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EMFIE AND MEFIE FORMULATIONS FOR THE ANALYSIS OF SCATTERING FROM DIELECTRIC AND COMPOSITE BODIES OF REVOLUTION

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ABSTRACT: *In this article, the electromagnetic scattering from dielectric and composite bodies of revolution are analyzed by the electric-magnetic field integral equation (EMFIE) and the magnetic-electric field integral equation (MEFIE), which are customarily overlooked in the literature. A standard method-of-moments (MoM) technique is applied for the numerical solution of the surface integral equations. Several dielectric and composite geometries are analyzed through the bandwidth and results are compared with those of well-established Müller and PMCHWT (a solution proposed by Poggio, Miller, Chang, Harrington, Wu, and Tsai) integral equation formulations. Investigated case studies indicate that the MoM* **Z***-matrices yielded by the EMFIE and MEFIE are as well-conditioned as those provided by the Müller and PMCWHT formulations.* © 2010 Wiley Periodicals, Inc. Microwave Opt Technol Lett 53:398–402, 2011; View this article online at wileyonlinelibrary.com. DOI 10.1002/mop.25720

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1. INTRODUCTION

Electromagnetic scattering from conducting, dielectric, and composite bodies has been intensively investigated. For arbitrary body shapes, it is quite difficult to obtain an analytic solution and, consequently, numerical methods must be used. Among the numerical techniques, those based on surface integral equations (SIEs) are generally adopted when the bodies are made of simple (i.e., linear, homogeneous, and isotropic) media, in which case the method-of-moments (MoM) is usually applied to obtain a numerical solution [1]. For an axisymmetric body [or body of revolution (BOR)], the solution can be formulated in terms of line integrals evaluated over the BORs generatrices, thus significantly reducing the computational effort [2–7].

For the analysis of scattering from perfectly electric conductor (PEC) BORs, the commonly adopted SIE formulations are the electric field integral equation (EFIE), for bodies that are modeled as open shells, and the combined field integral equation (CFIE), which prevents spurious resonances in the numerical analysis of closed PEC BORs [1, 2]. For dielectric and composite BORs, different linear combinations of EFIE and magnetic field integral equation (MFIE) have been investigated [3–7]. Among them, the most commonly used at the interface between dielectric media are the PMCHWT (a solution proposed by Poggio, Miller, Chang, Harrington, Wu, and Tsai) and Müller SIE formulations for the well-conditioned behavior of their matrices [1, 8]. However, other linear combinations of EFIE and MFIE are possible, like the electric-magnetic field integral equation (EMFIE) and the magnetic-electric field integral equation (MEFIE) investigated in this work.

To establish the capabilities of EMFIE and MEFIE in comparison with other SIE formulations, several dielectric and composite BORs (all of them involving spheres) are numerically analyzed by the MoM technique (using triangular basis functions and Galerkin's method) [1]. For the case studies to be presented, the equivalent surface currents, numerically determined by the EMFIE and MEFIE, have relative mean errors similar to those obtained from other formulations. The condition numbers of MoM **Z**-matrices are also investigated for a wide frequency range and results indicate that EMFIE and MEFIE may provide matrices that are better conditioned than those of other formulations. Furthermore, the EMFIE and MEFIE formulations reduce the time spent to fill the **Z**-matrix for coated conducting BORs.

In the following sections, EFIE and MFIE are briefly presented together with several linear combinations used in the analysis of conducting, dielectric, and composite BORs. Afterward, the MoM technique used in the numerical analysis is briefly addressed before the investigation comprising several dielectric and composite spherical BORs.

2. SURFACE INTEGRAL EQUATION FORMULATIONS

In this section, general SIE formulations are presented for a simple geometry: a conductor or dielectric BOR (medium 1) imbedded in free space (medium 0). All dielectric media are assumed linear, homogeneous, and isotropic, with permittivity ε_i and permeability μ_i, where the index $i = 0$ or 1 indicates the corresponding medium. The time variation is assumed to be $\exp(j\omega t)$. That will be sufficient to illustrate several linear combinations of EFIE and MFIE, particularly the EMFIE and MEFIE investigated in this work. General formulations for composite bodies and mixed boundary conditions can be found in [3–7].

After the equivalence principle is applied to establish an equivalent problem in terms of electric (**J**) and magnetic (**M**) surface currents, a set of four SIEs are established to solve for the electric (**E**) and magnetic (**H**) fields [1]. Assuming that the only sources radiating the incident electric ($\mathbf{E}^{\text{inc}}$) and magnetic ($\mathbf{H}^{\text{inc}}$) fields are outside medium 1, the SIEs in terms of tangential field components are [1–7]:

$$[\eta_0 \mathbf{L}_0(\mathbf{J}) + \mathbf{K}_0(\mathbf{M})]_{\tan} = \mathbf{E}^{\text{inc}}_{\tan}, \quad (1)$$

$$[\mathbf{L}_0(\mathbf{M}) - \eta_0 \mathbf{K}_0(\mathbf{J})]_{\tan} = \mathbf{H}^{\text{inc}}_{\tan}, \quad (2)$$

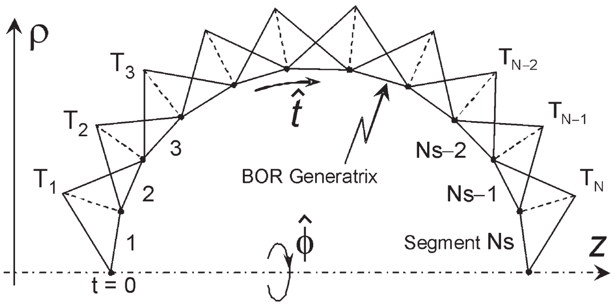

Figure 1 Triangular basis functions over the BOR generatrix

$$[\eta_1 \mathbf{L}_1(\mathbf{J}) + \mathbf{K}_1(\mathbf{M})]_{\tan} = 0, \tag{3}$$

$$[\mathbf{L}_1(\mathbf{M}) - \eta_1 \mathbf{K}_1(\mathbf{J})]_{\tan} = 0. \tag{4}$$

where η_i is the intrinsic impedance of the medium i, the surface integral operators $\mathbf{L}_i$ and $\mathbf{K}_i$ are given by

$$\mathbf{L}_i(\mathbf{X}) = \frac{j}{k_i} \iint_{s'} \left[k_i^2 \mathbf{X}(\mathbf{r}') G_i(\mathbf{r}, \mathbf{r}') - \nabla' \mathbf{X}(\mathbf{r}') \nabla' G_i(\mathbf{r}, \mathbf{r}')\right] ds', \tag{5}$$

$$\mathbf{K}_i(\mathbf{X}) = \upsilon_i \hat{n} \times \frac{\mathbf{X}(\mathbf{r})}{2} + \iint_{s'} [\mathbf{X}(\mathbf{r}') \times \nabla' G_i(\mathbf{r}, \mathbf{r}')] ds', \tag{6}$$

with integrals given by their Cauchy's principal value when $\mathbf{r} = \mathbf{r}'$, $=$ *cmmib*10n̂ is the outward surface normal, $\upsilon_i = +1$ (or -1) for an observation point $\mathbf{r}$ outside (or inside) the surface S', $k_i = 2\pi/\lambda_i$, and the free space Green's function G_i is given by

$$G_i(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_i|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|}. \tag{7}$$

Equations (1)–(4) represent a set of four equations that can be linearly combined in different ways to solve for the two unknowns $\mathbf{J}$ and $\mathbf{M}$.

If the BOR is a PEC and, consequently, $\mathbf{M} = 0$, than the CFIE given by the linear combination of (1) and (2) is generally the best choice to avoid interior resonances [1, 2]. However, if the BOR is an open PEC shell, than the EFIE given by (1) must be used [1].

If the BOR (medium 1) is a dielectric, several combinations among (1)–(4) can be applied to solve the problem, in principle [5]. Three possible combinations are the EFIE composed of (1) and (3), the MFIE composed of (2) and (4), and the CFIE with two integral equations derived from linear combinations of (1) and (2) and of (3) and (4). However, the formulations that are generally applied in the analysis of scattering from dielectric objects are the PMCHWT and Müller, given by the following combinations of (1) and (3) and of (2) and (4):

$$[\eta_0 \mathbf{L}_0(\mathbf{J}) + \mathbf{K}_0(\mathbf{M})]_{\tan} + \alpha[\eta_1 \mathbf{L}_1(\mathbf{J}) + \mathbf{K}_1(\mathbf{M})]_{\tan} = \mathbf{E}^{\text{inc}}_{\tan}, \tag{8}$$

$$[\mathbf{L}_0(\mathbf{M}) - \eta_0 \mathbf{K}_0(\mathbf{J})]_{\tan} + \beta[\mathbf{L}_1(\mathbf{M}) - \eta_1 \mathbf{K}_1(\mathbf{J})]_{\tan} = \mathbf{H}^{\text{inc}}_{\tan}, \tag{9}$$

where $\alpha = \beta = 1$ for PMCHWT formulation, whereas $\alpha = -\varepsilon_1/\varepsilon_0$ and $\beta = -\mu_1/\mu_0$ for Müller formulation. In principle, both formulations do not suffer from resonance problems and are capable of yielding well-conditioned MoM matrices and, consequently, stable numerical solutions [3–8].

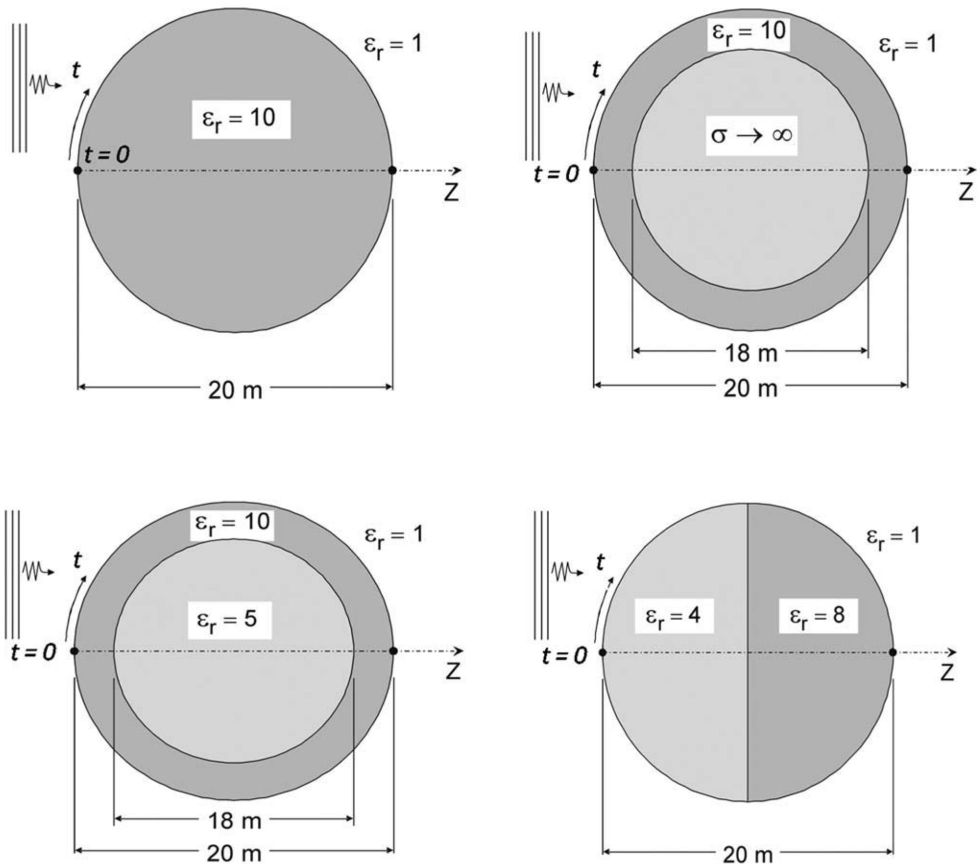

Figure 2 The spherical geometries investigated: (a) dielectric sphere, (b) coated conducting sphere, (c) coated dielectric sphere, and (d) bisectioned dielectric sphere

In this work, we investigate two other possible SIE formulations that are generally overlooked in the literature. One is composed of (1) and (4), i.e., an EFIE to describe the tangential field behavior over the interface side at (the external) medium 0 and an MFIE over the interface side at (the internal) medium 1. For this reason, we name it EMFIE formulation. The other formulation is composed of (2) and (3) and is named MEFIE. Using arguments similar to those in [2], it can be shown that no spurious resonant field can satisfy both an EFIE and a MFIE at the dielectric interface. Both EMFIE and MEFIE do not suffer from resonance problems, in principle. Several case studies comparing the EMFIE and MEFIE with other SIE formulations will be presented and discussed in Section 4.

Finally, for composite BORs (made of different dielectric and conducting media), the treatment at each interface follows what was previously discussed, depending if it lies between two dielectric media or between a dielectric and a conductor [5]. It is important to note that special care must be taken when dealing with junctions involving three or more media [9, 10].

3. MoM NUMERICAL SOLUTION

Any of the previously discussed integral equation formulations may be numerically evaluated by the MoM technique [1], appropriately suited to handle BORs [2–7]. In this work, the equivalent surface currents **J** and **M** are described in terms of triangular basis functions along the BORs generatrices, to represent the current variations along the contour, combined with a Fourier series, to describe azimuthal variations. A triangular function is defined over two consecutive segments of the generatrix description, as illustrated in Figure 1. Their characterization over generatrices and at junctions of composite BORs follows what is discussed in [3]. For instance, the unknown surface current **J** is described as:

$$\mathbf{J}(\mathbf{r}') = \sum_{n=-\infty}^{\infty} \left[\sum_{j=1}^{Nt} I_{jn}^{t} \frac{T_j^t(t')}{\rho(t')} \hat{t}' + \sum_{j=1}^{N\phi} I_{jn}^{\phi} \frac{T_j^{\phi}(t')}{\rho(t')} \hat{\phi}' \right] e^{jn\phi'}, \qquad (10)$$

where $\hat{t}'$ and $\hat{\phi}'$, together with the surface normal $= cmmib10\hat{n}$ form an orthogonal curvilinear coordinate system at the BOR surface, T_j^t and T_j^{ϕ} are triangular functions representing the $\hat{t}'$ and $\hat{\phi}'$ components of **J**, respectively, and I_{jn}^t and I_{jn}^{ϕ} are the unknown coefficients to be determined by the MoM procedure. The description of **M** follows likewise. In Eq. (10), the division by the coordinate ρ actually prevents singularity problems at the symmetry axis when (10) are substituted into (5) and (6) [2].

The integral equations are evaluated by the MoM with testing functions defined according to Galerkin's method [1]. All resulting integrals of the excitation (V) and impedance (Z) matrices were evaluated using Legendre-Gauss quadrature, with singularities removed according to the procedures discussed in [11] and [12]. The linear system was solved using standard LU decomposition with pivoting.

4. NUMERICAL EXAMPLES

To investigate the capabilities of the EMFIE and MEFIE in comparison to the commonly used PMCHWT and Müller formulations, several case studies were analyzed. The numerical examples comprise the scattering of a linearly polarized plane wave (propagating in the positive z direction) from a dielectric sphere, a coated conducting sphere, a coated dielectric sphere, and a bisectioned dielectric sphere, as illustrated in Figure 2. All of them have an outer spherical interface with a radius equal to 10 m. Numerical results were obtained for a wide range of frequencies, from $0.1f_0$ up to f_0, such that in the reference frequency f_0 the wavelength at the exterior medium is $\lambda_0 = 1$ m. Consequently, all spheres have a relatively large outer radius (equal to $10\lambda_0$) at f_0, which will help to clearly demonstrate the stability of the different SIE formulations and their capability of avoiding resonance problems.

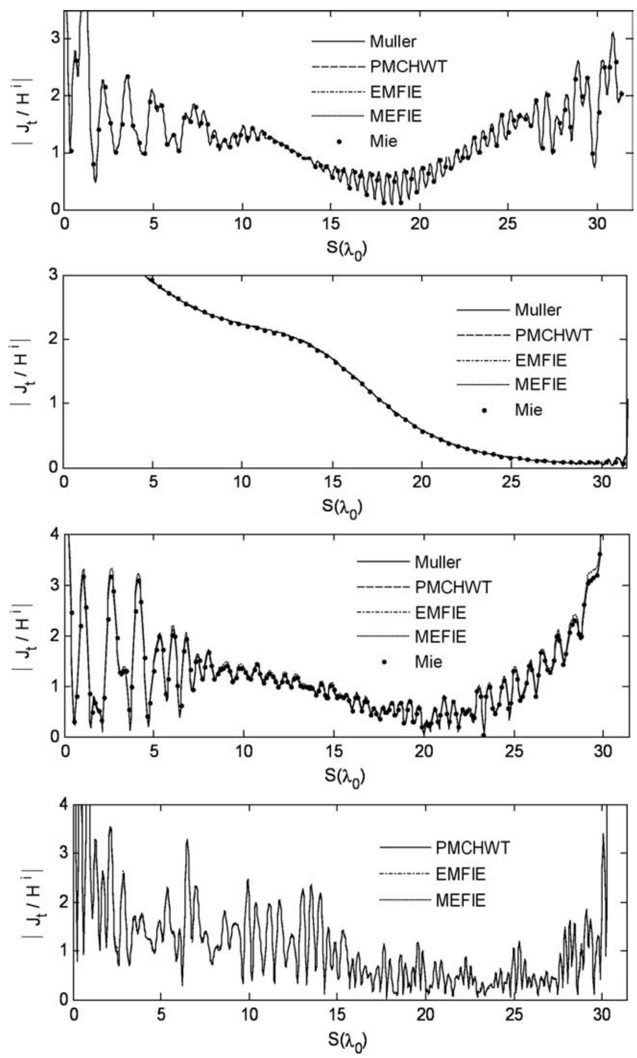

Figure 3 Electric current component J_t (plane $\phi = 0$) over the external surface of the dielectric and composite spheres illustrated in Figure 2 for $f = f_0$: (a) dielectric sphere, (b) coated conducting sphere, (c) coated dielectric sphere, and (d) bisectioned dielectric sphere

To ensure proper convergence of the numerical solutions, about 25 triangular functions per wavelength are used to represent each one of the four components of **J** and **M** (i.e., J_t, J_ϕ, M_t, and M_ϕ) at all BORs surfaces and interfaces throughout the bandwidth, with the wavelength being that of the interface medium with the highest relative permittivity ε_r. The values of ε_r vary from 4 to 10 for the dielectric bodies, as indicated in Figure 2. Note that for the coated conducting sphere of Figure 2(b) the CFIE was used over the internal conductor surface. Also note that Müller formulation was not used in the analysis of the bisectioned dielectric sphere of Figure 2(d) as it demands special care for the testing functions at junctions [9, 10], which was not attempted in the present work. The accuracy of the numerical results was determined with respect to analytical Mie-series solutions [13], except for the bisectioned sphere of Figure 2(d) where numerical PMCHWT results were adopted instead.

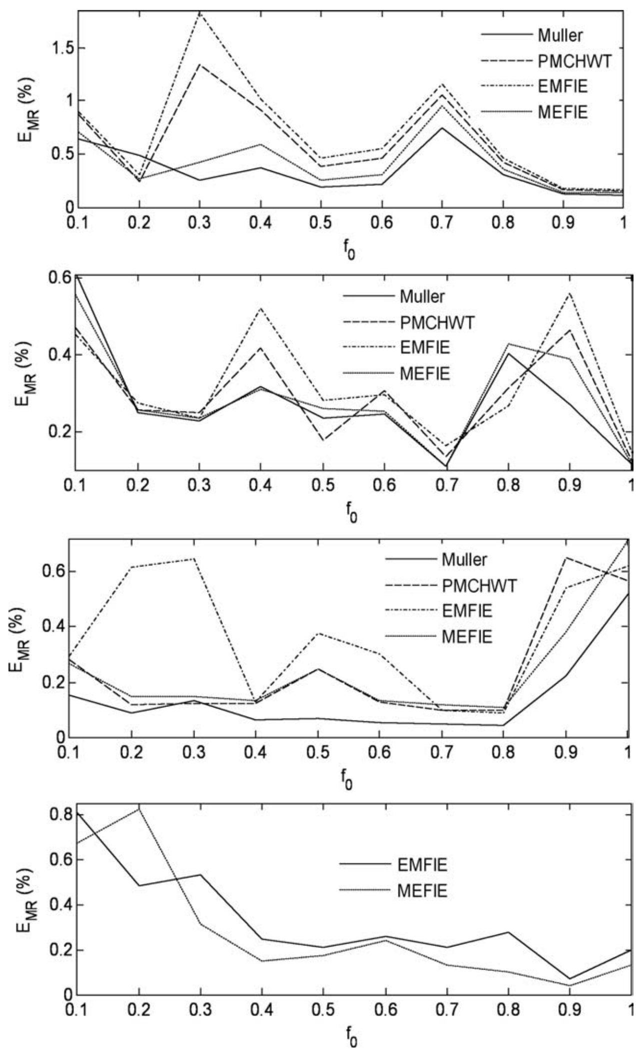

Figure 4 Relative mean error of the equivalent currents for the dielectric and composite spheres illustrated in Figure 2 over a wide frequency range: (a) dielectric sphere, (b) coated conducting sphere, (c) coated dielectric sphere, and (d) bisectioned dielectric sphere

The electric current component J_t over the external surfaces of the spheres of Figures 2(a)–2(d) is shown in Figures 3(a)–3(d), respectively, at the frequency f_0. Because of the lack of space the other current components are not shown. The normalized current amplitude is plotted from $t = 0$ to 10π m, according to the geometries illustrated in Figure 2. From Figure 3, one readily observes that all the investigated SIE formulations yield about the same accuracy for the J_t amplitude calculation and no resonance problems are apparently present, even for such large spheres with outer radius equal to $10\lambda_0$.

To better illustrate the numerical behavior of the EMFIE and MEFIE with respect to the PMCHWT and Müller formulations, Figures 4(a)–4(d) show mean relative error ($E_{\rm MR}$) of the currents for the geometries depicted in Figures 2(a)–2(d), respectively, across the bandwidth ranging from $0.1f_0$ to f_0. The $E_{\rm MR}$ is defined as

$$E_{\rm MR}(\%) = \frac{E_{{\rm R}_{Jt}} + E_{{\rm R}_{J\phi}} + E_{{\rm R}_{Mt}} + E_{{\rm R}_{M\phi}}}{4}, \qquad (11)$$

where $E_{\rm RX}$ is the mean relative error of the current component X, given by:

$$E_{\rm RX}(\%) = 100\,\frac{\sum_{i=1}^{NT} \left|X^{\rm MoM}(i) - X^{\rm Mie}(i)\right|}{{\rm NT}|X^{\rm Mie}(i)|_{\rm Max}}, \qquad (12)$$

where X represents the current component (J_t, J_ϕ, M_t, or M_ϕ) and $X^{\rm MoM}$ and $X^{\rm Mie}$ represent the MoM and Mie-series solutions for X, respectively. The summation is evaluated over the NT triangular basis functions representing X over all BOR interfaces and surfaces. For the bisectioned sphere of Figure 2(d), the PMCHWT results were used to establish the relative error of the EMFIE and MEFIE currents instead of a Mie series. Although the definition of $E_{\rm MR}$ is quite arbitrary, the results of Figure 4 (with $E_{\rm MR} < 2\%$) clearly indicate that all investigated SIE formulations provide about the same accuracy.

The condition number of the MoM **Z**-matrices involved in the calculations of the results presented in Figures 4(a)–4(d) is shown in Figures 5(a)–5(d), respectively. For the present numerical examples, one may observe that no severe fluctuations are present in Figures 5(a)–5(d), apparently indicating the absence of resonance problems. One may also observe that the MEFIE generally provides a smaller condition number than the other SIE formulations.

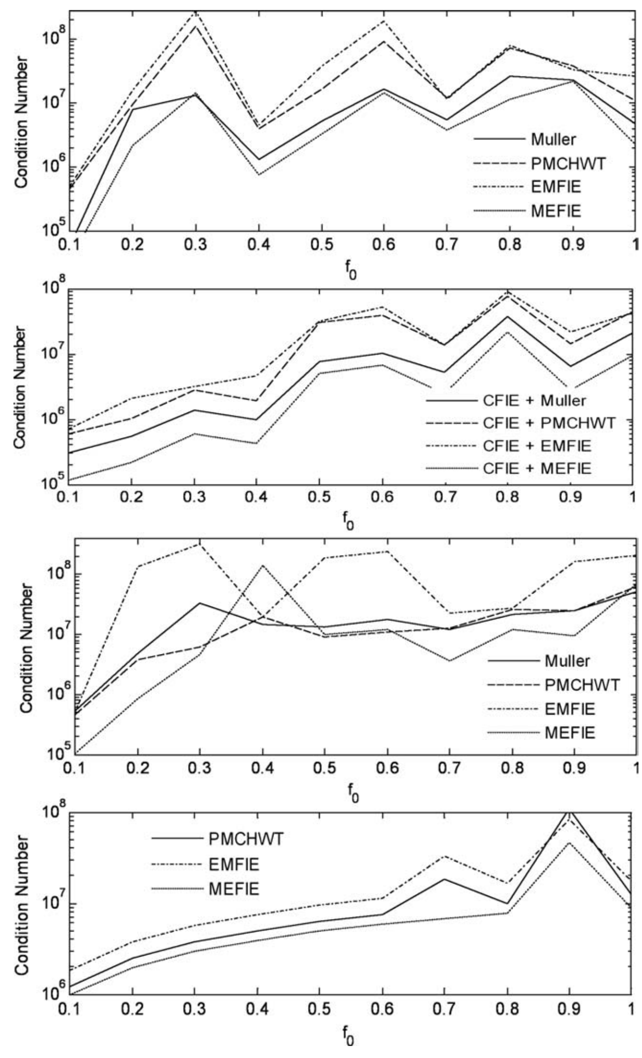

Figure 5 Condition number of the MoM **Z**-matrix for the dielectric and composite spheres illustrated in Figure 2 over a wide frequency range: (a) dielectric sphere, (b) coated conducting sphere, (c) coated dielectric sphere, and (d) bisectioned dielectric sphere

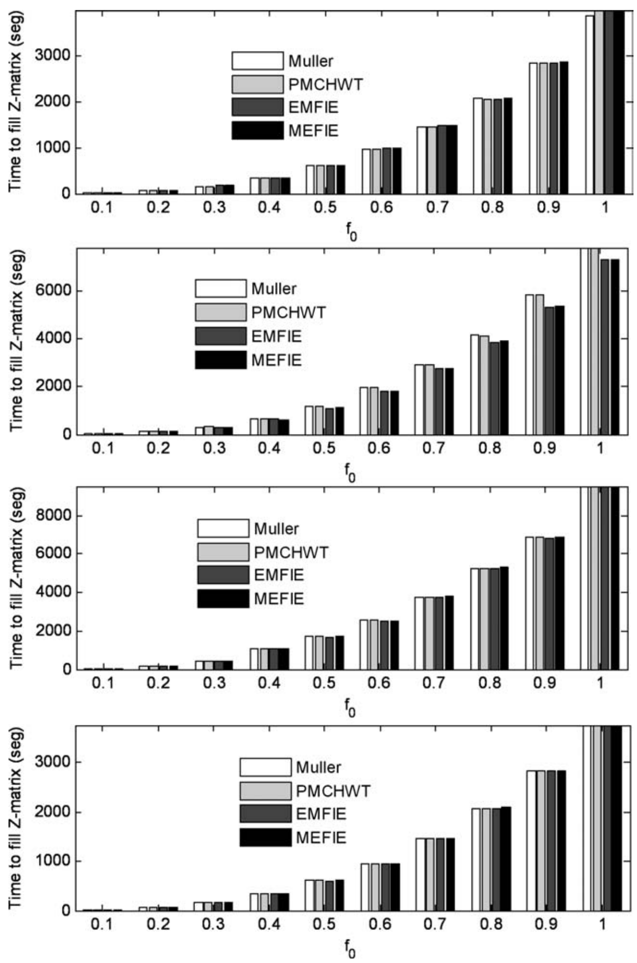

Figure 6 Time required to fill the MoM **Z**-matrix (in seconds) for the dielectric and composite spheres illustrated in Figure 2 over a wide frequency range: (a) dielectric sphere, (b) coated conducting sphere, (c) coated dielectric sphere, and (d) bisectioned dielectric sphere

An interesting feature of the EMFIE and MEFIE is that, for being simpler, they may provide a small reduction of the time required to fill the MoM **Z**-matrix. As can be observed in Figure 6, the time required by all formulations is very close to each other, except for the coated conducting sphere in Figure 6(b). In this case, the time required by EMFIE and MEFIE formulations is smaller than that required by classical Müller and PMCHWT formulations. This fact occurs because EMFIE and MEFIE formulations use only one SIE at each side of the surface. When the scatterer is composed of conducting and dielectric surfaces as in Figure 2(b), it is not necessary to solve some integral equation in the MoM solution, because conducting surfaces have only electric currents. The time saving depends on the number and size of dielectric and conducting surfaces. For the coated conducting sphere of Figure 2(b), the time required by EMFIE and MEFIE formulations to fill the **Z**-matrices was about 5% smaller than the time required by classical Müller and PMCHWT formulations across the bandwidth, as indicated in Figure 6(b).

5. CONCLUSIONS

Two SIE formulations that are generally overlooked in the literature were investigated in this work to analyze the scattering from bodies of revolution: the EMFIE and MEFIE. These formulations are simpler than the classical CFIE, Müller, and PMCWHT formulations and may provide a small reduction in the time required to fill the MoM **Z**-matrix when analyzing composite bodies made of dielectric and conducting media. The plane-wave scattering from several different dielectric and composite spheres was analyzed over a wide frequency range, such that at the higher frequency the spheres have an outer radius equal to 10 wavelengths. The investigation also comprised the mean error of the equivalent surface currents (relative to Mie-series solutions whenever possible) and the **Z**-matrix condition number across the bandwidth. The results demonstrated that EMFIE and MEFIE formulations may provide accuracies and numerical stabilities comparable with those of classical Müller and PMCWHT formulations. Also, no resonance problems were observed, as expected from the use of both EFIE and MFIE at surfaces and interfaces.

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