Universidade Federal de Minas Gerais - Programa de Pós-Graduação em Engenharia Elétrica

EEE945 - INTRODUÇÃO AOS PROCESSOS ESTOCÁSTICOS

HOMEWORK 1

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Problem 1. A famliy receives the newspaper every morning and, after being read, the newspaper is placed on a pile. Every afternoon, with probability 1/2, the pile is taken to the recycle bin. Moreover, the pile is always taken to the recycle bin whenever six papers are gathered. Is it possible to model the number of papers in the pile as a Markov chain? If yes, present the corresponding graph and the corresponding transition matrix.

Problem 2. Draw the graph for a Markov chain X_n with state space $\{1, 2, 3\}$ and transition matrix

$$P = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.1 & 0 & 0.9 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$$

Problem 3. Find $\Pr\{X_3 = 1 \mid X_0 = 1\}$ e $\Pr\{X_7 = 2 \mid X_4 = 0\}$ for the chain in Problem 2 above.

Problem 4. Present an example of a Markov chain on $\{1, \ldots, 5\}$ such that

$$\Pr\{X_2 = 5 \mid X_1 \in \{3, 4\}, X_0 = 2\} \neq \Pr\{X_2 = 5 \mid X_1 \in \{3, 4\}\}$$

and explain why the results do not violate the Markov property.

Problem 5. Find the invariant probability distributions for the Markov chains with transition matrices given below and draw the corresponding graphs.

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & p & 1-p \\ 0 & q & 1-q \end{bmatrix}$$

$$P = \begin{bmatrix} 0.2 & 0 & 0.8 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0.4 \\ 0.2 & 0 & 0.8 & 0 & 0 \\ 0.1 & 0.1 & 0 & 0.8 & 0 \\ 0 & 0.5 & 0 & 0 & 0.5 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.7 & 0.3 \\ 1 & 0 & 0 \end{bmatrix}$$