cdot \nabla u}{\rho f(\vec{x})} dA + \oint_{\partial\Omega} \frac{\psi}{\rho f(\vec{x})} \frac{\partial u}{\partial n} dl = \iint_{\Omega} \frac{k^2 g(\vec{x}) \psi u}{\rho} dA \quad (1)$$

where $k = 2\pi/\lambda$ is the wavenumber and ψ is the test function. For TM modes, u is the ϕ-component of the magnetic field multiplied by ρ (i.e., u = ρH_ϕ), $f(\vec{x})$ is the relative electric permittivity ϵ_r and $g(\vec{x})$ is the relative magnetic permeability μ_r. For TE modes, $u = \rho E_\phi$, $f(\vec{x}) = \mu_r$, and $g(\vec{x}) = \epsilon_r$.

3 The Meshless Approach

The meshless approach begins spreading nodes over the problem domain, Ω. All nodes have an associated shape function with compact support, which defines the influence domain of that node [4]. The numerical construction of shape function $\phi_i(\vec{x})$ is done through the Moving Least Square (MLS) approximation and involves several matrix manipulations, which are detailed in [4]. After the definition of $\phi_i(\vec{x})$, the approximation of u in a point $\vec{x}$ is given by:

$$u^h(\vec{x}) = \sum_{i=1}^{N} \phi_i(\vec{x}) \hat{u}_i \quad (2)$$

where $i = 1..N$ are the nodes whose influence domains include point $\vec{x}$ and $\hat{u}_i$ are the nodal parameters [4].

4 The MLPG Analysis

The proposed analysis is similar to MLPG4/LBIE (Local Boundary Integral Equation) [4], but it differs in what concerns the imposition of boundary conditions, which follows the ideas regarding the treatment of interface conditions found in [6]. In the MLPG method it is necessary to spread nodes inside Ω (interior nodes) and nodes exactly over global boundary $\partial\Omega$ (boundary nodes). Interior nodes use the test function ψ_i, which acts in a region near node i (the node's test domain Ω_i) where the integrations are carried out. In LBIE, Ω_i is generally a circle centered at the interior node i and the corresponding test function ψ_i must satisfy the following requirements:

$$\nabla^2\psi_i = -\delta(\vec{x} - \vec{x}_i), \quad \text{a Dirac delta at } \vec{x}_i; \quad (3)$$

$$\psi_i = 0, \text{ at the test domain boundary } \partial\Omega_i; \quad (4)$$

where $\vec{x}_i$ locates the node i in Ω. Equations (3) and (4) are satisfied by the following test function:

$$\psi_i(\vec{x}) = \frac{1}{2\pi} \ln\left(\frac{s_i}{|\vec{x} - \vec{x}_i|}\right) \quad (5)$$

where s_i is the radius of the circular domain Ω_i, chosen such that Ω_i does not intersect the global boundary $\partial\Omega$. The local weak form can be obtained replacing ψ by ψ_i in Equation (1), where the line integral vanishes due to Equation (4).

Boundary conditions are imposed through the boundary nodes. A simple technique (known as meshless collocation scheme) that requires no integration is adopted [6]. Boundary conditions are expressed in general form as:

$$a(\vec{x}_i)u(\vec{x}_i) + b(\vec{x}_i)\frac{\partial u(\vec{x}_i)}{\partial n} = h(\vec{x}_i) \qquad (6)$$

where $a = 1$ and $b = 0$ if $\vec{x}_i$ is at a Dirichlet boundary or $a = 0$ and $b = 1$ if it is at a Neumann boundary. h is the known imposed value. In a cavity with a perfect electric conductor wall, for TE modes the function u satisfies a Dirichlet boundary condition over the wall (i.e. $E_\phi = 0$), while for TM modes a Neumann condition is imposed ($\partial H_\phi/\partial n = 0$). Over the *z*-axis, the Dirichlet condition $u = 0$ is imposed for both modes, as $\rho = 0$.

The numerical solution of the problem is obtained by transforming Equation (1) and Equation (6) into a linear set of equations, built from MLS approximations of u, as defined in Equation (2). The wavenumbers *k* are obtained from the eigenvalues of

$$C - k_o^2 D = 0 \qquad (7)$$

where, for interior nodes,

$$C_{ij} = \iint_{\Omega_i} \frac{1}{\rho f(\vec{x})} \nabla\psi_i \cdot \nabla\phi_j dA \qquad (8)$$

$$D_{ij} = \iint_{\Omega_i} \frac{g(\vec{x})}{\rho} \psi_i \phi_j dA \qquad (9)$$

and, for boundary nodes, $C_{ij} = \phi_j(\vec{x}_i)$ (Dirichlet) or $C_{ij} = \partial\phi_j(\vec{x}_i)/\partial n$ (Neumann), and $D_{ij} = 0$.

5 Numerical Results and Conclusions

A spherical cavity with radius equal to 1 m and vacuum in its interior ($\varepsilon_r = \mu_r = 1$) is analyzed. Table 1 shows analytical [2] and numerical results (obtained with 630 nodes uniformly spaced over the domain) for the resonant wavenumbers of some particular modes (m = 0). The maximum relative error is 0.519%. Figure 1 shows the numerical field distributions inside the spherical cavity for TM011 (H_ϕ) and TE011 (E_ϕ) modes.

TM Modes				TE Modes			
Mode (n,p)	Analytical Solution	Numerical Solution	Error (%)	Mode (n,p)	Analytical Solution	Numerical Solution	Error (%)
(1,1)	2.744	2,758	0,519	(1,1)	4.493	4,491	0,048
(2,1)	3.870	3,889	0,502	(2,1)	5.763	5,760	0,045
(3,1)	4.973	4,996	0,459	(3,1)	6.988	6,984	0,053
(4,1)	6.062	6,085	0,380	(1,2)	7.725	7,720	0,069
(1,2)	6.117	6,146	0,481	(4,1)	8.183	8,178	0,060
(5,1)	7.140	7,164	0,334	(2,2)	9.095	9,089	0,068
(2,2)	7.443	7,476	0,444	(5,1)	9.356	9,351	0,058

Table 1: Resonant Wavenumbers k (rad/m), for m = 0.

This work combines, for the first time, an axisymmetric analysis with the Meshless Local Petrov Galerkin method. The proposed analysis is precise and simple, having higher convergence rates than collocation methods and not requiring a background mesh, which is indispensable for the EFG method. In the extended paper, the obtained convergence rates and other examples of axisymmetric cavities will be presented.

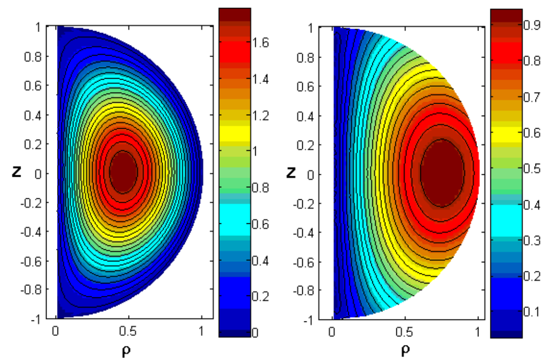

Figure 2: Numerical Field distribution: (a) TE011(E_ϕ) and (b) TM011 (H_ϕ).

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