Comparison of Different Integral Equations in Moment Methods Applied to Solve Scattering Problems from Bodies of Revolution

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Abstract – In this paper, several integral equations derived for scattering problems are evaluated by the MoM technique and compared to each other in the numerical analysis of scattering by bodies of revolution when illuminated by plane-waves. The numerical results are compared to Mie-series analytical solutions. Dielectric and composite bodies with different electrical size and different relative permittivity (for dielectrics) are analyzed. It is shown that Müller integral equation is the most accurate solution for dielectric and layered bodies analysis, PMCHWT is the most accurate for bodies with homogeneous regions analysis.

I. INTRODUCTION

One of the most efficient numerical techniques to analyze the scattering by a body of revolution (BOR) is the surface integral equation, evaluated by the moment method (MoM). For perfectly conducting BOR´s the problem has been exhaustively studied [1]. For dielectric BOR´s, many combinations of the electric (EFIE) and magnetic (MFIE) field integral equations have been investigated [2]-[5]. Some of these are the EFIE, MFIE, CFIE, PMCHWT, and Müller integral equations [2]-[5]. Such solutions (or proper combinations of them) have also been used in the analysis of scattering by composite BOR's (i.e., bodies formed by dielectric and conductors) [3]. It has been reported that for dielectric BOR´s with small relative permittivity (ε_R), Müller integral equation is the most accurate and well-conditioned solution, especially for $\varepsilon_R \approx 1$ [2],[4],[5]. Even as ε_R and, consequently, the number of basis functions increase, Müller integral equation is still the preferred option, [2] and [5], despite the increase of numerical errors, but other formulations can be more accurate [3]. For BOR´s with regions of different materials the most used formulation is PMCHWT [6]. The problem is that the reported investigations are generally conducted for electrically small BOR's (i.e., with dimensions of the order of the wavelength). The objective of the present study is to evaluate the accuracy of different integral equations (EFIE, MFIE, CFIE, PMCHWT, and Müller) in the analysis of large dielectric and composite BOR's.

II. INTEGRAL EQUATION FORMULATIONS

The EFIE and MFIE equations

$$\hat{n} \times \vec{E}_i^s(\vec{J}_i,\vec{M}_i) + \hat{n} \times \vec{E}_i^{inc} = -\vec{M}_i, \qquad (1)$$

$$\hat{n} \times \vec{H}_i^s(\vec{J}_i,\vec{M}_i) + \hat{n} \times \vec{H}_i^{inc} = \vec{J}_i, \qquad (2)$$

where $i = 1,2$ represent the external and internal media, are properly combined to yield integral equations suited for the analysis of the scattering by dielectric BOR's. In the present work integral equations are numerically evaluated by the MoM technique. Triangular basis functions are employed for the equivalent current representation and the Galerkin's method is adopted for the numerical evaluation.

III. CASE STUDIES

To investigate the different integral equation solutions, the plane-wave scattering from differently sized dielectric and composite spheres, were analyzed. For the dielectric regions, their relative permittivities (ε_R) were varied from 1 (free-space) to 100. The sphere generatrix is described by straight segments. Figure 1 presents the men relative errors, E_{RM}, for the surface equivalent currents with respect to Mie series for dielectric spheres with different radius values, $R(\lambda_o)$, and ε_R. It can be observed in the majority of case, in special for ε_R=1, that the most accurate formulation is Müller, as expected [2] and [5].

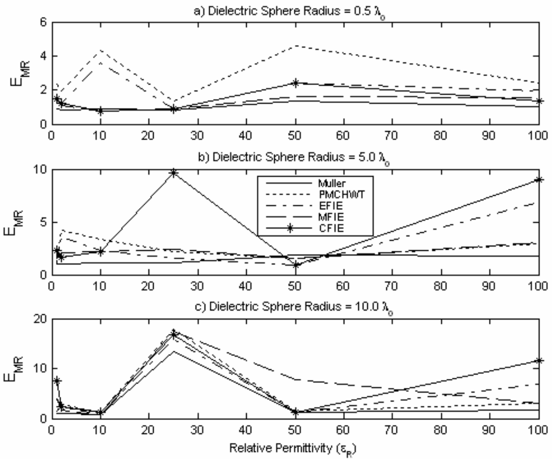

Fig. 1. Mean elative error for dielectric spheres.

IV. REFERENCES

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