Universidade Federal de Minas Gerais - Programa de Pós-Graduação em Engenharia Elétrica

EEE945 - INTRODUÇÃO AOS PROCESSOS ESTOCÁSTICOS

Homework 2

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Problem 1. A communication channel may exhibit two possible states. In the state "Good", this channel transmits packets successfully with probability 0.9. In the state "Bad", successful transmissions occur with probability 0.3. We know that, at each communication round, the transition probability from state "Good"to state "Bad"is 0.2 and from the state "Bad" to the state "Good"is 0.6. Find the average packet transmission rate for this channel.

Problem 2. Consider a Markov chain on $\{1, 2, \ldots, 7\}$ with transition matrix

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.7 & 0.1 & 0 & 0.1 & 0 & 0 & 0.1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 & 0.7 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}.$$

(1) Draw its graph and identify its communicating classes.
(2) Identify its recurrent and transient classes.
(3) Is this chain irreducible?
(4) Find its ergodic invariant probability distributions and identify their support.
(5) Find the eigenvectors to the right for the eigenvalues with absolute value less than 1 and explain how they are related with the communicating classes.
(6) Find the eigenvectors to the right for the eigenvalues with absolute value equal to 1 and explain how they are related with the communicating classes.
(7) From the analysis of the eigenvalues and eigenvectors of P, identify the period of each ergodic class.
(8) Find the time average of $f(X_n) = X_n^2 + X_n$ for $X_0 = 3$?
(9) What can we say about the time average of $f(X_n)$ if $X_0 = 2$?
(10) Suppose that $C(X_n, u) = (uX_n)^2 + uX_n$ represents a cost paid at time n for state X_n, where u is a some decision parameter that is chosen at time 0. Find u that minimizes the time average of the cost when $X_0 = 3$.

Problem 3. Let τ be the first time n such that $X_n \neq X_0$, where $\tau = +\infty$ if $X_n = X_0$, $\forall n \geq 0$. Find $\mathrm{E}[\ \tau \mid X_0 = i]$ as a function of P_{ii}.

Problem 4. Consider the following game: a fair coin is tossed until we get either 4 consecutive heads, in which case Player 1 is declared the winner, or 3 consecutive tails, in which case Player 2 wins. About this game, answer:

1) What is the expected duration of the game in terms of the total number of tosses?
2) What is the probability that Player 1 wins?