

Combined Formulation for Meshless-MoM Hybrid Method Applied to 2D Electromagnetic Scattering

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Abstract—In this work a new formulation for the hybrid Meshless - Method of Moments is presented. The proposed approach employs the Interpolating Element Free Galerkin Meshless Method (IEFGM) and three different combinations of Electric Field Integral Equation (EFIE) and Magnetic Filed Integral Equation (MFIE). The technique is applied to solve the TMz scattering by an infinite dielectric cylinder and numerical and analytical results are compared.

Index Terms—IEFGM, EFIE, CFIE, MFIE, Scattering.

I. INTRODUCTION

The use of hybrid techniques is a well known and established way to solve the electromagnetic scattering from composite bodies. This kind of solution combines two different techniques in order to explore the main advantage and strong features of each. The most common and used hybrid technique combines Finite Element Method (FEM) and the Method of Moments (MoM) [1]. However FEM is a mesh based method and the mesh generation is a difficult and time-consuming task in complex geometries. In this context, the development of hybrid methods combining Meshless methods, which does not require a mesh structure, and MoM is an attractive and interesting way to deal with complex geometries and inhomogeneities. This work presents a new formulation (IEFGM-CFIE) for hybrid IFEFGM-MoM technique [2].

II. IEFGM-CFIE FORMULATION

The TM_Z plane wave scattering by a z-directed infinitely long dielectric cylinder, with relative electric permittivity ϵ_r and relative magnetic permeability μ_r is the problem under analysis. The cylinder cross section is Ω and Γ is its boundary, in which the unit normal vector is \mathbf{n} . For the internal region of the scatter the IEFGM is used to solve the weak form of 2D Helmholtz equation [3], [4]:

$$\int_{\Omega} [\mu_r^{-1} (\nabla \times \mathbf{W}_E) \cdot (\nabla \times \mathbf{E}) - k_0^2 \epsilon_r \mathbf{W}_E \cdot \mathbf{E}] d\Omega = j\omega\mu_0 \int_{\Gamma} \mathbf{W}_E \cdot \mathbf{J} d\Gamma, \quad (1)$$

where k_0 is the vacuum wave number, \mathbf{E} is the electric field, \mathbf{J} is the surface electric current and \mathbf{W}_E is a IEFGM test function. The external equivalent problem is evaluated by EFIE and MFIE, respectively:

$$\int_{\Gamma} \mathbf{W} \cdot \left[\int_{\Gamma} (k_0 \eta_0 \mathbf{J} G + \nabla \times \mathbf{J} \mathbf{M}) d\Gamma' - \mathbf{M} - \mathbf{E}^i \right] d\Gamma = 0 \quad (2)$$

$$\int_{\Gamma} \mathbf{W} \cdot \left[\int_{\Gamma} (\omega \epsilon_0 \mathbf{M} G + \nabla \nabla \cdot \mathbf{M} G (\omega \mu_0)^{-1} - \nabla \times \mathbf{J} G) d\Gamma' + \mathbf{J} - \mathbf{H}^i \right] = 0 \quad (3)$$

where $G = 0.25H_0^{(2)}(k_0 R)$, is the Hankel function of the second order, R is the distance between observation and source points, \mathbf{W} is a MoM test function, \mathbf{M} is the magnetic surface current

and \mathbf{E}^i and \mathbf{H}^i are the electric and magnetic incident fields, respectively. \mathbf{E} in the interior and exterior regions is linked by:

$$\int_{\Gamma} \mathbf{W} \cdot (\mathbf{E} - \mathbf{n} \times \mathbf{M}) d\Gamma = 0. \quad (4)$$

Using (1) and different combinations of (2)-(4) the following formulations are possible in order to find \mathbf{E} , \mathbf{J} and \mathbf{M} : IEFGM-EFIE (F1) [2], IEFGM-MFIE (F2), IEFGM-CFIE (F3), IEFGM-CFIE1 (F4) and IEFGM-CFIE2 (F5).

III. NUMERICAL RESULTS

To evaluate the proposed techniques a dielectric cylinder with radius of $2\lambda_0$ (λ_0 is the vacuum wavelength) and $\epsilon_r = 2$ was chosen. The average relative error, E(%), was used to verify the agreement between numerical and analytical results. 14561 nodes were spread over Ω and Γ . The numerical result obtained for \mathbf{E} is shown in Fig.1 and E(%) error in Table I.

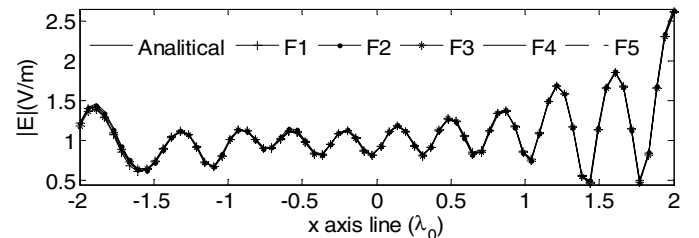


Fig. 1. \mathbf{E} along the cross-section diameter.

TABLE I
AVERAGE RELATIVE ERROR, E(%)

| | F1 | F2 | F3 | F4 | F5 |
|----------|------|------|------|------|------|
| E | 2.77 | 3.02 | 2.79 | 2.46 | 2.77 |
| J | 3.36 | 2.56 | 2.37 | 2.18 | 3.35 |
| M | 5.47 | 3.33 | 3.73 | 3.73 | 5.47 |

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