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Fringing Field Correction for
Spherical-Rectangular Microstrip Antennas

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Abstract

This paper presents the analysis of spherical-rectangular patches using both cavity method and method of moments. A detailed procedure of cavity method is presented, including the field distribution inside the cavity, equivalent magnetic current distributions, radiation fields, effective loss tangent, and input impedance. A new fringing field correction is proposed taking into account the curvature of the patch, resulting in accurate resonance frequency.

Keywords: microstrip antennas, spherical antennas, conformal antennas, spherical patches.

1 Introduction

Microstrip antennas have been widely used due to their low cost, electrical properties, as well as their ability to connect to microwave circuits. In many applications the microstrip antennas need to adjust to non planar surfaces, as in satellites, rockets, airplanes, or for wearable antennas. These conformal geometries, like cylindrical [1, 2], spherical [3, 4] or conical [5, 6], also allow different radiation and electrical properties, and are used accordingly. Spherical microstrip antennas are those printed onto a spherically shaped dielectric layer that covers a spherical conductor. Usual shapes of the spherical antennas include the circular, annular-ring, and rectangular (or quasi-rectangular) patches.
Nonplanar microstrip antennas have been described, for different antenna shapes, arrays, and analysis methods [7, 8]. Particularly, spherical microstrip antennas have been studied using several techniques, including cavity method [9]-[14], electric surface current method [15], Generalized Transmission Line Model (GTLM) [16, 17], and Method of Moments (MoM) [18]-[26]. Although cavity method is among the simplest ones, it provides invaluable physical insight about the field distribution and current excitation on the patch. It assumes the fields in the dielectric to be bounded in the volume between the antenna and the spherical conductor, where they are determined from expansion in cavity eigenmodes. Radiation properties are obtained from equivalent magnetic surface currents on the cavity side walls, neglecting the presence of the dielectric layer. Despite its simplicity the method results in accurate radiation patterns. Additionally, evaluation of input impedances requires the determination of equivalent loss tangent, including dielectric, radiation and conductor losses, and a correction on the resonant length of the antenna is required to account for fringe fields. Previous contributions using cavity method for spherical microstrip antenna include obtaining the radiation efficiency [9] and radiation patterns [10] for spherical-rectangular microstrip antennas, and for circular disk and annular ring [11]-[13]. In [14] a Mathematica based CAD was presented using cavity method to obtain input impedances and radiation patterns of spherical annular and circular microstrip antennas. On the other hand, the electric surface current method assumes the electric current on the surfaces of the patches to be known, either from cavity method or any other source, and computes the radiation fields and input impedance using the proper Green’s functions, including the presence of the dielectric layer. It was used in [15] for determining the radiation patterns of microstrip spherical circular disk antennas. The input impedance can also be obtained from GTLM, by modeling the antenna as a transmission line loaded with wall admittances, as in [16] for spherical circular, and in [17] for spherical annular antennas. The MoM is a full-wave method in which the electric current distribution on the patches are expanded into a set of known functions. The expansion coefficients are determined by imposing the boundary conditions on the patch surface. Appropriate symmetric products by a set of weighting functions.
transform the boundary condition equations into a linear system, from which the expansion coefficients are determined. MoM was used in [19]-[21] to obtain resonances, radiation patterns, and input impedances of spherical circular and annular microstrip antennas. It was also used in [22]-[25] to analyze spherical-rectangular microstrip antennas and arrays, and in [26, 27] to analyze spherical-trapezoidal microstrip antennas and arrays. Procedures for evaluation of Green’s functions in spherically layered media were presented in [28]-[30]. And results for circular and quasi-rectangular microstrip antennas for multilayered spherical media were shown in [31, 32]. Curvilinear Rao-Wilton-Glisson triangular basis functions were used in [33, 34] for modeling antennas in spherically layered media.

This paper presents the application of cavity method to spherical-rectangular patches, and introduces a new fringing field correction to take into account the patch curvature. Section 2 describes the geometry of the spherical microstrip patches. Section 3 shows the application of cavity method to these antennas. Section 4 describes the application of MoM to spherical microstrip antennas. Section 5 describes how the fringing field corrections used for planar microstrip antennas can be modified to account for the curvature of spherical patches. Results are presented in section 6 comparing the solutions.

2 Geometry of the Antennas

Figure 1 shows the geometry of the spherical-rectangular microstrip patch. A metallic sphere of radius $r_1$ is covered by a dielectric layer of thickness $h$ ($r_2 = r_1 + h$), relative permittivity $\varepsilon_r$, and loss tangent $\tan\delta_d$. The patch is printed onto the dielectric surface at $r = r_2$, limited by angles $\theta_1$, $\theta_2$, $\phi_1$, and $\phi_2$, in spherical coordinates. And fed by coaxial cables centered at $\theta_f$ and $\phi_f$. For simplicity the inner conductor of the coaxial cable feeding the patch is modeled as a strip [35] at $\theta_f$, $r_1 < r < r_2$, and $\phi_1 < \phi < \phi_2$, as presented in Fig. 2.
Figure 1: Geometry of the spherical-rectangular microstrip patch.

Figure 2: Feeding strip representing the coaxial cable.
3 Application of Cavity Method

The cavity method assumes the fields in the dielectric layer to be confined within the region below the patch, i.e., in a cavity formed by the patch, the metallic sphere, and perfect magnetic conductors on the side walls [9]-[14].

3.1 Cavity fields

Usually the thickness of the dielectric layer is small compared to the wavelength, and the fields in the cavity are almost independent of \( r \). Assuming no radial variation (\( \partial / \partial r = 0 \)), only TM\(^r\) fields are excited in the cavity, and the radial electric field is the solution of:

\[
\frac{1}{r_{12}^{12}} \sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial E_r}{\partial \theta} \right) + \frac{1}{r_{12}^{12}} \sin^2 \theta \frac{\partial^2 E_r}{\partial \phi^2} + k_d^2 E_r = jw \mu J_f
\]  

(1)

where \( r_{12} = (r_1 + r_2)/2 \), \( k_d = \sqrt{\mu \varepsilon_d} \) is the wave number in the dielectric, \( \varepsilon_d = \varepsilon_o \varepsilon_r (1 - j \tan \delta_d) \) is the complex permittivity of the dielectric, and \( J_f \) is the volume density of the feed current in radial direction. The electric field in the cavity can be expanded into TM\(^r\) modes, already satisfying the boundary conditions on the patch, metallic sphere and perfect magnetic walls:

\[
E_r(\theta, \phi) = \sum_u \sum_l e_{lu} E_{r,lu}(\theta, \phi)
\]  

(2)

where \( e_{lu} \) are the coefficients of the expansion, and the eigenmodes \( E_{r,lu} \) are solution of the source-free problem:

\[
\frac{1}{r_{12}^{12}} \sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial E_{r,lu}}{\partial \theta} \right) + \frac{1}{r_{12}^{12}} \sin^2 \theta \frac{\partial^2 E_{r,lu}}{\partial \phi^2} + k_{lu}^2 E_{r,lu} = 0
\]  

(3)

These modes are obtained by imposing the boundary conditions on the cavity walls \( \theta = \theta_1 \), \( \phi = \phi_1 \), \( \phi = \phi_2 \), and are given by:

\[
E_{r,lu} = \left[ P_l^u(\cos \theta) \frac{d}{d\theta} Q_l^u(\cos \theta_1) - \frac{d}{d\theta} P_l^u(\cos \theta_1) Q_l^u(\cos \theta) \right] \cos (u(\phi - \phi_1))
\]  

(4)

where \( P_l^u(\cdot) \) and \( Q_l^u(\cdot) \) are associated Legendre functions, \( u = \frac{p \pi}{\Delta \phi} \), \( p = 0, 1, 2, \ldots \), and \( \Delta \phi = \phi_2 - \phi_1 \). The values of eigenvalue \( l \), for a given \( u \), are obtained from the transcendental equation
below, that ensures the boundary condition on the side wall \( \theta = \theta_2 \):

\[
\frac{d}{d\theta} P_{l}^u(\cos \theta_2) \frac{d}{d\theta} Q_{l}^u(\cos \theta_1) - \frac{d}{d\theta} P_{l}^u(\cos \theta_1) \frac{d}{d\theta} Q_{l}^u(\cos \theta_2) = 0
\] (5)

and

\[
k_{lu} = \sqrt{\frac{l(l+1)}{r_{12}}}
\] (6)

While the modal electric field has only \( r \)-component (4), the magnetic field has \( \theta \) and \( \phi \)-
components given by [36]:

\[
H_{\theta,lu} = \frac{-1}{j w \mu r_{12} \sin \theta} \frac{\partial E_{r,lu}}{\partial \phi} = \frac{u}{j w \mu r_{12} \sin \theta} \left[ P_{l}^u(\cos \theta) \frac{d}{d\theta} Q_{l}^u(\cos \theta_1) - \frac{d}{d\theta} P_{l}^u(\cos \theta_1) \frac{d}{d\theta} Q_{l}^u(\cos \theta) \right] \sin (u(\phi - \phi_1))
\] (7)

\[
H_{\phi,lu} = \frac{1}{j w \mu r_{12}} \frac{\partial E_{r,lu}}{\partial \theta} = \frac{1}{j w \mu r_{12}} \left[ \frac{d}{d\theta} P_{l}^u(\cos \theta) \frac{d}{d\theta} Q_{l}^u(\cos \theta_1) - \frac{d}{d\theta} P_{l}^u(\cos \theta_1) \frac{d}{d\theta} Q_{l}^u(\cos \theta) \right] \cos (u(\phi - \phi_1))
\] (8)

Assuming the feed current to be uniform across the strip, see Fig. 2, its volume distribution,
corresponding to a total current \( I_0 \), is given by:

\[
J_f(r, \theta, \phi) = \frac{I_0}{r_{12}^2 \sin(\theta_f) \Delta \phi_f} \delta(\theta - \theta_f)
\] (9)

for \( r_1 < r < r_2 \) and \( \phi_1 f < \phi < \phi_2 f \), where \( \Delta \phi_f = \phi_2 f - \phi_1 f \). Substituting the eigenmode
expansion (2) in (1) and using (4):

\[
\sum_u \sum_l e_{lu} \left( k_d^2 - k_{lu}^2 \right) E_{r,lu}(\theta, \phi) = j w \mu J_f
\] (10)

Using the orthogonality of the eigenmodes \( E_{r,lu} \), the expansion coefficients \( e_{lu} \) are given by:

\[
e_{lu} = \frac{j w \mu}{k_d^2 - k_{lu}^2} \frac{< J_f, E_{r,lu} >}{\| E_{r,lu} \|^2}
\] (11)

where the inner product is defined as:

\[
< f, g > = \int_0^\pi \int_{-\pi}^\pi f(\theta, \phi) g^*(\theta, \phi) \sin \theta \ d\phi \ d\theta
\] (12)
Substituting the expansion coefficients $e_{lu}$ (11) in (2), the electric field in the cavity is given by:

$$E_r(\theta, \phi) = \sum_u \sum_l \frac{j\omega}{k^2 - k^2_{lu}} J_f^u(\cos \theta_f) Q_l^u(\cos \theta_1) - \frac{d}{d\theta} P_l^u(\cos \theta_1) Q_l^u(\cos \theta_f)$$

where the inner product $\langle J_f, E_{r,lu} \rangle$ is given by:

$$\langle J_f, E_{r,lu} \rangle = \frac{I_o}{r_{12}^2} \int_0^{\phi_2} \left[ P_l^u(\cos \theta) \frac{d}{d\theta} Q_l^u(\cos \theta_1) - \frac{d}{d\theta} P_l^u(\cos \theta_1) Q_l^u(\cos \theta_1) \right] \sin \frac{u\Delta \phi_f}{2\pi} \cos (u(\phi_f - \phi_1))$$

where $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. And the squared norm of $E_{r,lu}$ is given by:

$$\|E_{r,lu}\|^2 = \frac{\Delta \phi}{\tau_u} \int_0^{\phi_2} \left[ P_l^u(\cos \theta) \frac{d}{d\theta} Q_l^u(\cos \theta_1) - \frac{d}{d\theta} P_l^u(\cos \theta_1) Q_l^u(\cos \theta) \right]^2 \sin \theta \ d\theta$$

where $\tau_u = 1$ if $u = 0$, and $\tau_u = 2$ if $u \neq 0$.

### 3.2 External Fields

The fields external to the cavity can be obtained with the aid of the equivalence principle [36]. Equivalent magnetic current surface densities are obtained from the electric fields at the four side walls of the cavity. Due to the small thickness of the dielectric, these surface current densities can be concentrated as surface current distributions on the metallic sphere ($r = r_1$):

$$\tilde{M}_{s1}(\theta, \phi) = -E_r(\theta_1, \phi) \frac{\hbar}{r_1} \delta(\theta - \theta_1) \hat{a}_\phi \quad \phi_1 < \phi < \phi_2$$

$$\tilde{M}_{s2}(\theta, \phi) = E_r(\theta_2, \phi) \frac{\hbar}{r_1} \delta(\theta - \theta_2) \hat{a}_\phi \quad \phi_1 < \phi < \phi_2$$

$$\tilde{M}_{s3}(\theta, \phi) = E_r(\theta, \phi_1) \frac{\hbar}{r_1 \sin \theta} \delta(\theta - \phi_1) \hat{a}_\theta \quad \theta_1 < \theta < \theta_2$$

$$\tilde{M}_{s4}(\theta, \phi) = -E_r(\theta, \phi_2) \frac{\hbar}{r_1 \sin \theta} \delta(\phi - \phi_2) \hat{a}_\theta \quad \theta_1 < \theta < \theta_2$$

The external fields generated by the magnetic sources (16)-(19), ignoring the presence of the dielectric layer, can be obtained using field expansion in $\text{TM}_{mn}$ and $\text{TE}_{mn}$ external modes, obtained using the vector potentials:

$$A_r^x(r, \theta, \phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} e_{nm} \hat{H}_n^{(2)}(k_o r) \bar{D}_m^{|m|}(\cos \theta) e^{jm\phi}$$

$$F_r^o(r, \theta, \phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} f_{nm} \hat{H}_n^{(2)}(k_o r) \bar{D}_m^{|m|}(\cos \theta) e^{jm\phi}$$
where $\hat{H}_n^{(2)}(.)$ is the Schelkunoff Hankel function of second kind, $\tilde{P}_{mn}^{[m]}(.)$ is the normalized associated Legendre function, as defined in (78), superscript "o" in $A_r^o$ and $F_r^o$ stands for free space, and $k_o = w\sqrt{\mu/\varepsilon_o}$ is the wave number in free space. The electric field can be obtained from the vector potentials [36]:

$$E_r = \frac{1}{jw\varepsilon} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) A_r$$  \hspace{1cm} (22)

$$E_\theta = -\frac{1}{r \sin \theta} \frac{\partial F_r}{\partial \phi} + \frac{1}{jw\varepsilon r} \frac{\partial^2 A_r}{\partial r \partial \theta}$$  \hspace{1cm} (23)

$$E_\phi = \frac{1}{jw\mu} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) F_r$$  \hspace{1cm} (24)

$$H_r = \frac{1}{jw\mu} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) F_r$$  \hspace{1cm} (25)

$$H_\theta = \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} + \frac{1}{jw\mu r} \frac{\partial^2 F_r}{\partial r \partial \theta}$$  \hspace{1cm} (26)

$$H_\phi = -\frac{1}{r} \frac{\partial A_r}{\partial \theta} + \frac{1}{jw\mu r \sin \theta} \frac{\partial^2 F_r}{\partial r \partial \phi}$$  \hspace{1cm} (27)

The radiated power associated to each external $mn$-mode (of unitary amplitude) is given by:

$$P_{r,mn} = \frac{1}{2} \int_0^\pi \int_0^{2\pi} \Re \left[ E_{mn} \times H_{mn}^* \right] \cdot \hat{a}_r \ r^2 \ \sin \theta \ d\phi \ d\theta$$  \hspace{1cm} (28)

which results in:

$$P_{r,mn}^{TM} = \frac{\eta_o}{2} S(n)$$  \hspace{1cm} (29)

$$P_{r,mn}^{TE} = \frac{1}{2\eta_o} S(n)$$  \hspace{1cm} (30)

where $\eta_o = \sqrt{\mu/\varepsilon_o}$ is the intrinsic impedance of free space and $S(n)$ is given by (77).

In the normalized vector Legendre domain, see Appendix A:

$$\tilde{E}_o(r,n,m) = \begin{bmatrix} E_o^u(r,n,m) \\ E_o^d(r,n,m) \end{bmatrix} = \begin{bmatrix} -j\eta_o \sqrt{S(n)} \hat{H}_n^{(2)_o}(k_o r) \ e_{nm} / r \\ \sqrt{S(n)} \hat{H}_n^{(2)_o}(k_o r) \ f_{nm} / r \end{bmatrix}$$  \hspace{1cm} (31)

$$\tilde{H}_o(r,n,m) = \begin{bmatrix} H_o^u(r,n,m) \\ H_o^d(r,n,m) \end{bmatrix} = \begin{bmatrix} -j \sqrt{S(n)} \hat{H}_n^{(2)_o}(k_o r) \ e_{nm} / (\eta_o r) \\ -\sqrt{S(n)} \hat{H}_n^{(2)_o}(k_o r) \ f_{nm} / (\eta_o r) \end{bmatrix}$$  \hspace{1cm} (32)

The coefficients $e_{nm}$ and $f_{nm}$ are determined from the boundary conditions at $r = r_1$ interface:

$$\hat{M}_s(\theta, \phi) = -\hat{a}_r \times \hat{E}(r_1, \theta, \phi)$$  \hspace{1cm} (33)
which can also be written in the normalized vector Legendre domain, as:

\[
\begin{bmatrix}
M_u(n, m) \\
M_d(n, m)
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix} \begin{bmatrix}
E_u(r_1, n, m) \\
E_d(r_1, n, m)
\end{bmatrix}
\] (34)

Substituting (31) into (34), the coefficients \(e_{nm}\) and \(f_{nm}\) are given by:

\[
e_{nm} = \frac{r_1}{jn_0 \sqrt{S(n)} H_n^{(2)}(k_0 r_1)} M_d(n, m)
\] (35)

\[
f_{nm} = \frac{r_1}{\sqrt{S(n)} H_n^{(2)}(k_0 r_1)} M_u(n, m)
\] (36)

where \(M_u(n, m)\) and \(M_d(n, m)\) are obtained from substituting (16)-(19) into (75).

### 3.3 Input Impedance

The input impedance at the probe is given by the variational expression [36]:

\[
Z_{in} = -\frac{1}{I_o^2} \iint_S E(\mathbf{J}_f) \cdot \mathbf{J}_f ds
\] (37)

Substituting (9) and (13) results:

\[
Z_{in} = -\frac{h}{I_o^2} \sum_u \sum_l \frac{jw\mu}{k_d^2 - k_{lu}^2} < J_f, E_{r,lu} >^2
\] (38)

\(Z_{in}\) shown above already includes the effects of dielectric losses, as \(k_d^2 = w^2\mu \varepsilon_d = w^2\mu \varepsilon_r \varepsilon_o (1 - j \tan \delta_d)\). Additionally, the conductor and radiation losses can be taken into account by substituting \(\tan \delta_d\) by an effective loss tangent (\(\tan \delta_{eff}\)) that includes all losses in the dielectric (\(\tan \delta_d\)), in the conductor (\(\tan \delta_c\)), and also radiation losses (\(\tan \delta_r\)), i.e.:

\[
\tan \delta_{eff} = \tan \delta_d + \tan \delta_c + \tan \delta_r
\] (39)

The loss tangent due to conductor loss is given by [14]:

\[
\tan \delta_c = \frac{P_c}{w(W_e + W_m)}
\] (40)

where \(P_c\) is the power loss in the conductor, and \(W_e\) and \(W_m\) are the average electric and magnetic energy stored in the cavity region, calculated at the resonance frequency of each
mode (where $W_e = W_m$), and given by:

\[ P_{c,lu} = \frac{R_s}{2} \int \int_S |\bar{H}_{lu}|^2 \, ds \] (41)

\[ W_{c,lu} = \frac{1}{4} \varepsilon_o \varepsilon_r \int \int \int_V |\bar{E}_{lu}|^2 \, dv \] (42)

where $R_s$ is the surface resistance of the conductor, $S$ is the internal cavity surface, $V$ is the cavity volume, and the components of electrical and magnetic fields of $lu$ mode are given by (4), (7) and (8). Similarly the loss tangent due to radiation is given by:

\[ \tan \delta_r = \frac{P_r}{w(W_e + W_m)} \] (43)

where $P_r$ is the power loss due to radiation, obtained at the mode resonance. It can be obtained from the coefficients of the external modes (35) and (36) and their radiating powers (29) and (30) when the equivalent magnetic currents in (35) and (36) are due only to cavity mode $lu$:

\[ P_{r,lu} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ |e_{nm}|^2 P_{r,lm}^{TM} + |f_{nm}|^2 P_{r,lm}^{TE} \right] \] (44)

Substituting $\tan \delta_d$ by $\tan \delta_{eff}$ in (38):

\[ Z_{in} = -\frac{h}{P^2} \sum_u \sum_i \frac{jw\mu}{k_{eff}^2 - k_{lu}^2} < J_f, E_{r,lu} >^2 \] (45)

where $k_{eff}^2 = w^2 \mu \varepsilon_o (1 - j \tan \delta_{eff})$.

4 Application of Method of Moments

We will now briefly discuss the application of MoM to the same geometry presented in Fig. 1. Detailed description can be found in [27]. The spherical patches are also fed as shown in Fig. 2. As mentioned earlier the MoM seeks to determine the distribution of electric currents on the patches that satisfies the boundary condition [18]. And the radiation properties follow from the current distribution [36].

Let the surface electrical current on the patches be expanded into a set of functions, known
as basis functions, as follows:

\[
J_\theta(\theta) = \sum_{q=1}^{Q_\theta} a_q J_{\theta q}(\theta)
\] (46)

\[
J_\phi(\phi) = \sum_{p=1}^{P_\phi} b_p J_{\phi p}(\phi)
\] (47)

where \(a_q\) and \(b_p\) are the expansion coefficients that will be determined using MoM. The basis functions are given by

\[
J_{\theta q}(\theta) = \sin\left(\frac{q\pi}{\Delta \theta}(\theta - \theta_1)\right)
\] (48)

\[
J_{\phi p}(\phi) = \sin\left(\frac{p\pi}{\Delta \phi}(\phi - \phi_1)\right)
\] (49)

where \(\Delta \theta = \theta_2 - \theta_1\). These basis functions present a sinusoidal variation across the patch.

The tangential components of the electric field in spectral domain are related to the surface electric current on the patches through the dyadic Green’s functions [27]:

\[
\tilde{E}_t^\theta(r, n, m) = \tilde{G}_t^{\theta o}(r, n) \tilde{J}(n, m)
\] (50)

\[
\tilde{E}_t^d(r, n, m) = \tilde{G}_t^{\theta d}(r, n) \tilde{J}(n, m)
\] (51)

The surface current distribution on the patches are determined from imposing the boundary condition of zero tangential component of electric field on the patch surface. The MoM will transform the resulting integral equations into a linear system where the unknowns are the coefficients \((a_q\) and \(b_p\)) of the surface electrical currents. The boundary conditions on the patches are:

\[
E_\theta(J_\theta) + E_\theta(J_\phi) + E_\theta(J_f) = 0
\] (52)

\[
E_\phi(J_\theta) + E_\phi(J_\phi) + E_\phi(J_f) = 0
\] (53)

Expanding the surface electrical currents on the patch as in (46)-(47) leads to:

\[
\sum_{q=1}^{Q_\theta} a_q E_\theta(J_{\theta q}) + \sum_{p=1}^{P_\phi} b_p E_\theta(J_{\phi p}) = -E_\theta(J_f)
\] (54)

\[
\sum_{q=1}^{Q_\theta} a_q E_\phi(J_{\theta q}) + \sum_{p=1}^{P_\phi} b_p E_\phi(J_{\phi p}) = -E_\phi(J_f)
\] (55)
Defining the symmetric product:
\[
<f, g>_s = \int_0^\pi \int_{-\pi}^{\pi} f(\theta, \phi) g(\theta, \phi) r_2^2 \sin \theta \, d\phi \, d\theta
\]  
(56)

and taking these products of (54) and (55) by a set of testing functions, which are chosen the same as the basis functions, results in a Galerkin method. The resulting linear system \([Z][I] = [V]\) can be written as [27]:

\[
\begin{bmatrix}
[Z_{\theta \theta}] & [Z_{\theta \phi}] \\
[Z_{\phi \theta}] & [Z_{\phi \phi}]
\end{bmatrix}
\begin{bmatrix}
[a] \\
[b]
\end{bmatrix}
= \begin{bmatrix}
[V_{\theta}] \\
[V_{\phi}]
\end{bmatrix}
\]  
(57)

where the elements of matrix \([Z]\) are of the following form:

\[
Z_{\text{test,basis}} = <\bar{E}(\bar{J}_{\text{basis}}), \bar{J}_{\text{test}}>_s = r_2^2 \sum_{m=-\infty}^{\infty} \sum_{n=-|m|}^{\infty} \left[ \bar{J}_{\text{test}}(n, -m) \right]^t \overline{G_{EF}(r_2, n)} \left[ \bar{J}_{\text{basis}}(n, m) \right]  
\]  
(58)

where \(\text{test}\) and \(\text{basis}\) refer to either \(\theta\) or \(\phi\). In (57) \([a]\) and \([b]\) are column matrices with the unknown coefficients. The elements of the matrix \([V]\), with the aid of reciprocity theorem, can be obtained from:

\[
V_{\text{test}} = -<\bar{E}(J_f), \bar{J}_{\text{test}}>_s = -\frac{r_2}{j \omega \epsilon} \sum_{m=-\infty}^{\infty} \sum_{n=-|m|}^{\infty} \frac{\sqrt{S(n)} \, J_u(n, m)}{D_1(n, r_1, r_2)} \times \\
\left( \frac{\eta_d \, \tilde{H}^{(2)}_n(k_o r_2)}{\eta_o \, \tilde{H}^{(2)}_n(k_o r_2)} - \frac{D_3(n, r_1, r_2)}{D_1(n, r_1, r_2)} \right)^{-1} \int_{r_1}^{r_2} D_3(n, r_1, r) \bar{J}_f(r, n, -m) \, dr
\]  
(59)

The coefficients of the currents on the patches, \(a_q\) and \(b_p\), are obtained from solving the MoM linear system.

The input impedance is given by the variational expression [36]:

\[
Z_{in} = -\frac{1}{I_o^2} \iint_S \bar{E}(J) \cdot \bar{J}_f ds
\]  
(60)

Expanding the surface current distribution on the patch \(\bar{J}\) in basis functions, and using (59) together with reciprocity theorem, results:

\[
Z_{in} = \frac{1}{I_o^2} \left[ \sum_{q=1}^{Q_\theta} a_q V^\theta_q + \sum_{p=1}^{P_\phi} b_p V^\phi_p \right]  
\]  
(61)
5 Fringing Field Correction

Despite the simplicity of the formulation, the cavity method presents accurate results for radiation pattern, as it will be shown in section 6. The biggest limitation results from considering that the fields in the dielectric layer exist only below the patch, while in fact the fields extend beyond the patch limits, as shown in Fig. 3a for a planar microstrip antenna. This supposition results in inaccurate resonance frequency. To compensate for this drawback a correction of the dimensions of the planar patch is used for simulation purposes, including fringing field corrections (or extensions) $\Delta L_\infty$. The cavity method is now applied as if the patch had dimension $L+2\Delta L_\infty$, and the fields were concentrated below it, as shown in Fig. 3b. A possible correction proposed in [37] is used for planar microstrip patch of resonant length $L$ and width $W$.

$$
\Delta L_\infty = 0.412 h \frac{(\varepsilon_{\text{eff}} + 0.3) (W/h + 0.264)}{(\varepsilon_{\text{eff}} - 0.258) (W/h + 0.8)}
$$

where $\varepsilon_{\text{eff}}$ stands for an effective dielectric constant given by:

$$
\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \frac{(\varepsilon_r - 1)}{2\sqrt{1 + 10 h/W}}
$$
The planar correction $\Delta L_\infty$ shown above has taken into account the effects of the dielectric thickness ($h$), dielectric constant ($\varepsilon_r$), and the width of the patch ($W$). Unfortunately $\Delta L_\infty$ is not a good approximation for fringing fields of spherical microstrip patches, and the resonance frequency of spherical patches obtained from cavity method with planar correction $\Delta L_\infty$ is usually shifted from measurement data or results from more accurate methods.

This section describes how the planar correction $\Delta L_\infty$ can be modified to take into account the curvature of the spherical patch. Results from method of moments procedure presented in section 4 will be treated as references, as it is a full-wave solution already validated [27]. Therefore we should seek for an spherical correction ($\Delta L_r$) such that the application of cavity method (section 3) will result in the same resonance frequencies provided by the method of moments. It should also be understood that the curvature effects on fringing fields are different for different dielectric constants.

In the following procedure $L$ is the resonant length of the spherical-rectangular microstrip antenna, which can be either in $\theta$ or $\phi$ directions. Application of cavity method with planar fringing field correction leads to a resonance frequency $f_{cav}(r_2, \varepsilon_r)$ for a resonance length of $(L + 2 \Delta L_\infty) = \lambda_{cav}/2$, where $\lambda_{cav}$ is the wavelength at $f_{cav}$. Therefore the phase velocity values $v_p = f_{cav} \lambda_{cav} = f_{cav} 2(L + 2 \Delta L_\infty)$. It is wanted to determine the right spherical correction $\Delta L_r$ such that application of cavity method for a length $(L + 2 \Delta L_r)$ would result in the correct resonance frequency $f_{MoM}(r_2, \varepsilon_r)$, as the method of moments is been used as a reference. Assuming that the antenna will present the same effective dielectric constant with either fringing field corrections, $v_p = f_{MoM} 2(L + 2 \Delta L_r)$. Therefore $\Delta L_r$ should be such that:

$$(L + 2 \Delta L_r) f_{MoM}(r_2, \varepsilon_r) = (L + 2 \Delta L_\infty) f_{cav}(r_2, \varepsilon_r)$$

(64)

Therefore:

$$\Delta L_r = \frac{1}{2} \left[ \frac{f_{cav}(r_2, \varepsilon_r)}{f_{MoM}(r_2, \varepsilon_r)} (L + 2 \Delta L_\infty) - L \right]$$

(65)

The spherical fringing field correction $\Delta L_r$ shown above is valid only for the first TM mode (first resonance in the given direction), as the field distribution changes for different modes,
leading to different fringing field effects. The spherical correction $\Delta L_r$ could be easily obtained if the function $f_{\text{cav}}/f_{\text{MoM}}$ were readily available for any given radius $r_2$ and any dielectric constant $\varepsilon_r$. A simple model for $f_{\text{cav}}/f_{\text{MoM}}$ is obtained in the least squares sense as follows:

1. The function $f_{\text{cav}}/f_{\text{MoM}}$ is modeled as:

$$\frac{f_{\text{cav}}(r_2, \varepsilon_r)}{f_{\text{MoM}}(r_2, \varepsilon_r)} = A(\varepsilon_r) \frac{r_2}{L} + B(\varepsilon_r) + C(\varepsilon_r) \frac{L}{r_2}$$ (66)

where

$$A(\varepsilon_r) = a_1 + \frac{a_2}{\varepsilon_r}$$ (67)
$$B(\varepsilon_r) = b_1 + \frac{b_2}{\varepsilon_r}$$ (68)
$$C(\varepsilon_r) = c_1 + \frac{c_2}{\varepsilon_r}$$ (69)

2. The values of $f_{\text{MoM}}$ and $f_{\text{cav}}$ are obtained for several values of radius $r_2$ and dielectric constant $(\varepsilon_r)$ using method of moments (section 4) and cavity method (section 3) using planar correction $\Delta L_\infty$, respectively. The values of the radius $r_2$ ranged from $\lambda_d/5$ to $2\lambda_d$, where $\lambda_d$ is the dielectric wavelength. And the dielectric constant ranged from 1 to 10.

3. For each combination of $r_2$ and $\varepsilon_r$, (66) represents a line of a linear system with unknowns $a_1, a_2, b_1, b_2, c_1,$ and $c_2$:

$$\frac{r_2}{L}a_1 + \frac{r_2}{\varepsilon_r L}a_2 + b_1 + \frac{1}{\varepsilon_r}b_2 + \frac{L}{r_2}c_1 + \frac{L}{\varepsilon_r r_2}c_2 = \frac{f_{\text{cav}}(r_2, \varepsilon_r)}{f_{\text{MoM}}(r_2, \varepsilon_r)}$$ (70)

4. As there are many more lines (combinations of $r_2$ and $\varepsilon_r$) than unknowns, the linear system is overdetermined, and solved in the least squares sense, resulting in:

$$A(\varepsilon_r) = 0.00357 \cdot \frac{0.00089}{\varepsilon_r}$$ (71)
$$B(\varepsilon_r) = 0.9695 + \frac{0.0450}{\varepsilon_r}$$ (72)
$$C(\varepsilon_r) = 0.0081 + \frac{0.0067}{\varepsilon_r}$$ (73)
Therefore the spherical fringing field correction $\Delta L_r$ is given by (65) where $f_{\text{cav}}/f_{\text{MoM}}$ is given by (66), with coefficients $A(\varepsilon_r)$, $B(\varepsilon_r)$, and $C(\varepsilon_r)$ given by (71)-(73), and planar fringing field correction $\Delta L_\infty$ given by (62).

6 Results and Comments

Consider the spherical-rectangular patch, as shown in Fig. 1. A spherical conductor of radius $r_1 = 18.5$ cm is covered by a layer of foam ($\varepsilon_r = 1$) of thickness $h = 4.5$ mm. A patch of width $W_\theta = 20$ cm in $\theta$-direction, and $W_\phi = 20$ cm in $\phi$-direction, is centered at $\theta_c = 90^\circ$ and $\phi_c = 0^\circ$. The patch is fed at $\theta_f = 90^\circ$, at a point $d_\phi = 8$ cm from the patch center, and the probe is modeled as a strip of width $W_f = 6.5$ mm. As a result the patch is $\phi$-polarized.

Figure 4 shows the normalized radiation patterns using the cavity method and MoM. Figure 4a shows the radiation in E-plane ($xy$ plane), as a function of angle $\phi$. It can be observed a maximum radiation in the direction of the antenna center, and that both methods present almost indistinguishable results. Figure 4b shows the radiation pattern in the H-plane ($\phi = 0^\circ$ plane), as a function of $\theta$. Again the results for both methods are very close, and the maximum radiation happens in front of the antenna ($\theta = 90^\circ$). It can also be observed a backlobe with maximum at $\theta = -90^\circ$, which means the opposite direction to the maximum radiation.

Figures 5 and 6 show the input impedance (resistance and reactance) of the same spherical-rectangular patch. Results presented are from application of method of moments, from measurements [25], and from application of cavity method with planar fringing field corrections ($\Delta L_\infty$). An excellent agreement can be observed between the results from method of moments and the measurements, with a small frequency shift at resonance. On the other hand it can be observed that the application of cavity method with planar correction does not take into account the fringing fields accurately, as the resonance frequency is significantly apart from previous results.

In order to correct the resonance frequency of the patch predicted by cavity method, the planar fringing field correction ($\Delta L_\infty$) is now substituted by the spherical one ($\Delta L_r$). The
Figure 4: Normalized radiation patterns of a spherical-rectangular patch at E-plane (xy plane) and H-plane (\( \phi = 0^\circ \) plane). \( \theta_c = 90^\circ, \phi_c = 0^\circ, \ r_1 = 18.5 \) cm, \( h = 4.5 \) mm, \( \varepsilon_r = 1, \ W_\theta = W_\phi = 20 \) cm, \( d_\theta = 0, \ d_\phi = 8 \) cm, and \( W_f = 6.5 \) mm.

Figure 5: Input Resistance of a spherical-rectangular patch. \( \theta_c = 90^\circ, \phi_c = 0^\circ, \ r_1 = 18.5 \) cm, \( h = 4.5 \) mm, \( \varepsilon_r = 1, \ W_\theta = W_\phi = 20 \) cm, \( d_\theta = 0, \ d_\phi = 8 \) cm, and \( W_f = 6.5 \) mm.
new results of cavity method with spherical correction are compared to those from method of moments in Fig. 7. It can be observed that the use of spherical correction has brought the resonance frequency of cavity method very close to results from MoM. This shows that, despite its simplicity, the cavity method with the aid of the spherical correction is able to provide accurate results for spherical microstrip patches, taking into account the fringing fields.

Additional examples are presented in Figs. 8 and 9, comparing results using method of moments and those using cavity method with spherical correction. Figure 8 presents the results for a new dielectric constant equal to 2.2. Figure 9 presents the results for two different radii of the sphere. In both cases the cavity method with spherical correction led to accurate results.

The spherical fringing field correction $\Delta L_r$, given by (65), takes into account variations of the dielectric thickness $h$ through the planar correction $\Delta L_\infty$ (62), but it was not part of the least square optimization. In order to evaluate results for different dielectric thickness Fig. 10 shows a comparison of input impedance using method of moments, cavity method with planar correction, and cavity method with spherical correction, for $h = 3$ mm, keeping $r_2 = 189.5$ mm. It can be observed that the use spherical correction led to more accurate results when compared to those using planar correction. In this case the shift of resonance frequency when using spherical correction is equal to 6 MHz. For different dielectric thicknesses a shift of
Figure 7: Input Impedance of a spherical-rectangular patch. $\theta_c = 90^\circ$, $\phi_c = 0^\circ$, $r_1 = 18.5$ cm, $h = 4.5$ mm, $\varepsilon_r = 1$, $W_\theta = W_\phi = 20$ cm, $d_\theta = 0$, $d_\phi = 8$ cm, and $W_f = 6.5$ mm.

Figure 8: Input Impedance of a spherical-rectangular patch with $\varepsilon_r = 2.2$, $\theta_c = 90^\circ$, $\phi_c = 0^\circ$, $r_1 = 18.5$ cm, $h = 4.5$ mm, $W_\theta = W_\phi = 20$ cm, $d_\theta = 0$, $d_\phi = 8$ cm, and $W_f = 6.5$ mm.
Figure 9: Input Impedance of a spherical-rectangular patch with $r_1 = 30$ cm and $r_1 = 45$ cm, $\theta_c = 90^\circ$, $\phi_c = 0^\circ$, $h = 4.5$ mm, $\varepsilon_r = 1$, $W_\theta = W_\phi = 20$ cm, $d_\theta = 0$, $d_\phi = 8$ cm, and $W_f = 6.5$ mm.

Some final comments on the computational resources demanded by the cavity method and MoM. MoM does not require as much memory as differential methods, like Finite Element Method. But when compared to cavity method, it does require more space for storing the linear system, as well as to store results of some integrals that happen many times. Comparing the computational time, it was observed that MoM took over 100 times more time than cavity method, which needed a fraction of a second per frequency point.

7 Conclusions

This paper presented a new approximate model for fringing field correction for spherical-rectangular microstrip antennas. The patch was analyzed using both cavity method and method of moments. It was shown that both methods are equivalent regarding the radiation diagrams. And that the use of spherical fringing field correction can make cavity method results to be comparable to those from method of moments, in a fraction of the computational time required.
A Normalized Vector Legendre Series

A normalized vector Legendre series of a two-dimensional field $\tilde{J}(\theta, \phi)$ is given by [15],[22]:

$$
\tilde{J}(\theta, \phi) = \left[ \begin{array}{c} J_\theta(\theta, \phi) \\ J_\phi(\theta, \phi) \end{array} \right] = \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \frac{1}{\sqrt{S(n)}} \overline{L}(n, m, \theta)\tilde{J}(n, m)e^{jm\phi}
$$

(74)

where

$$
\tilde{J}(n, m) = \left[ \begin{array}{c} J_u(n, m) \\ J_d(n, m) \end{array} \right] = \frac{1}{\sqrt{S(n)}} \int_{0}^{\pi} \int_{-\pi}^{\pi} \overline{L}(n, m, \theta)\tilde{J}(\theta, \phi) e^{-jm\phi} \sin \theta \, d\theta \, d\phi
$$

(75)

and

$$
\overline{L}(n, m, \theta) = \left[ \begin{array}{cc} \frac{\partial}{\partial \theta} \tilde{P}_n^{\text{im}}(\cos \theta) & -jm \frac{\tilde{P}_n^{\text{im}}(\cos \theta)}{\sin \theta} \\ jm \frac{\tilde{P}_n^{\text{im}}(\cos \theta)}{\sin \theta} & \frac{\partial}{\partial \theta} \tilde{P}_n^{\text{im}}(\cos \theta) \end{array} \right]
$$

(76)

$$
S(n) = 4\pi \frac{n(n+1)}{2n+1}
$$

(77)

The normalized associated Legendre functions $\tilde{P}_n^m(.)$ relates to the associated Legendre functions $P_n^m(.)$ by:

$$
\tilde{P}_n^m(\cos \theta) = \sqrt{(n-m)!/(n+m)!} P_n^m(\cos \theta)
$$

(78)
References


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Conflict of Interest and Authorship Conformation Form

Please check the following as appropriate:

X  All authors have participated in (a) conception and design, or analysis and interpretation of the data; (b) drafting the article or revising it critically for important intellectual content; and (c) approval of the final version.

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