Circuitos Elétricos III

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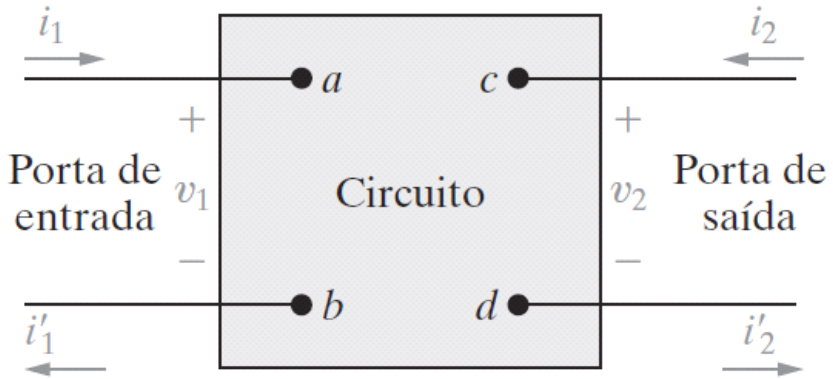
Quadripolos

Quadripolo

- Motivação: análise das tensões e correntes em dois pares de terminais (duas portas) - i_1, i_2, v_1 e v2.



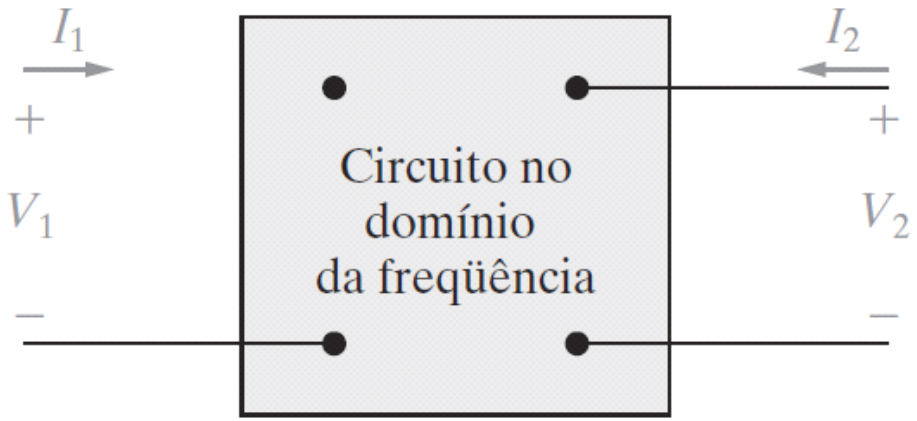
- Restrições:
  - Não há energia armazenada no circuito;
  - Não há fontes independentes no circuito;
  - $i_1 = i'_1$ e $i_2 = i'_2$

Quadripolo no domínio da freqüência

- Das 4 variáveis I_1, I_2, V_1, V_2, se 2 variáveis forem especificadas, pode-se obter as outras (somente 2 são independentes).
- Pode-se descrever o quadripolo por meio de 2 equações simultâneas.
- Há 6 modos de combinar as 4 variáveis.

$$V_1 = z_{11}I_1 + z_{12}I_2\,,$$

$$V_2 = z_{21}I_1 + z_{22}I_2\,;$$

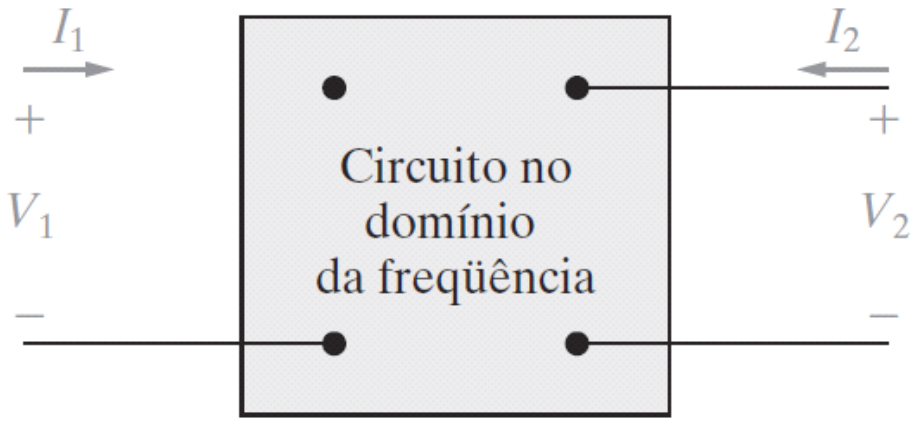


Quadripolo no domínio da freqüência

- Os coeficientes das variáveis de corrente/tensão são os parâmetros do quadripolo.
- Parâmetros z: impedâncias

$$V_1 = z_{11}I_1 + z_{12}I_2\,,$$

$$V_2 = z_{21}I_1 + z_{22}I_2\,;$$

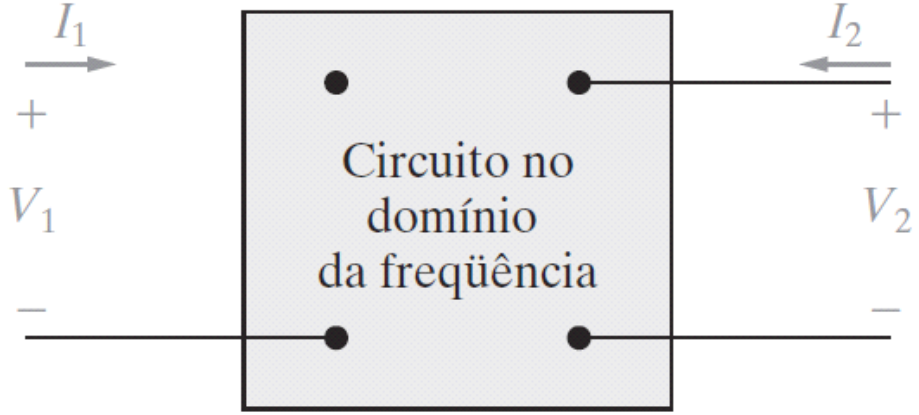


Quadripolo no domínio da freqüência

- Parâmetros y: admitâncias

$$I_1 = y_{11}V_1 + y_{12}V_2\,,$$

$$I_2 = y_{21}V_1 + y_{22}V_2\,;$$



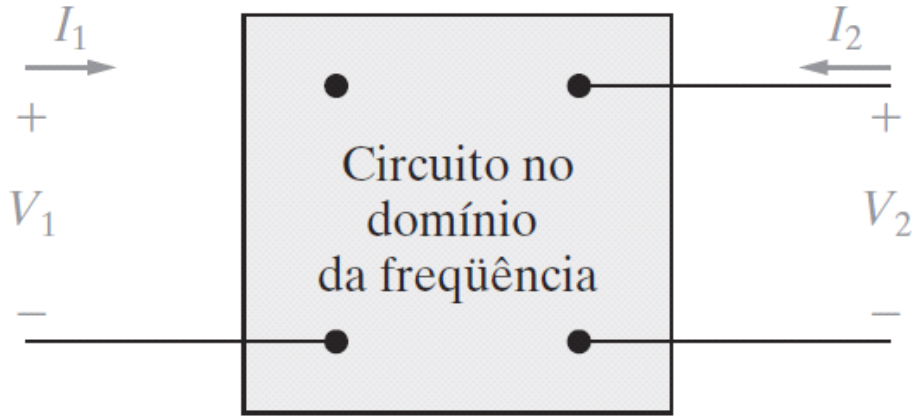
- Parâmetros x e y são também chamados de imitâncias.

Quadripolo no domínio da freqüência

- Parâmetros a

$$V_1 = a_{11}V_2 - a_{12}I_2\,,$$

$$I_1 = a_{21}V_2 - a_{22}I_2\,;$$

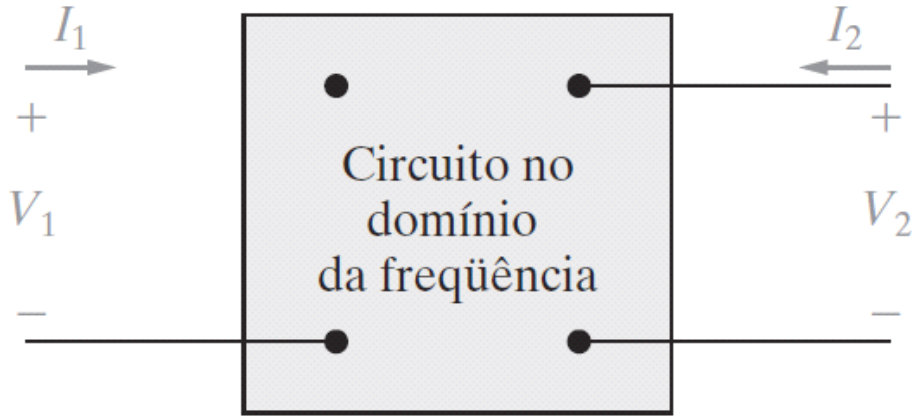


Quadripolo no domínio da freqüência

- Parâmetros b

$$V_2 = b_{11}V_1 - b_{12}I_1\,,$$

$$I_2 = b_{21}V_1 - b_{22}I_1\,;$$



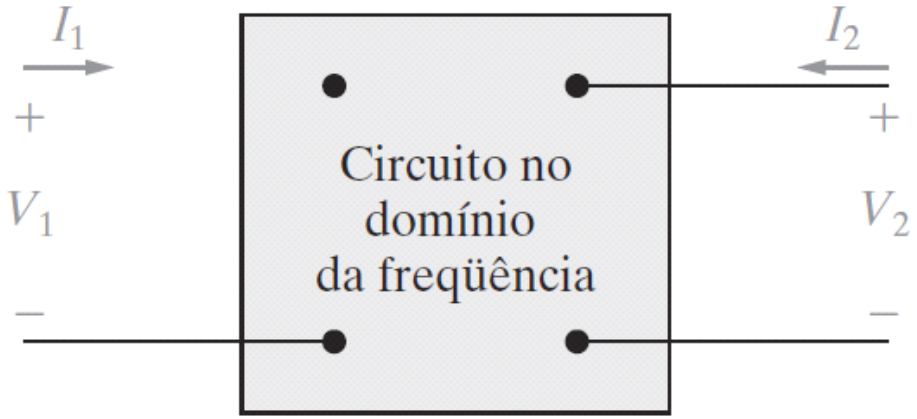
- Parâmetros a e b são também chamados de parâmetros de transmissão.

Quadripolo no domínio da freqüência

- Parâmetros h

$$V_1 = h_{11} I_1 + h_{12} V_2 ,$$

$$I_2 = h_{21} I_1 + h_{22} V_2 ;$$

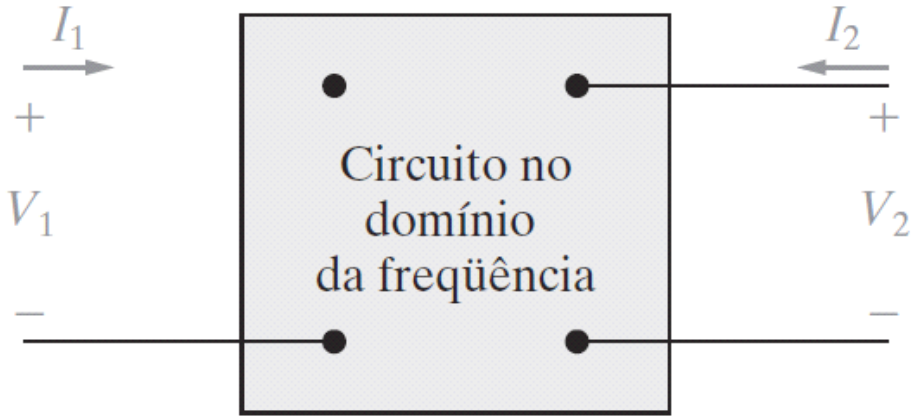


Quadripolo no domínio da freqüência

- Parâmetros g

$$I_1 = g_{11} V_1 + g_{12} I_2 ,$$

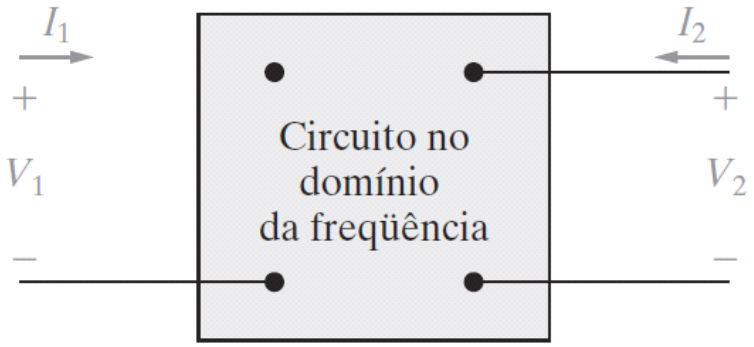
$$V_2 = g_{21} V_1 + g_{22} I_2 .$$



- Parâmetros a e b são também chamados de parâmetros híbridos.

Parâmetros z do Quadripolo

- impedâncias:

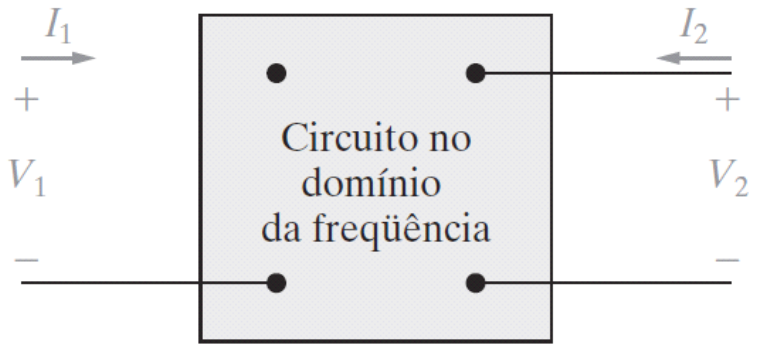


$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega, \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega,$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega, \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega.$$

Parâmetros y do Quadripolo

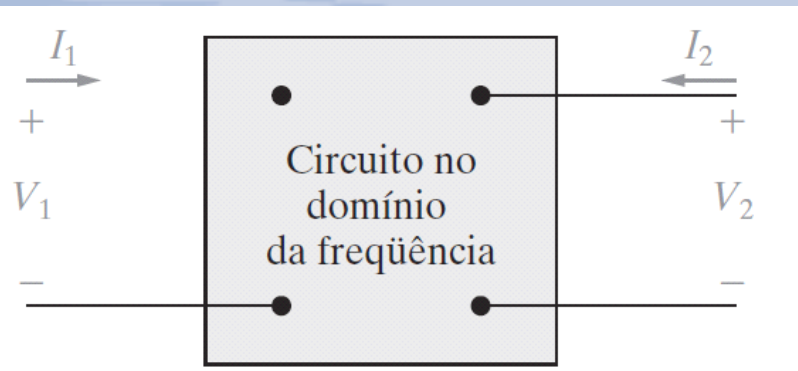
- admitâncias



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \text{S}, \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \text{S},$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \text{S}, \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \text{S}.$$

Parâmetros a do Quadripolo



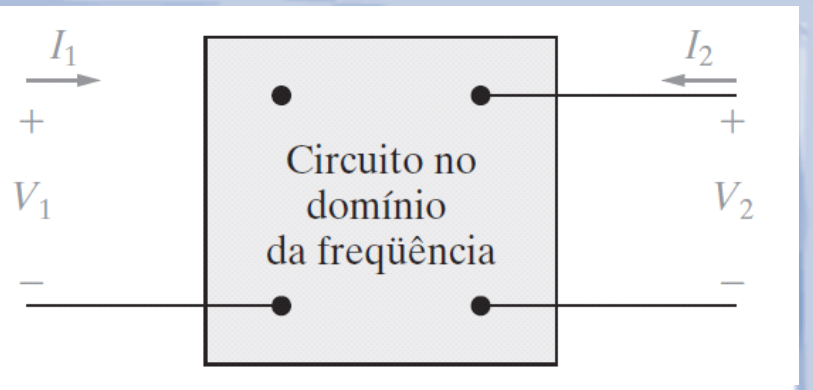
- a11 e a22: adimensionais;
- a12: impedância; a21: adimitância

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \qquad a_{12} = \left.-\frac{V_1}{I_2}\right|_{V_2=0} \Omega,$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} \mathrm{S}, \qquad a_{22} = \left.-\frac{I_1}{I_2}\right|_{V_2=0}.$$

Parâmetros b do Quadripolo

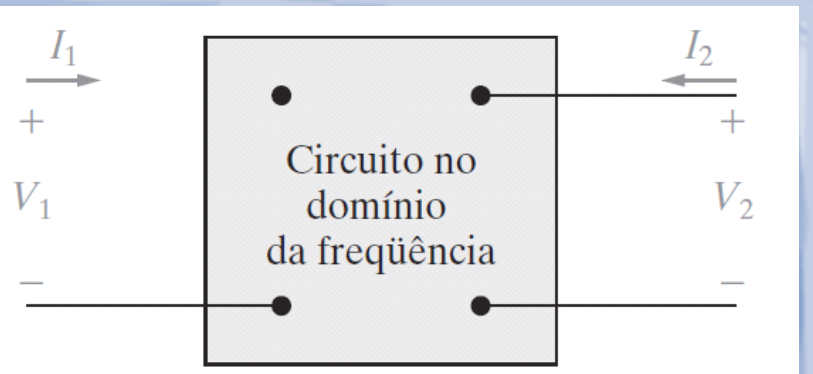
- b11 e b22: adimensionais;
- b12: impedância; b21: adimitância



$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}, \qquad b_{12} = \left.-\frac{V_2}{I_1}\right|_{V_1=0} \Omega,$$

$$b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0} \mathrm{S}, \qquad b_{22} = \left.-\frac{I_2}{I_1}\right|_{V_1=0}.$$

Parâmetros h do Quadripolo



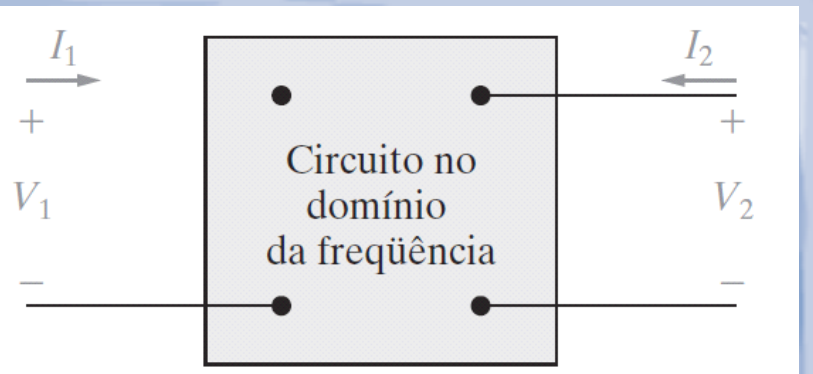
- h11: impedância; h22: admitância;
- h12 e h21: adimensionais.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \Omega, \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0},$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}, \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \text{S}.$$

Parâmetros g do Quadripolo

- g11: admitância; g22: impedância;
- g12 e g21: adimensionais.



$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \text{S}, \qquad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0},$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}, \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \Omega.$$

Relações entre parâmetros do quadripolo

TABELA 18.1 Tabela de conversão de parâmetros

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{\Delta h}{h_{22}} = \frac{1}{g_{11}}$$

$$z_{12} = -\frac{y_{12}}{\Delta y} = \frac{\Delta a}{a_{21}} = \frac{1}{b_{21}} = \frac{h_{12}}{h_{22}} = -\frac{g_{12}}{g_{11}}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{1}{a_{21}} = \frac{\Delta b}{b_{21}} = -\frac{h_{21}}{h_{22}} = \frac{g_{21}}{g_{11}}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{a_{22}}{a_{21}} = \frac{b_{11}}{b_{21}} = \frac{1}{h_{22}} = \frac{\Delta g}{g_{11}}$$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{a_{22}}{a_{12}} = \frac{b_{11}}{b_{12}} = \frac{1}{h_{11}} = \frac{\Delta g}{g_{22}}$$

$$y_{12} = -\frac{z_{12}}{\Delta z} = -\frac{\Delta a}{a_{12}} = -\frac{1}{b_{12}} = -\frac{h_{12}}{h_{11}} = \frac{g_{12}}{g_{22}}$$

$$y_{21} = -\frac{z_{21}}{\Delta z} = -\frac{1}{a_{12}} = -\frac{\Delta b}{b_{12}} = \frac{h_{21}}{h_{11}} = -\frac{g_{21}}{g_{22}}$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{a_{11}}{a_{12}} = \frac{b_{22}}{b_{12}} = \frac{\Delta h}{h_{11}} = \frac{1}{g_{22}}$$

$$b_{21} = \frac{1}{z_{12}} = -\frac{\Delta y}{y_{12}} = \frac{a_{21}}{\Delta a} = \frac{h_{22}}{h_{12}} = -\frac{g_{11}}{g_{12}}$$

$$b_{22} = \frac{z_{11}}{z_{12}} = \frac{y_{22}}{y_{12}} = \frac{a_{11}}{\Delta a} = \frac{\Delta h}{h_{12}} = -\frac{1}{g_{12}}$$

$$h_{11} = \frac{\Delta z}{z_{22}} = \frac{1}{y_{11}} = \frac{a_{12}}{a_{22}} = \frac{b_{12}}{b_{11}} = \frac{g_{22}}{\Delta g}$$

$$h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} = \frac{\Delta a}{a_{22}} = \frac{1}{b_{11}} = -\frac{g_{12}}{\Delta g}$$

$$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}} = -\frac{1}{a_{22}} = -\frac{\Delta b}{b_{11}} = -\frac{g_{21}}{\Delta g}$$

$$h_{22} = \frac{1}{z_{22}} = \frac{\Delta y}{y_{11}} = \frac{a_{21}}{a_{22}} = \frac{b_{21}}{b_{11}} = \frac{g_{11}}{\Delta g}$$

$$g_{11} = \frac{1}{z_{11}} = \frac{\Delta y}{y_{22}} = \frac{a_{21}}{a_{11}} = \frac{b_{21}}{b_{22}} = \frac{h_{22}}{\Delta h}$$

$$g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{\Delta a}{a_{11}} = -\frac{1}{b_{22}} = -\frac{h_{12}}{\Delta h}$$

Relações entre parâmetros do quadripolo

TABELA 18.1 Tabela de conversão de parâmetros

$$a_{11} = \frac{z_{11}}{z_{21}} = -\frac{y_{22}}{y_{21}} = \frac{b_{22}}{\Delta b} = -\frac{\Delta h}{h_{21}} = \frac{1}{g_{21}}$$

$$a_{12} = \frac{\Delta z}{z_{21}} = -\frac{1}{y_{21}} = \frac{b_{12}}{\Delta b} = -\frac{h_{11}}{h_{21}} = \frac{g_{22}}{g_{21}}$$

$$a_{21} = \frac{1}{z_{21}} = -\frac{\Delta y}{y_{21}} = \frac{b_{21}}{\Delta b} = -\frac{h_{22}}{h_{21}} = \frac{g_{11}}{g_{21}}$$

$$a_{22} = \frac{z_{22}}{z_{21}} = -\frac{y_{11}}{y_{21}} = \frac{b_{11}}{\Delta b} = -\frac{1}{h_{21}} = \frac{\Delta g}{g_{21}}$$

$$b_{11} = \frac{z_{22}}{z_{12}} = -\frac{y_{11}}{y_{12}} = \frac{a_{22}}{\Delta a} = \frac{1}{h_{12}} = -\frac{\Delta g}{g_{12}}$$

$$b_{12} = \frac{\Delta z}{z_{12}} = -\frac{1}{y_{12}} = \frac{a_{12}}{\Delta a} = \frac{h_{11}}{h_{12}} = -\frac{g_{22}}{g_{12}}$$

$$g_{21} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = \frac{1}{a_{11}} = \frac{\Delta b}{b_{22}} = -\frac{h_{21}}{\Delta h}$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{1}{y_{22}} = \frac{a_{12}}{a_{11}} = \frac{b_{12}}{b_{22}} = \frac{h_{11}}{\Delta h}$$

$$\Delta z = z_{11}z_{22} - z_{12}z_{21}$$

$$\Delta y = y_{11}y_{22} - y_{12}y_{21}$$

$$\Delta a = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21}$$

- Se conhecemos um conjunto de parâmetros de um quadripolo, podemos determinar todos os outros conjuntos.

Obtendo os parâmetros z em função dos parâmetros y

$$I_1 = y_{11}V_1 + y_{12}V_2,$$

$$I_2 = y_{21}V_1 + y_{22}V_2;$$

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{22}}{\Delta y} I_1 - \frac{y_{12}}{\Delta y} I_2,$$

$$V_2 = \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\Delta y} = -\frac{y_{21}}{\Delta y} I_1 + \frac{y_{11}}{\Delta y} I_2.$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega,$$

$$z_{11} = \frac{y_{22}}{\Delta y},$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega,$$

$$z_{12} = -\frac{y_{12}}{\Delta y},$$

Obtendo os parâmetros z em função dos parâmetros y

$$I_1 = y_{11}V_1 + y_{12}V_2,$$

$$I_2 = y_{21}V_1 + y_{22}V_2;$$

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{22}}{\Delta y} I_1 - \frac{y_{12}}{\Delta y} I_2,$$

$$V_2 = \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\Delta y} = -\frac{y_{21}}{\Delta y} I_1 + \frac{y_{11}}{\Delta y} I_2.$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega,$$

$$z_{21} = -\frac{y_{21}}{\Delta y},$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega.$$

$$z_{22} = \frac{y_{11}}{\Delta y}.$$

Obtendo os parâmetros z em função dos parâmetros a

- Rearranjamos a equação abaixo:

$$V_1 = a_{11}V_2 - a_{12}I_2\,,$$
$$I_1 = a_{21}V_2 - a_{22}I_2\,;\longrightarrow \qquad V_2 = \frac{1}{a_{21}}I_1 + \frac{a_{22}}{a_{21}}I_2\,.$$

- Substituindo:

$$V_1 = \frac{a_{11}}{a_{21}}I_1 + \left(\frac{a_{11}a_{22}}{a_{21}} - a_{12}\right)I_2\,.$$

- Comparando com as equações dos parâmetros z, temos:

$$V_1 = z_{11}I_1 + z_{12}I_2\,,$$
$$V_2 = z_{21}I_1 + z_{22}I_2\,;\longrightarrow$$

$$z_{11} = \frac{a_{11}}{a_{21}}, \qquad z_{21} = \frac{1}{a_{21}},$$
$$z_{12} = \frac{\Delta a}{a_{21}}. \qquad z_{22} = \frac{a_{22}}{a_{21}}.$$

Obtendo os parâmetros z em função dos parâmetros a

- Rearranjamos a equação abaixo:

$$V_1 = a_{11}V_2 - a_{12}I_2\,,$$

$$I_1 = a_{21}V_2 - a_{22}I_2\,;\longrightarrow \qquad V_2 = \frac{1}{a_{21}}I_1 + \frac{a_{22}}{a_{21}}I_2\,.$$

- Substituindo:

$$V_1 = \frac{a_{11}}{a_{21}}I_1 + \left(\frac{a_{11}a_{22}}{a_{21}} - a_{12}\right)I_2\,.$$

- Comparando com as equações dos parâmetros z, temos:

$$V_1 = z_{11}I_1 + z_{12}I_2\,,$$

$$V_2 = z_{21}I_1 + z_{22}I_2\,;\longrightarrow$$

$$z_{11} = \frac{a_{11}}{a_{21}}, \qquad z_{21} = \frac{1}{a_{21}},$$

$$z_{12} = \frac{\Delta a}{a_{21}}. \qquad z_{22} = \frac{a_{22}}{a_{21}}.$$

Quadripolos recíprocos

- Um quadripolo é recíproco quando obedecem às seguintes relações:

$$z_{12} = z_{21},$$

$$y_{12} = y_{21},$$

$$a_{11}a_{22} - a_{12}a_{21} = \Delta a = 1,$$

$$b_{11}b_{22} - b_{12}b_{21} = \Delta b = 1,$$

$$h_{12} = -h_{21},$$

$$g_{12} = -g_{21}.$$

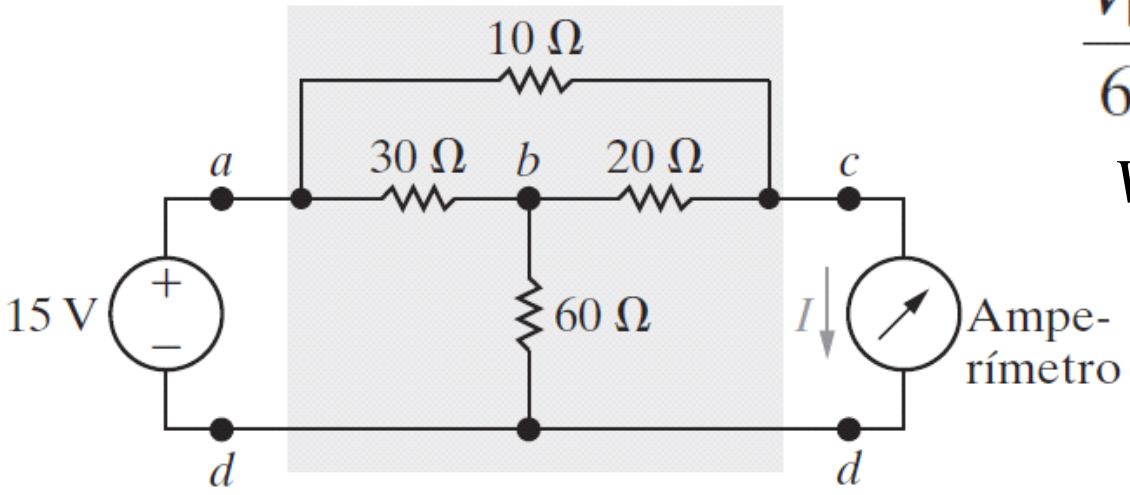
Quadripolos recíprocos

- Um quadripolo é recíproco “se a permuta entre a fonte ideal de tensão em uma porta e um amperímetro ideal em outra porta produzir a mesma leitura no amperímetro”.

- Alternativamente: Um quadripolo é recíproco “se a permuta entre a fonte ideal de corrente em uma porta e um voltímetro ideal em outra porta produzir a mesma leitura no voltímetro”.

Quadripolos recíprocos

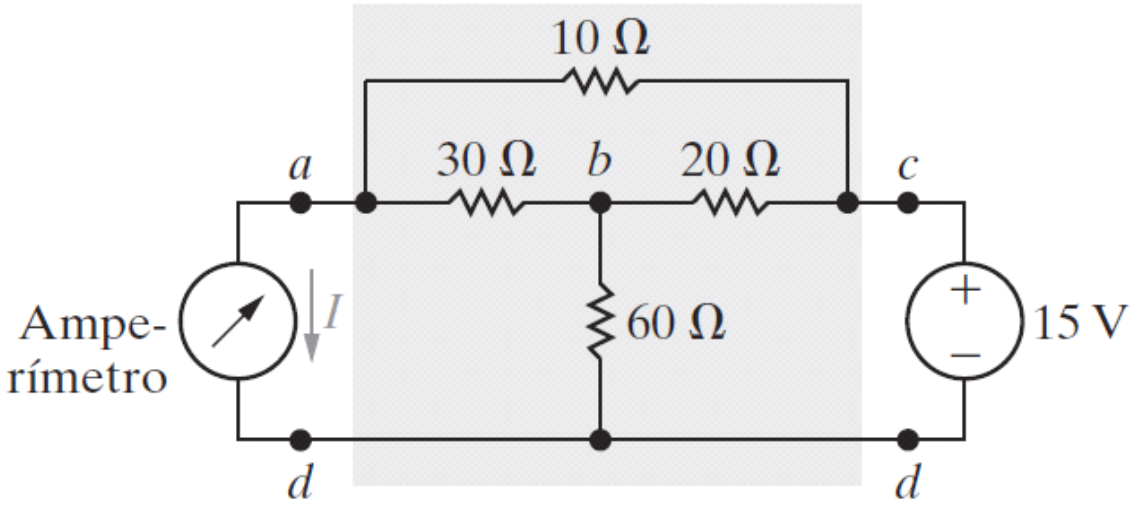
- Exemplo:



$$\frac{V_{bd}}{60} + \frac{V_{bd} - 15}{30} + \frac{V_{bd}}{20} = 0,$$

$$V_{bd} = 5\text{V}$$

$$I = \frac{5}{20} + \frac{15}{10} = 1{,}75 \text{ A}.$$



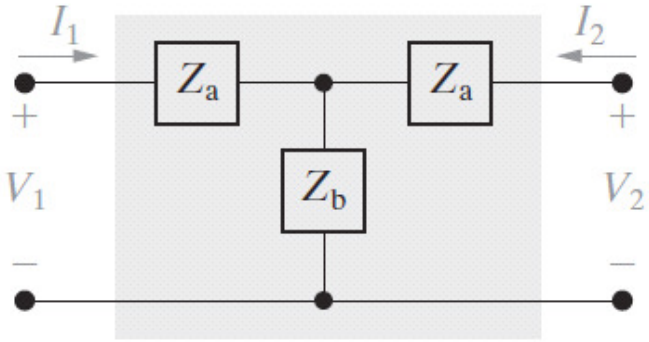
$$\frac{V_{bd}}{60} + \frac{V_{bd}}{30} + \frac{V_{bd} - 15}{20} = 0.$$

$$V_{bd} = 7{,}5\,V$$

$$I_{ad} = \frac{7{,}5}{30} + \frac{15}{10} = 1{,}75 \text{ A}.$$

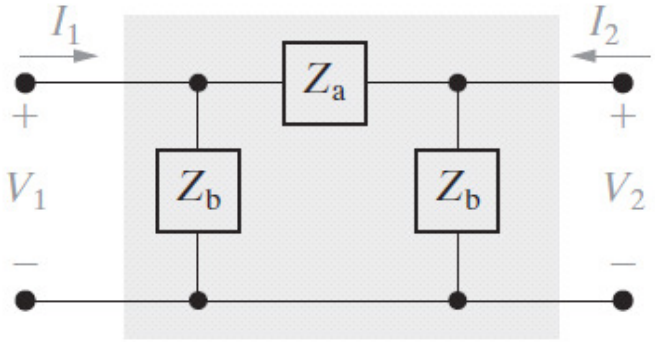
Quadripolos simétrico

- “Um quadripolo recíproco é simétrico se sua entrada puder ser trocada pela saída sem alteração dos valores de correntes e tensões nos terminais.”



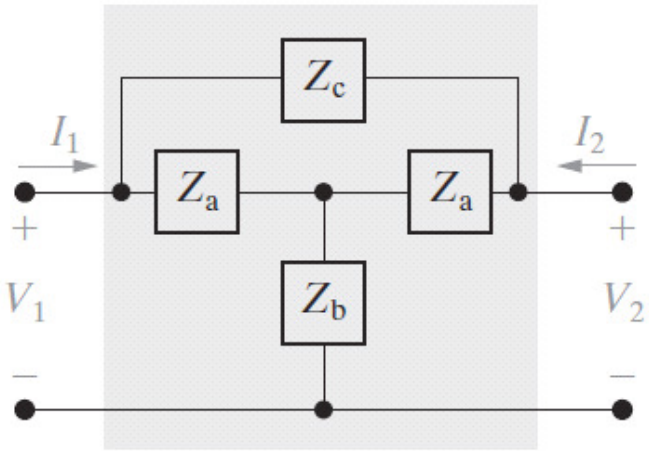
(a)

Circuito T simétrico



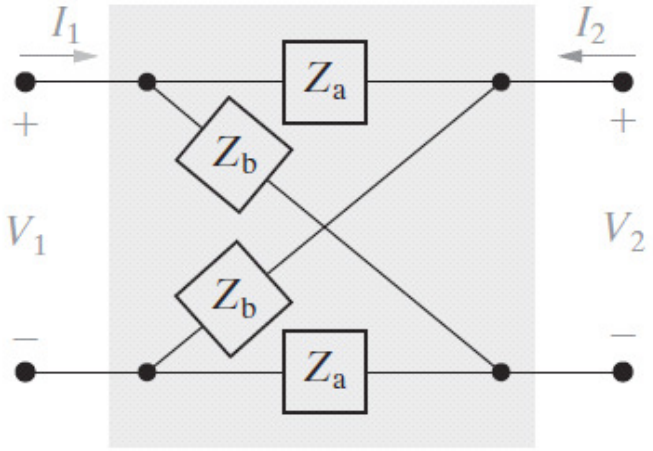
(b)

Circuito π simétrico



(c)

Circuito T simétrico com ponte



(d)

Treliça simétrica

Quadripolos simétrico

- Em um quadripolo simétrico, além das relações dos parâmetros para quadripolos recíprocos, estes devem obedecer às seguintes relações:

$$z_{11} = z_{22},$$

$$y_{11} = y_{22},$$

$$a_{11} = a_{22},$$

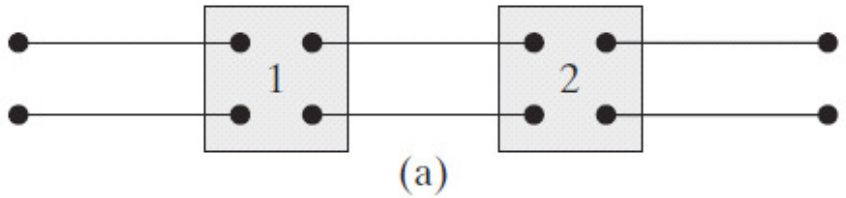
$$b_{11} = b_{22},$$

$$h_{11}h_{22} - h_{12}h_{21} = \Delta h = 1,$$

$$g_{11}g_{22} - g_{12}g_{21} = \Delta g = 1.$$

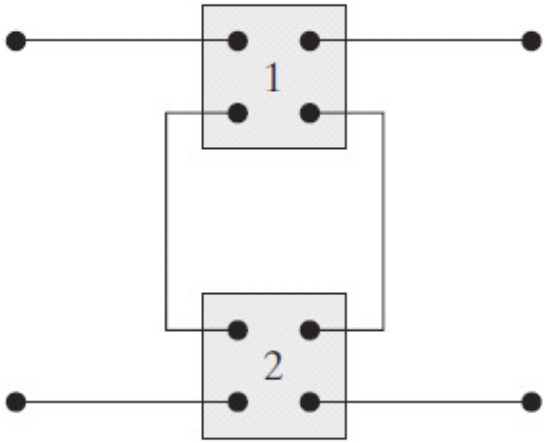
Interconexão de quadripolos

cascata



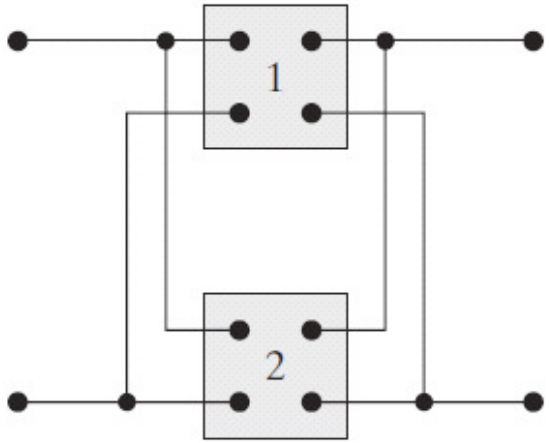
(a)

série



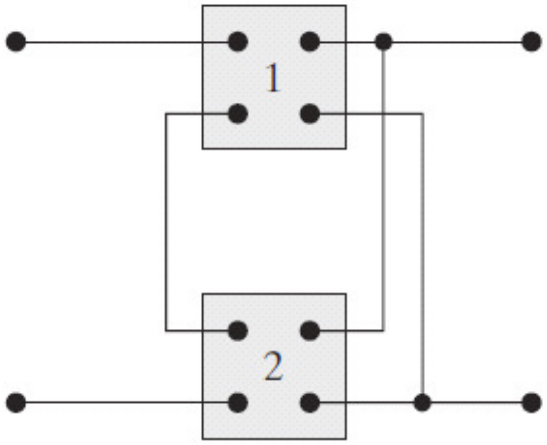
(b)

paralelo



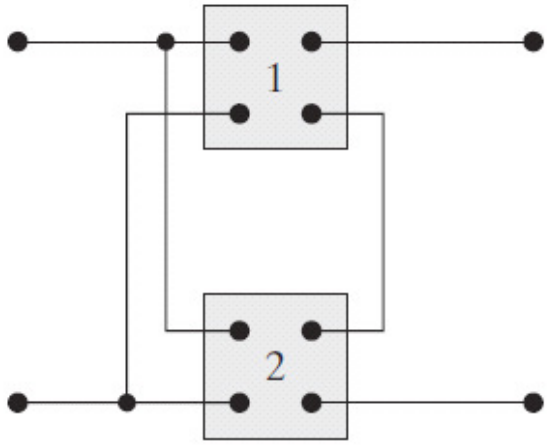
(c)

série-paralelo



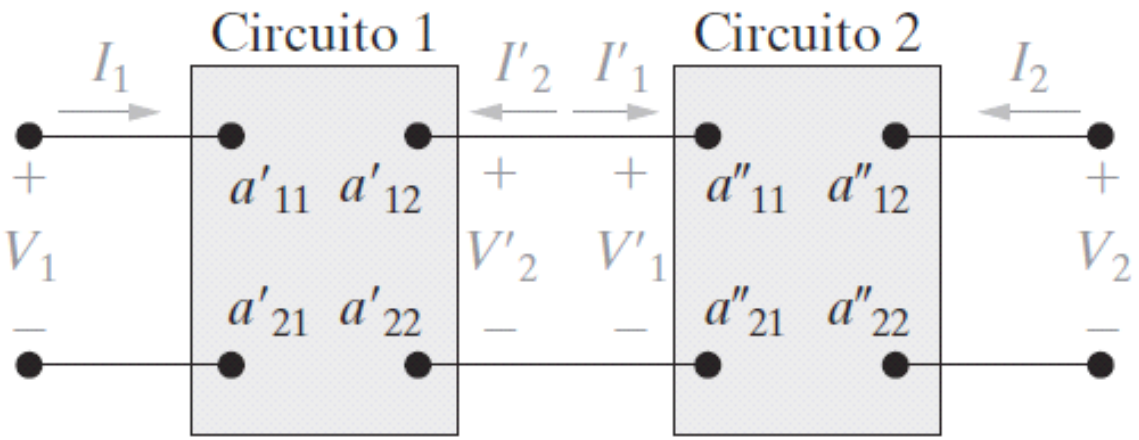
(d)

paralelo-série



(e)

Interconexão de quadripolos: Exemplo

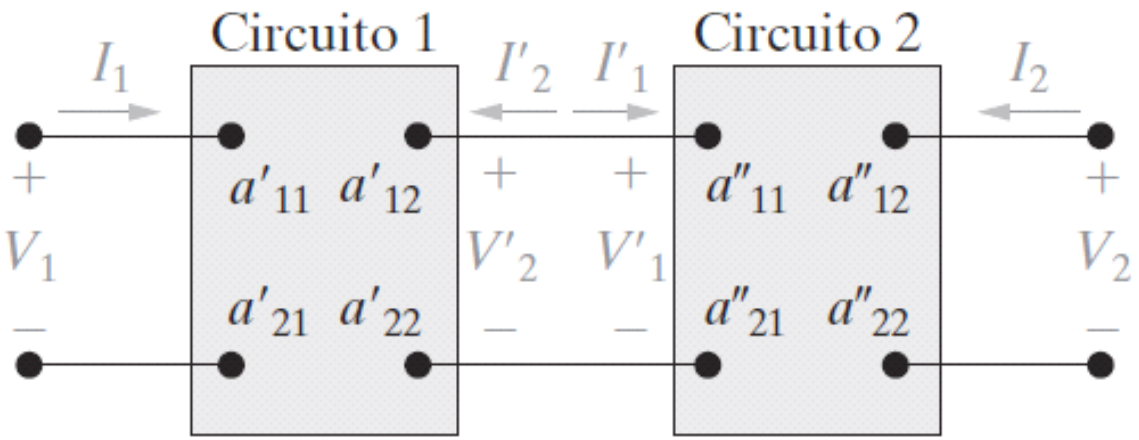


- Para o circuito acima, desejamos encontrar:

$$V_1 = a_{11}V_2 - a_{12}I_2$$

$$I_1 = a_{21}V_2 - a_{22}I_2,$$

Interconexão de quadripolos: Exemplo



- Do primeiro quadripolo, temos:

$$V_1 = a'_{11}\boxed{V'_2} - a'_{12}\boxed{I'_2}$$

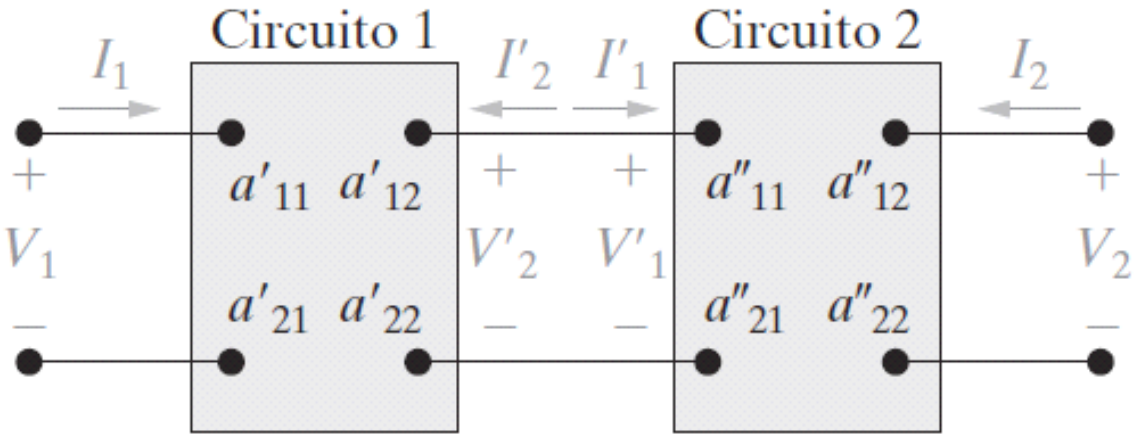
$$I_1 = a'_{21}\boxed{V'_2} - a'_{22}\boxed{I'_2}$$

- Mas, $\boxed{V'_2 = V'_1}$ e $\boxed{I'_2 = -I'_1}$, logo:

$$V_1 = a'_{11}\boxed{V'_1} + a'_{12}\boxed{I'_1}$$

$$I_1 = a'_{21}\boxed{V'_1} + a'_{22}\boxed{I'_1}$$

Interconexão de quadripolos: Exemplo



- Do segundo quadripolo, temos:

$$V'_1 = a''_{11} V_2 - a''_{12} I_2,$$

$$I'_1 = a''_{21} V_2 - a''_{22} I_2.$$

Interconexão de quadripolos: Exemplo

(1)

$$V_1 = a'_{11}\, V'_1 + a'_{12}\, I'_1$$

$$I_1 = a'_{21}\, V'_1 + a'_{22}\, I'_1$$

(2)

$$V'_1 = a''_{11} V_2 - a''_{12} I_2,$$

$$I'_1 = a''_{21} V_2 - a''_{22} I_2.$$

- Substituindo (2) em (1), temos:

a_{11} a_{12}

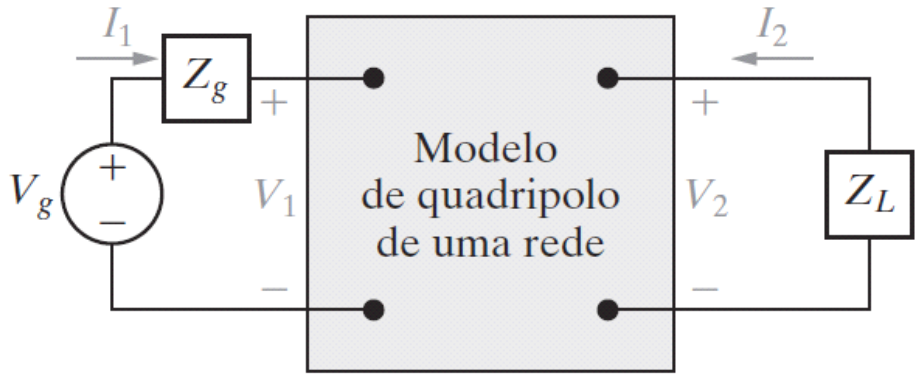
$$V_1 = (a'_{11}\, a''_{11} + a'_{12}\, a''_{21}) V_2 - (a'_{11}\, a'_{12} + a''_{22}\, a'_{12}) I_2,$$

$$I_1 = (a'_{21}\, a''_{11} + a'_{22}\, a''_{21}) V_2 - (a'_{21}\, a''_{12} + a'_{22}\, a''_{22}) I_2.$$

a_{21} a_{22}

Quadripolos com carga

- Casos reais: empregamos fontes e cargas



- Parâmetros usuais:
  - Impedância (ou admitância) de entrada: V_1/I_1 (I_1/V_1)
  - Corrente de saída: I_2
  - Tensão e impedância de Thevenin (V_{th}, Z_{th}) vistas da saída.
  - Ganho de corrente: I_2/I_1
  - Ganho de tensão: V_2/V_1
  - Ganho de tensão total: V_2/V_g

Quadripolos com carga

TABELA 18.2 Equações de quadripolos com cargas em seus terminais

Parâmetros z	Parâmetros y
$Z_{\text{ent}} = z_{11} - \dfrac{z_{12}z_{21}}{z_{22} + Z_L}$	$Y_{\text{ent}} = y_{11} - \dfrac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$
$I_2 = \dfrac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$I_2 = \dfrac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$
$V_{\text{Th}} = \dfrac{z_{21}}{z_{11} + Z_g}V_g$	$V_{\text{Th}} = \dfrac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$
$Z_{\text{Th}} = z_{22} - \dfrac{z_{12}z_{21}}{z_{11} + Z_g}$	$Z_{\text{Th}} = \dfrac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$
$\dfrac{I_2}{I_1} = \dfrac{-z_{21}}{z_{22} + Z_L}$	$\dfrac{I_2}{I_1} = \dfrac{y_{21}}{y_{11} + \Delta y Z_L}$
$\dfrac{V_2}{V_1} = \dfrac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$	$\dfrac{V_2}{V_1} = \dfrac{-y_{21}Z_L}{1 + y_{22}Z_L}$
$\dfrac{V_2}{V_g} = \dfrac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$\dfrac{V_2}{V_g} = \dfrac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$

Quadripolos com carga

Parâmetros *a*	Parâmetros *b*
$Z_{ent} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}}$	$Z_{ent} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$
$I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$	$I_2 = \frac{-V_g\Delta b}{b_{11}Z_g + b_{21}Z_gZ_L + b_{22}Z_L + b_{12}}$
$V_{Th} = \frac{V_g}{a_{11} + a_{21}Z_g}$	$V_{Th} = \frac{V_g\Delta b}{b_{22} + b_{21}Z_g}$
$Z_{Th} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g}$	$Z_{Th} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$
$\frac{I_2}{I_1} = \frac{-1}{a_{21}Z_L + a_{22}}$	$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L}$
$\frac{V_2}{V_1} = \frac{Z_L}{a_{11}Z_L + a_{12}}$	$\frac{V_2}{V_1} = \frac{\Delta bZ_L}{b_{12} + b_{22}Z_L}$
$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$	$\frac{V_2}{V_g} = \frac{\Delta bZ_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_gZ_L}$