Efficient Evaluation of Singular Integrals in Moment Methods Applied to Solve Scattering Problems from Bodies of Revolution

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Abstract— This work proposes an efficient procedure for evaluating singular integrals arising in the moment method analysis of scattering from bodies of revolution. Perfect electric conductor, dielectric and composite bodies with different electrical sizes and different relative permittivities are analyzed by the CFIE, Müller and PMCHWT integral equation, respectively. It is demonstrated that an efficient singularity removal reduces the number of basis functions necessary to represent the equivalent current distributions.

I. Introduction

The efficiency of the moment method (MoM) in the numerical analysis of electromagnetic scattering from objects made of simple media (i.e., perfect conductors and dielectrics) has been already demonstrated [1]-[5]. Among the necessary precautions for a numerically stable analysis are the choice of suitable basis functions for proper surface current representation, efficient algorithms for solving the linear system of equations, and numerical procedures to adequately evaluate the integrals of the matrices elements, in particular of the Z-matrix. The evaluation of singular integrals in the Z-matrix self terms is not a straightforward task. One way to do this is to apply singularity extraction technique and to calculate the extracted singular integral in closed form. These technique is used in [5] with RWG functions and EFIE and MFIE to dielectric bodies of arbitrary shape, and in [3] with triangular and pulse functions and EFIE for conducting bodies of revolution (BOR´s).

II. Integral Evaluation

The Z-matrix elements arising in the MoM analyses of BOR's, together with the Galerkin's method are of the form

$$\iiint w(t)\, f(t')\, G(t,t',\varphi)\, dt\, dt'\, d\varphi \qquad (1)$$

where $w(t)$ is the test function, $f(t')$ is the basis function, G is a certain element of the Green's function dyad, and t, t', and $\varphi = \phi - \phi'$ are the coordinates locating the source and observer over the BOR surface [1]-[3]. Singularities occur whenever an observation is made upon the source location (i.e., $t = t'$ and $\varphi = 0$). In this work we propose an accurate and robust numerical procedure, based on [3], to remove the integrand singularities arising in the CFIE, PMCHWT and Müller integral equation solutions. The equivalent current representation adopted is the triangular basis functions for $\hat{t}$ and $\hat{\phi}$ directions. These current representation leads to integrals equations whose the singularities are of order $1/R$, $1/(R\rho)$, $1/(R\rho')$, $1/(R\rho'\rho)$, and $1/R^3$, where $R = |\boldsymbol{r} - \boldsymbol{r}'|$ is the distance between observation and source points. In the proposed method, all singularities are extracted and calculated in close form. Numerical integration (Gaussian quadrature) is applied only for regular integrands.

III. Case Studies

In order to demonstrate the efficiency of the proposed procedure, the plane-wave scatterings from differently-sized conducting, dielectric and composite spheres were analyzed by the CFIE [1] for conducting spheres, Müller for homogeneous and layered dielectric spheres, and PMCHWT for bisected spheres [2]. For dielectric media, the relative permittivity (ε_R) was varied from 1 (free-space) to 100. Using the Mie-series solution, the mean relative error (E_{MR}) for the equivalent currents was calculated. Figure 1 shows the computed electric current in $\hat{t}$ direction over the external surface of a coated conducting sphere with $a = 10\lambda_o$ (λ_o is the wavelength in vacuum), $b = 11\lambda_o$ and $\varepsilon_R = 20$. In this case, 1099 triangular basis functions (TBF) were used to represent the current over each BOR generatrix. When singularity remotion is used, $E_{MR} = 1.24\%$, while $E_{MR} = 40\%$ on the contrary.

In another example, a conducting sphere with $a = 10\lambda_o$ was analyzed with 471 TBF. With singularity removal, $E_{MR} = 0.91\%$. To attain de same E_{MR} without the proposed singularity removal, 600 TBF were needed. Similar results were verified for dielectric and composites spheres. In some cases, when the singularity removal was not used, the desired accuracy could not be reached at all.

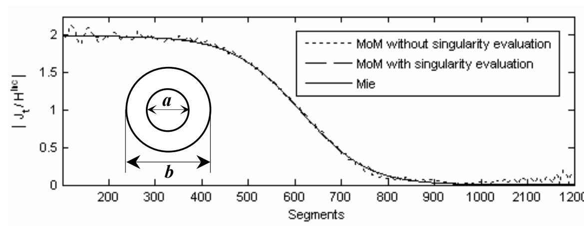

Fig. 1. Electric surface current in $\hat{t}$ direction for the CEP sphere with $a = 10\lambda_o$ coated with a dielectric layer with $b = 11\lambda_o$ and $\varepsilon_R = 20$.

IV. References

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