

# Integral Operators Formulation for Transient Radiation from Parabolic Antennas Using Modified Raised Cosine Feeder

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**Abstract**—A closed-form solution in time-domain for integral operators associated to a modified raised cosine feeder is developed and analytical results have been generated and compared to the original raised cosine model.

## I. INTRODUCTION

The temporal characterization of parabolic reflector antennas is a technique highly used when the transmitted pulse via the antenna is ultra-short or when high data rates are employed [1]. The analysis of these problems in frequency-domain becomes impractical due to the large operation bandwidths, so this way, the scattered field from conducting surfaces has been approached directly in time domain [2]. In the present work, integral operators are developed to obtain the transient response of reflector antennas using the aperture method. Such operators are valid near and far from the reflector and by means of a temporal convolution, can be applied to define the antenna radiated field with arbitrary sources.

Initially, the integral operators are found in frequency-domain and then converted into time-domain using Inverse Laplace Transform. The formulation developed here is based on [1] and [3]. In [1], the transient field generated by a parabolic reflector antenna uses an elemental Huygen's source at the focus while in [3], the feeder utilized is the raised cosine model. This paper intends to investigate the same transient field developed in those previous works but this time, considering a modified raised cosine model feeder.

One of the advantages of using a modified raised cosine model as a feeder is due to the simpler closed-form solution for integral operators obtained when compared to the original raised cosine model. Analytical results were obtained and compared to the results found for the raised cosine model introduced in [3] using the same parameters.

## II. FORMULATION

### A. Frequency-Domain Field

The electric field radiated in free space can be expressed as equation 16 of [1] where  $s$  is the complex frequency,  $c$  is the free-space velocity of light,  $Z_0$  is the free-space impedance,

$Y_0$  is the free-space admittance,  $R$  is the distance between the source and the observer and  $\vec{E}_a$  and  $\vec{H}_a$  are the electric and magnetic aperture fields respectively.

Applying the parameters associated to the paraboloid in equation 16 of [1], the integral operator in frequency-domain can be written as [1]

$$\vec{E}_r^\delta(\vec{r}, s) = \frac{FV_0}{\pi c} \iint_{S_a} \frac{\vec{g}(\vec{\rho}', \phi')}{R(\rho'^2 + 4F^2)} e^{-\frac{s}{c}(R+F+\frac{D^2}{16F})} dS' \quad (1)$$

where  $V_0$  is a constant,  $D$  is the surface diameter,  $F$  is the focal distance and  $g(\vec{\rho}', \phi')$  is a generalized function related to the feeder.

### B. Time-Domain Field

In order to obtain the radiated field in time-domain, the Inverse Laplace Transform was applied and the equation 1 becomes [1]

$$\vec{E}_r^u(\vec{r}, t) = \frac{FV_0}{\pi c} \iint_{S_a} \frac{\vec{g}(\vec{\rho}', \phi')}{R(\rho'^2 + 4F^2)} \delta\left(t - \frac{R+F+\frac{D^2}{16F}}{c}\right) dS' \quad (2)$$

In this paper, it was used a modified raised cosine feeder with  $n = 1$  and the argument equal to  $\frac{\theta_F}{2}$ . Hence, after some extensive mathematical manipulations, the integral operator can be written as

$$\vec{E}_r^u(\vec{r}, t) = E_{r_x}^u(\vec{r}, t)\hat{x} + E_{r_y}^u(\vec{r}, t)\hat{y} + E_{r_z}^u(\vec{r}, t)\hat{z} \quad (3)$$

where

$$\vec{E}_x^u(\vec{r}, t) = -\frac{FV_0}{\pi} \Pi(t) \left\{ \int_{\alpha_1}^{\alpha_2} \left[ \frac{\delta_3 \cos^2(\theta_F) \sin^2(\psi + \phi)}{(\delta_1 + \delta_2 \cos(\psi))^{\frac{3}{2}}} \right] d\psi + \int_{\alpha_1}^{\alpha_2} \left[ \frac{\delta_3 \sin^2(\theta_F)}{(\delta_1 + \delta_2 \cos(\psi))^{\frac{3}{2}}} \right] d\psi \right\} \quad (4)$$

$$\vec{E}_y^u(\vec{r}, t) = \frac{FV_0}{\pi} \Pi(t) \left\{ \int_{\alpha_1}^{\alpha_2} \left[ \frac{\delta_3 \cos^2(\theta_F) \cos(\psi + \phi) \sin(\psi + \phi)}{(\delta_1 + \delta_2 \cos(\psi))^{\frac{3}{2}}} \right] d\psi + \int_{\alpha_1}^{\alpha_2} \left[ \frac{\delta_3}{(\delta_1 + \delta_2 \cos(\psi))^{\frac{3}{2}}} \right] d\psi \right\} \quad (5)$$

$$\vec{E}_z^u(\vec{r}, t) = -\frac{FV_0}{\pi}\Pi(t)\left\{\int_{\alpha_1}^{\alpha_2}\left[\frac{\delta_3\cos(\theta_P)\sin(\theta_P)}{(\delta_1+\delta_2\cos(\psi))^{\frac{3}{2}}}\right]d\psi\right\} \quad (6)$$

where  $\Pi(t)$  collects the time duration of the operator  $\vec{E}_r^u(\vec{r}, t)$ ,  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  are parameters associated to the parabolic antenna and  $\rho$ ,  $\xi'$ ,  $\alpha_1$  and  $\alpha_2$  are defined according to [1].

### III. CASE STUDIES

The results obtained here show the magnitude of the integral operators in near and far field regions. It was considered a parabolic reflector antenna with  $D = 7.5$  m and  $F = 0.4D$  at the plane  $\phi = 0$ . Figure 1 presents the magnitude of the operator considering the  $\cos\left(\frac{\theta_F}{2}\right)$  feeder and a radial distance equal to 50 m while Figure 2 shows the same operator, however the computed feeder now is  $\cos(\theta_F)$  [3]. In Figures 3 and 4, the magnitude of the integral operators are obtained in the far field region, considering a radial distance equal to 5000 m and the  $\cos(\theta_F)$  and  $\cos\left(\frac{\theta_F}{2}\right)$  feeders, respectively.

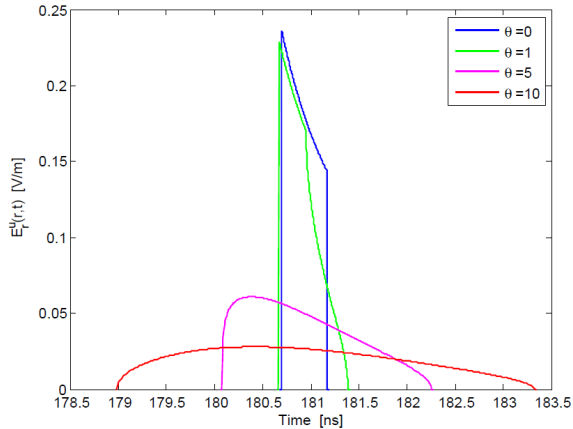


Fig. 1. Magnitude of the integral operator at  $r = 50$  m generated by a parabolic antenna using the  $\cos\left(\frac{\theta_F}{2}\right)$  feeder.

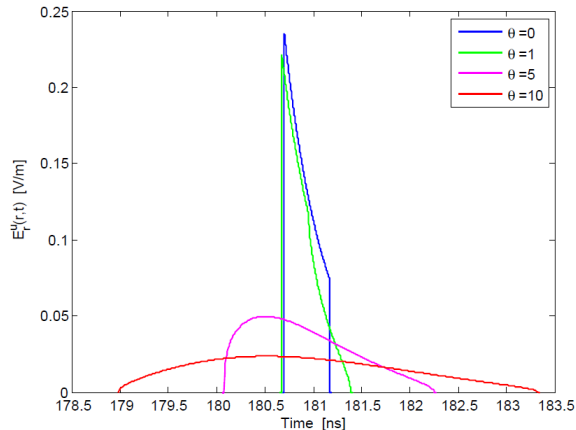


Fig. 2. Magnitude of the integral operator at  $r = 50$  m generated by a parabolic antenna using the  $\cos(\theta)$  feeder.

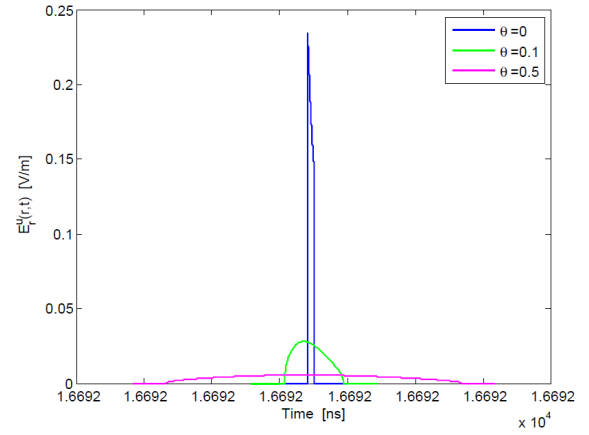


Fig. 3. Magnitude of the integral operator at  $r = 5000$  m generated by a parabolic antenna using the  $\cos\left(\frac{\theta_F}{2}\right)$  feeder.

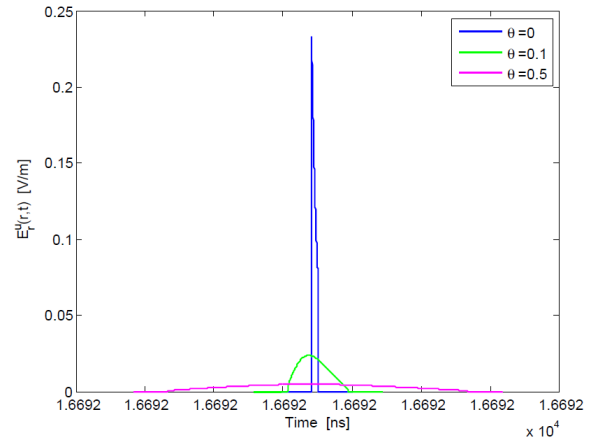


Fig. 4. Magnitude of the integral operator at  $r = 5000$  m generated by a parabolic antenna using the  $\cos(\theta)$  feeder.

### IV. CONCLUSION

The time-domain formulation developed in this paper can be applied to generalized feeders and once the integral operators are known, the radiated field from the parabolic reflector antenna can be easily found by means of a temporal convolution. The huge advantage of this technique is the low time consumption to generate radiated fields from parabolic antennas using generic sources. Analytical results were obtained for two different feeders in the near and far field regions.

### ACKNOWLEDGMENT

This work was partially supported by CAPES and CNPq.

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