Universidade Federal de Minas Gerais - Programa de Pós-Graduação em
Engenharia Elétrica

EEE945 - INTRODUÇÃO AOS PROCESSOS ESTOCÁSTICOS

FINAL EXAM

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Problem 1. (30 pts) Consider the stochastic process given by

$$X_{n+1} = X_n + X_{n-1}W_n \ \ ,$$

where W_n is a white Gaussian noise, independent from X_0.

a) Explain why X_n is not a Markov chain.

b) Find a Markov chain that describes the process X_n.

c) Discuss the irreducibility of the chain given above.

Problem 2. (60 pts) Consider the Markov process that describes the health state of a person: when the person is in the healthy state H, he/she may get sick with rate s and then move to the state S, in addition, the person may suddenly die and move to state D, which happens with rate d_H; when in the sick state S, the person recovers with rate r and dies with rate d_S.

a) Draw the graph that describes this Markov process.
b) Find its communicating classes and classify them as recurrent or transient.
c) Find at least one of its ergodic invariant distributions.
d) What is the expected time until a person leaves the healthy state H?
e) Given that a person hasn't been sick for two months, what is the expected time that he/she leaves the healthy state H?
f) What is the probability that a person dies given that he/she is in the sick state S?

Problem 3. (30 pts) Consider the Markov chain Z_n on $\{1, 2, 3, 4, 5\}$ with transition matrix

$$P = \begin{bmatrix} 0.9 & 0.1 & 0 & 0 & 0 \\ 0.1 & 0.9 & 0 & 0 & 0 \\ 0.5 & 0 & 0.3 & 0.2 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

At every time step, we incur in a cost $f(Z_n) = \max\{Z_n - 1, 0\}$.

a) Explain the convergence of the empirical average cost $1/N \sum_{n=1}^{N} f(Z_n)$ as N goes to infinity.

b) Find the average cost when $Z_0 = 1$.