Universidade Federal de Minas Gerais - Programa de Pós-Graduação em Engenharia Elétrica

EEE945 - INTRODUÇÃO AOS PROCESSOS ESTOCÁSTICOS

Homework 3

Instructors: Alexandre R. Mesquita and Eduardo M. A. M. Mendes

Note: You must hand in all Matlab code for simulations.

Problem 1. Consider the Markov process with the generator matrix

$$A = \begin{bmatrix} -12 & 1 & 11 & 0 & 0 \\ 0 & -2 & 0 & 0 & 2 \\ 6 & 0 & -8 & 2 & 0 \\ 0 & 3 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Draw its graph representation and find its transition matrix P_t and its invariant probability distributions. *Hint:* Use diagonalization or Cayley-Hamilton's theorem to compute matrix exponentials. Alternatively, you may use Matlab's symbolic math toolbox.

Problem 2. For the process above, find the expected time before it leaves the set $\{0, 2\}$.

Problem 3. Simulate the Michaelis-Menten model presented in class. Derive the ODEs for the concentration of species, simulate them and compare with the stochastic simulation.

Problem 4. Consider a queue with arrival rate λ, service rate μ and a maximum size 8. When the queue is at its maximum size, newly-arrived objects are discarded. Simulate this queue and obtain its stationary distribution (from simulation) for

a) $\lambda = 2$ and $\mu = 1$;
b) $\lambda = 1$ and $\mu = 2$;
c) $\lambda = 1$ and $\mu = 1$.

Problem 5. Simulate the differential stochastic equations:

$$dx_t = -2\frac{x_t}{\sqrt{|x_t|}}\, dt + x_t dB_t$$

and

$$dx_t = -2\frac{x_t}{\sqrt{|x_t|}}\, dt + \sqrt{|x_t|} dB_t$$

where $x_0 = 1$ and B_t is the Wiener process.