Universidade Federal de Minas Gerais - Programa de Pós-Graduação em Engenharia Elétrica

EEE945 - INTRODUÇÃO AOS PROCESSOS ESTOCÁSTICOS

Homework 3

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Note: You must hand in all simulation codes.

Problem 1. Consider the Markov process with the generator matrix

$$A = \begin{bmatrix} -10 & 1 & 9 & 0 & 0 \\ 0 & -2 & 0 & 0 & 2 \\ 6 & 0 & -8 & 2 & 0 \\ 0 & 3 & 0 & -3 & 0 \\ 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

Draw its graph representation and find its transition matrix P_t and its invariant probability distributions. *Hint:* Use diagonalization or Cayley-Hamilton's theorem to compute matrix exponentials. Alternatively, you may use Matlab's symbolic math toolbox.

Problem 2. For the process above, find the expected time before it leaves the set $\{0, 1\}$.

Problem 3. Consider the Markov process on $\{1, 2, \ldots, 6\}$ with generator matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -4 & 2 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 5 & -5 \end{bmatrix}.$$

(1) Draw its graph and identify its communicating classes.
(2) Identify its recurrent and transient classes.
(3) Is this process irreducible?
(4) Find its ergodic invariant probability distributions and identify their support.
(5) Identify the period of each ergodic class.
(6) Find the time average of $f(X_t) = X_t$ for $X_0 = 6$?

Problem 4. Simulate the Michaelis-Menten model presented in class. Derive the ODEs for the concentration of species, simulate them and compare with the stochastic simulation.

Problem 5. Consider a queue with arrival rate λ, service rate μ and a maximum size 8. When the queue is at its maximum size, newly-arrived objects are discarded. Simulate this queue and obtain its stationary distribution (from simulation) for

a) $\lambda = 2$ and $\mu = 1$;
b) $\lambda = 1$ and $\mu = 2$;
c) $\lambda = 1$ and $\mu = 1$.