modified PID control schemes have proved their usefulness in providing satisfactory control, although they may not provide optimal control in many given situations.

**Outline of the chapter.** Section 10–1 has presented introductory material for the chapter. Section 10–2 deals with tuning methods for the basic PID control, commonly known as Ziegler–Nichols tuning rules. Section 10–3 discusses modified PID control schemes, such as PI-D control and I-PD control. Section 10–4 introduces two-degrees-of-freedom PID control schemes. Section 10–5 introduces the concept of robust control using a two-degrees-of-freedom control system as an example.

## 10–2 TUNING RULES FOR PID CONTROLLERS

**PID control of plants.** Figure 10–1 shows a PID control of a plant. If a mathematical model of the plant can be derived, then it is possible to apply various design techniques for determining parameters of the controller that will meet the transient and steady-state specifications of the closed-loop system. However, if the plant is so complicated that its mathematical model cannot be easily obtained, then analytical approach to the design of a PID controller is not possible. Then we must resort to experimental approaches to the tuning of PID controllers.

The process of selecting the controller parameters to meet given performance specifications is known as controller tuning. Ziegler and Nichols suggested rules for tuning PID controllers (meaning to set values $K_p$, $T_i$, and $T_d$) based on experimental step responses or based on the value of $K_p$ that results in marginal stability when only the proportional control action is used. Ziegler–Nichols rules, which are presented in the following, are very convenient when mathematical models of plants are not known. (These rules can, of course, be applied to the design of systems with known mathematical models.)

**Ziegler–Nichols rules for tuning PID controllers.** Ziegler and Nichols proposed rules for determining values of the proportional gain $K_p$, integral time $T_i$, and derivative time $T_d$ based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on site by experiments on the plant. (Numerous tuning rules for PID controllers have been proposed since the Ziegler–Nichols proposal. They are available in the literature. Here, however, we introduce only the Ziegler–Nichols tuning rules.)

There are two methods called Ziegler–Nichols tuning rules. In both methods, they aimed at obtaining 25% maximum overshoot in step response (see Figure 10–2).

**First Method.** In the first method, we obtain experimentally the response of the plant to a unit-step input, as shown in Figure 10–3. If the plant involves neither inte-

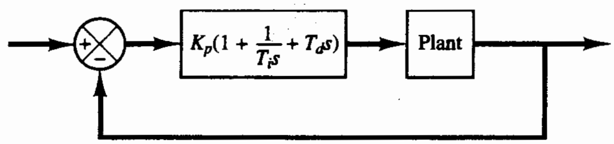



**Figure 10–1** PID control of a plant.

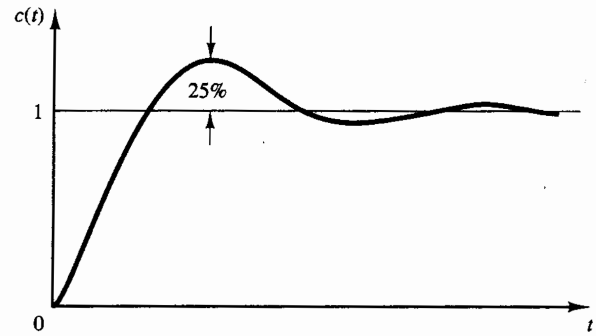



**Figure 10–2**
Unit-step response curve showing 25% maximum overshoot.

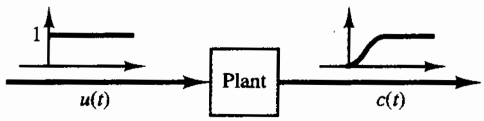



**Figure 10–3**
Unit-step response of a plant.

grator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look like an S-shaped curve, as shown in Figure 10–4. (If the response does not exhibit an S-shaped curve, this method does not apply.) Such step-response curves may be generated experimentally or from a dynamic simulation of the plant.

The S-shaped curve may be characterized by two constants, delay time $L$ and time constant $T$. The delay time and time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line $c(t) = K$, as shown in Figure 10–4. The transfer function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of $K_p$, $T_i$, and $T_d$ according to the formula shown in Table 10–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

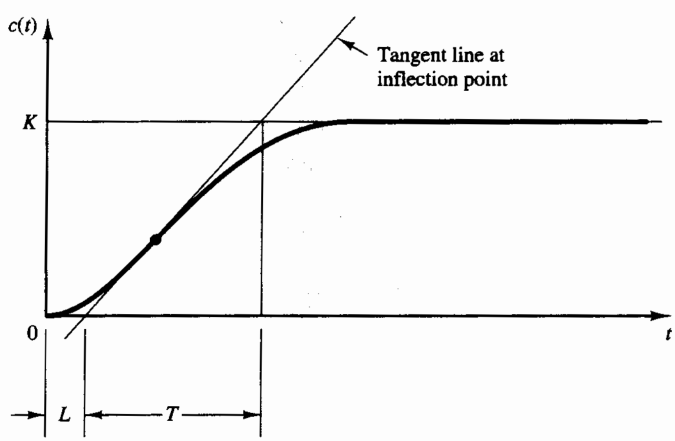



**Figure 10–4**
S-shaped response curve.

**Table 10–1** Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

| Type of Controller | $K_p$ | $T_i$ | $T_d$ |
|---|---|---|---|
| P | $\frac{T}{L}$ | $\infty$ | 0 |
| PI | $0.9\frac{T}{L}$ | $\frac{L}{0.3}$ | 0 |
| PID | $1.2\frac{T}{L}$ | $2L$ | $0.5L$ |

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right)$$

$$= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

**Second method.** In the second method, we first set $T_i = \infty$ and $T_d = 0$. Using the proportional control action only (see Figure 10–5), increase $K_p$ from 0 to a critical value $K_{cr}$ where the output first exhibits sustained oscillations. (If the output does not exhibit sustained oscillations for whatever value $K_p$ may take, then this method does not apply.) Thus, the critical gain $K_{cr}$ and the corresponding period $P_{cr}$ are experimentally determined (see Figure 10–6). Ziegler and Nichols suggested that we set the values of the parameters $K_p$, $T_i$, and $T_d$ according to the formula shown in Table 10–2.

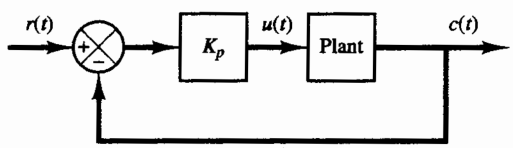



**Figure 10–5** Closed-loop system with a proportional controller.

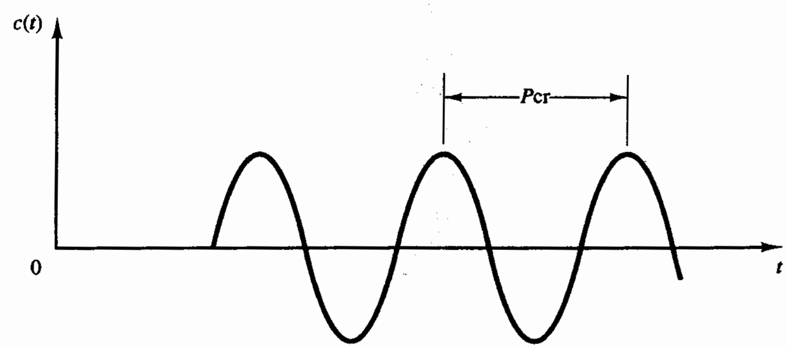



**Figure 10–6** Sustained oscillation with period $P_{cr}$.

**Table 10–2** Ziegler–Nichols Tuning Rule Based on Critical Gain $K_{cr}$ and Critical Period $P_{cr}$ (Second Method)

| Type of Controller | $K_p$ | $T_i$ | $T_d$ |
|---|---|---|---|
| P | $0.5K_{cr}$ | $\infty$ | 0 |
| PI | $0.45K_{cr}$ | $\frac{1}{1.2}P_{cr}$ | 0 |
| PID | $0.6K_{cr}$ | $0.5P_{cr}$ | $0.125P_{cr}$ |

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125\,P_{cr}s\right) \\ &= 0.075K_{cr}P_{cr}\frac{\left(s + \dfrac{4}{P_{cr}}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

**Comments.** Ziegler–Nichols tuning rules (and other tuning rules presented in the literature) have been widely used to tune PID controllers in process control systems where the plant dynamics are not precisely known. Over many years, such tuning rules proved to be very useful. Ziegler–Nichols tuning rules can, of course, be applied to plants whose dynamics are known. (If plant dynamics are known, many analytical and graphical approaches to the design of PID controllers are available, in addition to Ziegler–Nichols tuning rules.)

If the transfer function of the plant is known, a unit-step response may be calculated or the critical gain $K_{cr}$ and critical period $P_{cr}$ may be calculated. Then, using those calculated values, it is possible to determine the parameters $K_p$, $T_i$, and $T_d$ from Table 10–1 or 10–2. However, the real usefulness of Ziegler–Nichols tuning rules (and other tuning rules) becomes apparent when the plant dynamics are not known so that no analytical or graphical approaches to the design of controllers are available.

Generally, for plants with complicated dynamics but no integrators, Ziegler–Nichols tuning rules can be applied. However, if the plant has an integrator, these rules may not be applied in some cases. To illustrate such a case where Ziegler–Nichols rules do not apply, consider the following case: Suppose that a unity-feedback control system has a plant whose transfer function is

$$G(s) = \frac{(s + 2)(s + 3)}{s(s + 1)(s + 5)}$$

Because of the presence of an integrator, the first method does not apply. Referring to

Figure 10–3, the step response of this plant will not have an S-shaped response curve; rather, the response increases with time. Also, if the second method is attempted (see Figure 10–5), the closed-loop system with a proportional controller will not exhibit sustained oscillations whatever value the gain $K_p$ may take. This can be seen from the following analysis. Since the characteristic equation is

$$s(s+1)(s+5) + K_p(s+2)(s+3) = 0$$

or

$$s^3 + (6+K_p)s^2 + (5+5K_p)s + 6K_p = 0$$

the Routh array becomes

$$\begin{array}{lcc} s^3 & 1 & 5+5K_p \\ s^2 & 6+K_p & 6K_p \\ s^1 & \dfrac{30+29K_p+5K_p^2}{6+K_p} & 0 \\ s^0 & 6K_p & \end{array}$$

The coefficients in the first column are positive for all values of positive $K_p$. Thus, in the present case the closed-loop system will not exhibit sustained oscillations and, therefore, the critical gain value $K_{cr}$ does not exist. Hence, the second method does not apply.

If the plant is such that Ziegler–Nichols rules can be applied, then the plant with a PID controller tuned by Ziegler–Nichols rules will exhibit approximately 10% ~ 60% maximum overshoot in step response. On the average (experimented on many different plants), the maximum overshoot is approximately 25%. (This is quite understandable because the values suggested in Tables 10–1 and 10–2 are based on the average.) In a given case, if the maximum overshoot is excessive, it is always possible (experimentally or otherwise) to make fine tuning so that the closed-loop system will exhibit satisfactory transient responses. In fact, Ziegler–Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning.

**EXAMPLE 10–1** Consider the control system shown in Figure 10–7 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters $K_p$, $T_i$, and $T_d$. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25%.

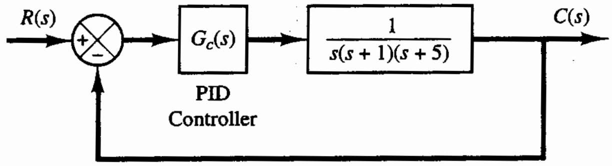



**Figure 10–7** PID-controlled system.

Since the plant has an integrator, we use the second method of Ziegler–Nichols tuning rules. By setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5)+K_p}$$

The value of $K_p$ that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$s^3 + 6s^2 + 5s + K_p = 0$$

the Routh array becomes as follows:

$$\begin{array}{lll} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30-K_p}{6} & \\ s^0 & K_p & \end{array}$$

Examining the coefficients of the first column of the Routh table, we find that sustained oscillation will occur if $K_p = 30$. Thus, the critical gain $K_{cr}$ is

$$K_{cr} = 30$$

With gain $K_p$ set equal to $K_{cr}$ ( $= 30$), the characteristic equation becomes

$$s^3 + 6s^2 + 5s + 30 = 0$$

To find the frequency of the sustained oscillation, we substitute $s = j\omega$ into this characteristic equation as follows:

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

or

$$6(5-\omega^2) + j\omega(5-\omega^2) = 0$$

from which we find the frequency of the sustained oscillation to be $\omega^2 = 5$ or $\omega = \sqrt{5}$. Hence, the period of sustained oscillation is

$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

Referring to Table 10–2, we determine $K_p$, $T_i$, and $T_d$ as follows:

$$K_p = 0.6K_{cr} = 18$$

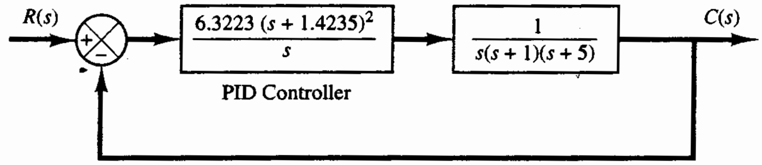
$$T_i = 0.5P_{cr} = 1.405$$

$$T_d = 0.125P_{cr} = 0.35124$$

The transfer function of the PID controller is thus

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right) \\ &= \frac{6.3223(s+1.4235)^2}{s} \end{aligned}$$

**Figure 10–8**
Block diagram of the system with PID controller designed by use of Ziegler–Nichols tuning rule (second method).




The PID controller has a pole at the origin and double zero at $s = -1.4235$. A block diagram of the control system with the designed PID controller is shown in Figure 10–8.

Next, let us examine the unit-step response of the system. The closed-loop transfer function $C(s)/R(s)$ is given by

$$\frac{C(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$

The unit-step response of this system can be obtained easily with MATLAB. See MATLAB Program 10–1. The resulting unit-step response curve is shown in Figure 10–9. The maximum overshoot in the unit-step response is approximately 62%. The amount of maximum overshoot is excessive. It can be reduced by fine tuning the controller parameters. Such fine tuning can be made on the computer. We find that by keeping $K_p = 18$ and by moving the double zero of the PID controller to $s = -0.65$, that is, using the PID controller

$$G_c(s) = 18\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 13.846\,\frac{(s + 0.65)^2}{s} \qquad (10\text{–}1)$$

the maximum overshoot in the unit-step response can be reduced to approximately 18% (see Figure 10–10). If the proportional gain $K_p$ is increased to 39.42, without changing the location of the double zero ($s = -0.65$), that is, using the PID controller

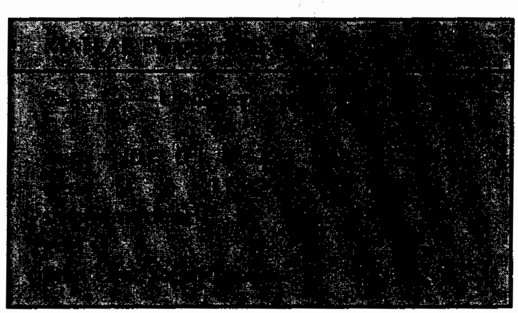
$$G_c(s) = 39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 30.322\,\frac{(s + 0.65)^2}{s} \qquad (10\text{–}2)$$

then the speed of response is increased, but the maximum overshoot is also increased to approximately 28%, as shown in Figure 10–11. Since the maximum overshoot in this case is fairly close to 25% and the response is faster than the system with $G_c(s)$ given by Equation (10–1), we may consider $G_c(s)$ as given by Equation (10–2) as acceptable. Then the tuned values of $K_p$, $T_i$, and $T_d$ become

$$K_p = 39.42, \qquad T_i = 3.077, \qquad T_d = 0.7692$$

It is interesting to observe that these values respectively are approximately twice the values suggested by the second method of the Ziegler–Nichols tuning rule. The important thing to note here is that the Ziegler–Nichols tuning rule has provided a starting point for fine tuning.

It is instructive to note that, for the case where the double zero is located at $s = -1.4235$, increasing the value of $K_p$ increases the speed of response, but as far as the percentage maximum



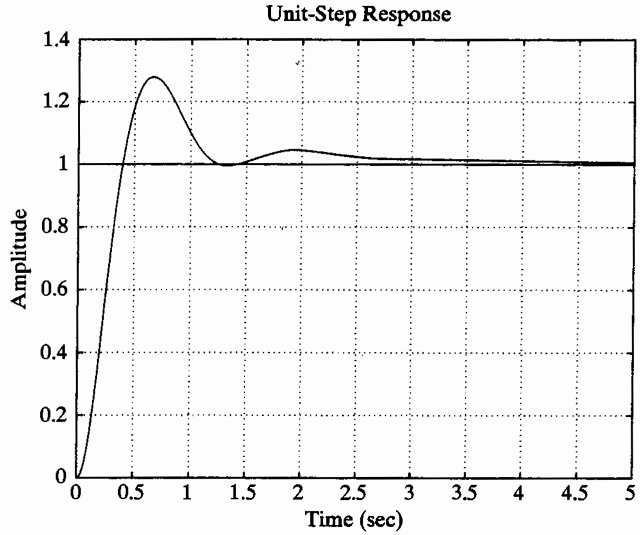



**Figure 10–11** Unit-step response of the system shown in Figure 10–7 with PID controller having parameters $K_p = 39.42$, $T_i = 3.077$, and $T_d = 0.7692$.

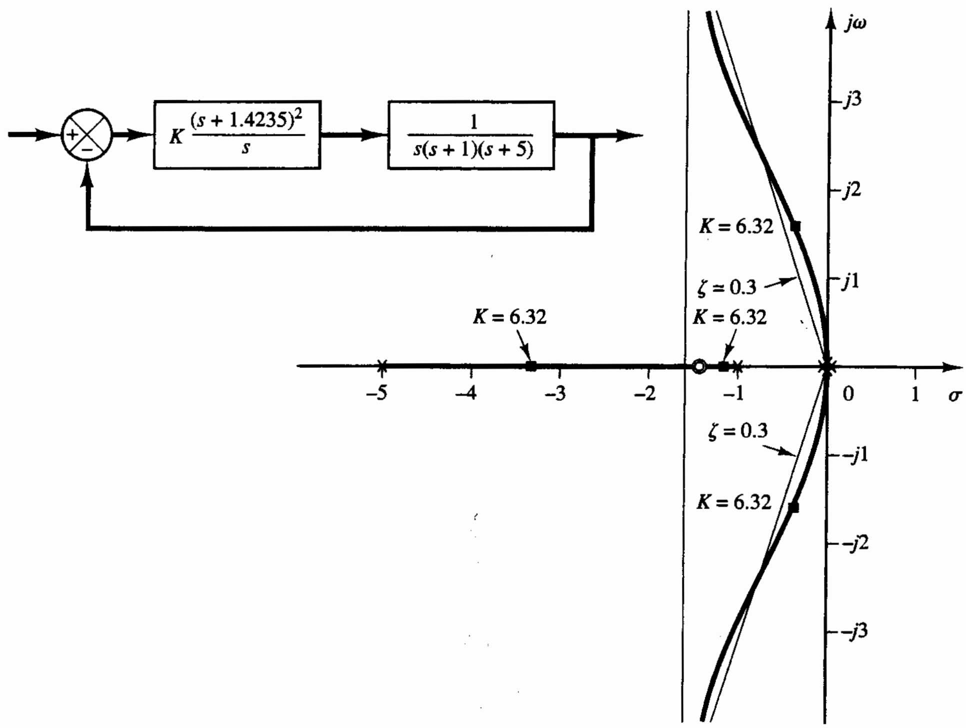



**Figure 10–12** Root-locus diagram of system when PID controller has double zero at $s = -1.4235$.

In Figure 10–13, notice that, in the case where the system has gain $K = 30.322$, the closed-loop poles at $s = -2.35 \pm j4.82$ act as dominant poles. Two additional closed-loop poles are very near the double zero at $s = -0.65$, with the result that these closed-loop poles and the double zero almost cancel each other. The dominant pair of closed-loop poles indeed determines the nature of the response. On the other hand, when the system has $K = 13.846$, the closed-loop poles at $s = -2.35 \pm j2.62$ are not quite dominant because the two other closed-loop poles near the dou-

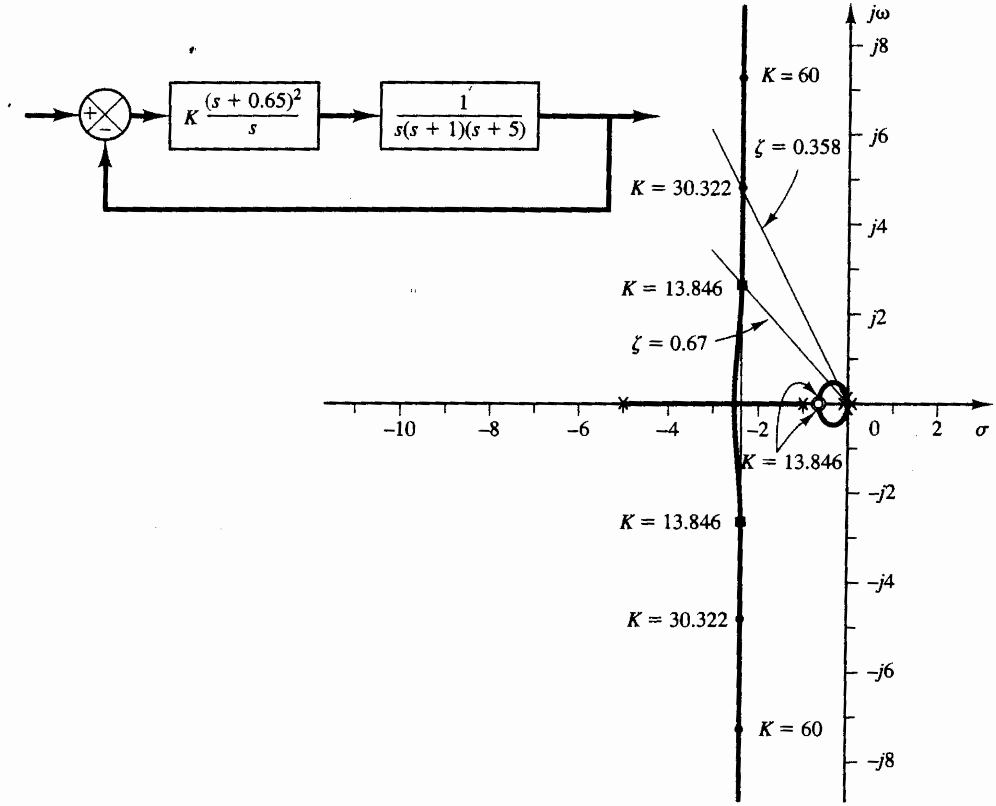



**Figure 10–13** Root-locus diagram of system when PID controller has double zero at $s = -0.65$. $K = 13.846$ corresponds to $G_c(s)$ given by Equation (10–1) and $K = 30.322$ corresponds to $G_c(s)$ given by Equation (10–2).

ble zero at $s = -0.65$ have considerable effect on the response. The maximum overshoot in the step response in this case (18%) is much larger than the case where the system is of second-order having only dominant closed-loop poles. (In the latter case the maximum overshoot in the step response would be approximately 6%.)

## 10–3 MODIFICATIONS OF PID CONTROL SCHEMES

Consider the basic PID control system shown in Figure 10–14(a), where the system is subjected to disturbances and noises. Figure 10–14(b) is a modified block diagram of the same system. In the basic PID control system such as the one shown in Figure 10–14(b), if the reference input is a step function, then, because of the presence of the derivative term in the control action, the manipulated variable $u(t)$ will involve an impulse function (delta function). In an actual PID controller, instead of the pure derivative term $T_d s$ we employ

$$\frac{T_d s}{1 + \gamma T_d s}$$

where the value of $\gamma$ is somewhere around 0.1. Therefore, when the reference input is a step function, the manipulated variable $u(t)$ will not involve an impulse function, but will involve a sharp pulse function. Such a phenomenon is called *set-point kick*.

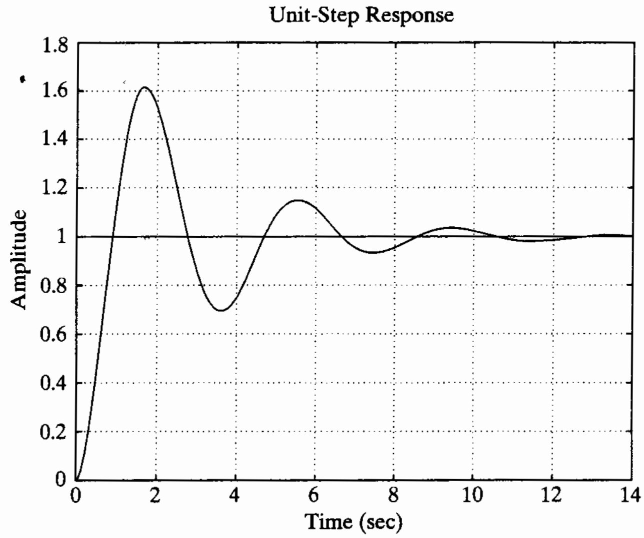



**Figure 10–9** Unit-step response curve of PID-controlled system designed by use of Ziegler–Nichols tuning rule (second method).

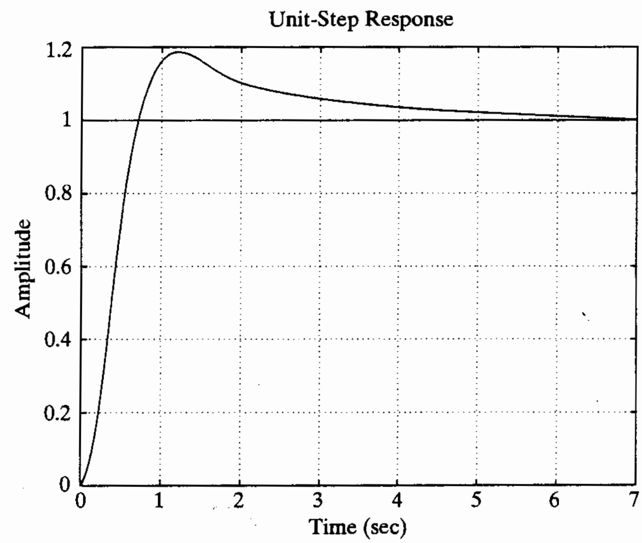



**Figure 10–10** Unit-step response of the system shown in Figure 10–7 with PID controller having parameters $K_p = 18$, $T_i = 3.077$, and $T_d = 0.7692$.

overshoot is concerned, varying gain $K_p$ has very little effect. The reason for this may be seen from the root-locus analysis. Figure 10–12 shows the root-locus diagram for the system designed by use of the second method of Ziegler–Nichols tuning rules. Since the dominant branches of root loci are along the $\zeta = 0.3$ lines for a considerable range of $K$, varying the value of $K$ (from 6 to 30) will not change the damping ratio of the dominant closed-loop poles very much. However, varying the location of the double zero has a significant effect on the maximum overshoot, because the damping ratio of the dominant closed-loop poles can be changed significantly. This can also be seen from the root-locus analysis. Figure 10–13 shows the root-locus diagram for the system where the PID controller has the double zero at $s = -0.65$. Notice the change of the root-locus configuration. This change in the configuration makes it possible to change the damping ratio of the dominant closed-loop poles.