

Universidade Federal de Minas Gerais - Programa de Pós-Graduação em  
Engenharia Elétrica

**EEE945 - INTRODUÇÃO AOS PROCESSOS ESTOCÁSTICOS**

**HOMEWORK 5**

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**Problem 1.** Consider the Markov chain on  $\mathbb{R}$  given by:

$$X_{n+1} = aX_n + W_n$$

where  $a < 1$  and  $\{W_n\}_{n=0}^{\infty}$  are i.i.d. with  $W_n \sim \gamma(w)$ ,  $\gamma(w) > 0$  on  $(-1, 1)$  and  $E[W_n^k] < \infty$ . Using the Foster-Lyapunov theorem, show that  $X_n$  is positive recurrent and that its  $k$ -th moment is bounded.

**Problem 2.** Simulate the differential stochastic equations:

$$dx_t = -2 \frac{x_t}{\sqrt{|x_t|}} dt + x_t dB_t$$

and

$$dx_t = -2 \frac{x_t}{\sqrt{|x_t|}} dt + \sqrt{|x_t|} dB_t$$

where  $x_0 = 1$  and  $B_t$  is the Wiener process.